GPS Observables

Pseudorange (code) observables
Phase observables
GPS data files

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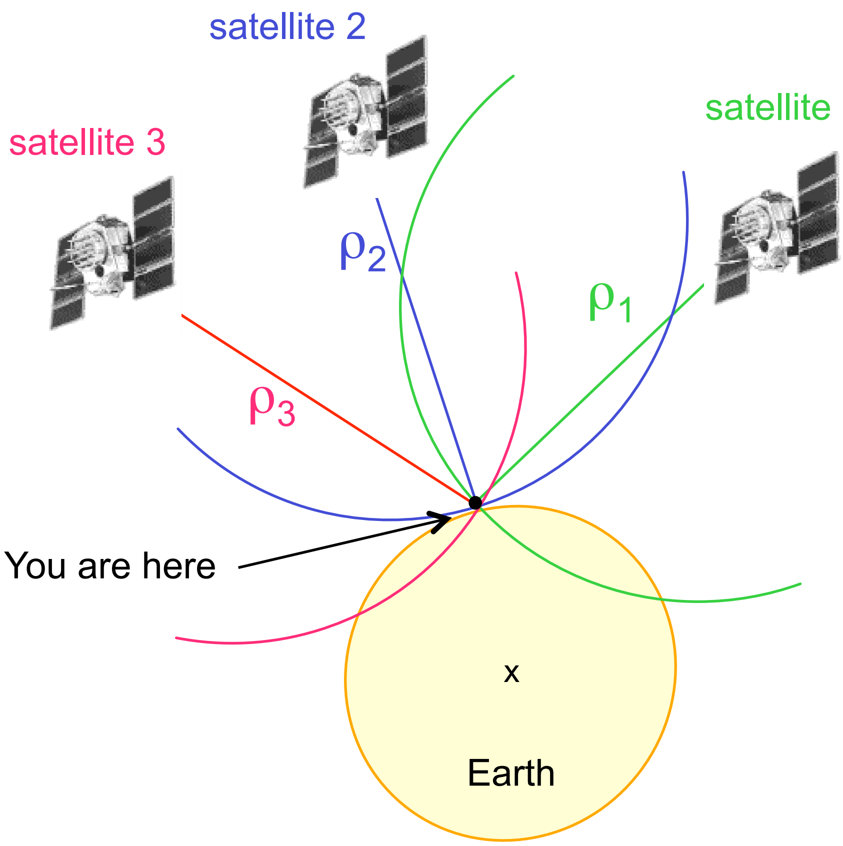
GPS Basic Principle



- Principle of GPS positioning:
  - Satellite 1 sends a signal at time t_{e1}
  - Ground receiver receives it signal at time t_r
  - The range measurement ρ_1 to satellite 1 is:
    - $\rho_1 = (t_r - t_{e1})$ x speed of light
    - We are therefore located on a sphere centered on satellite 1, with radius ρ1
  - 3 satellites => intersection of 3 spheres

- The mathematical model is:

$$\rho_r^s = \sqrt{(X_s - X_r)^2 + (Y_s - Y_r)^2 + (Z_s - Z_r)^2}$$

  - If the position of the satellites in an Earth-fixed frame (X_s, Y_s, Z_s) is known,
  - Then one can solve for (X_r, Y_r, X_r) (if at least 3 simultaneous range measurements)

- GPS receivers:
  - Measure t_r
  - Decode t_e
  - Compute range ρ_r^s

- **Observable = range measurement.**

Satellite-receiver time offset

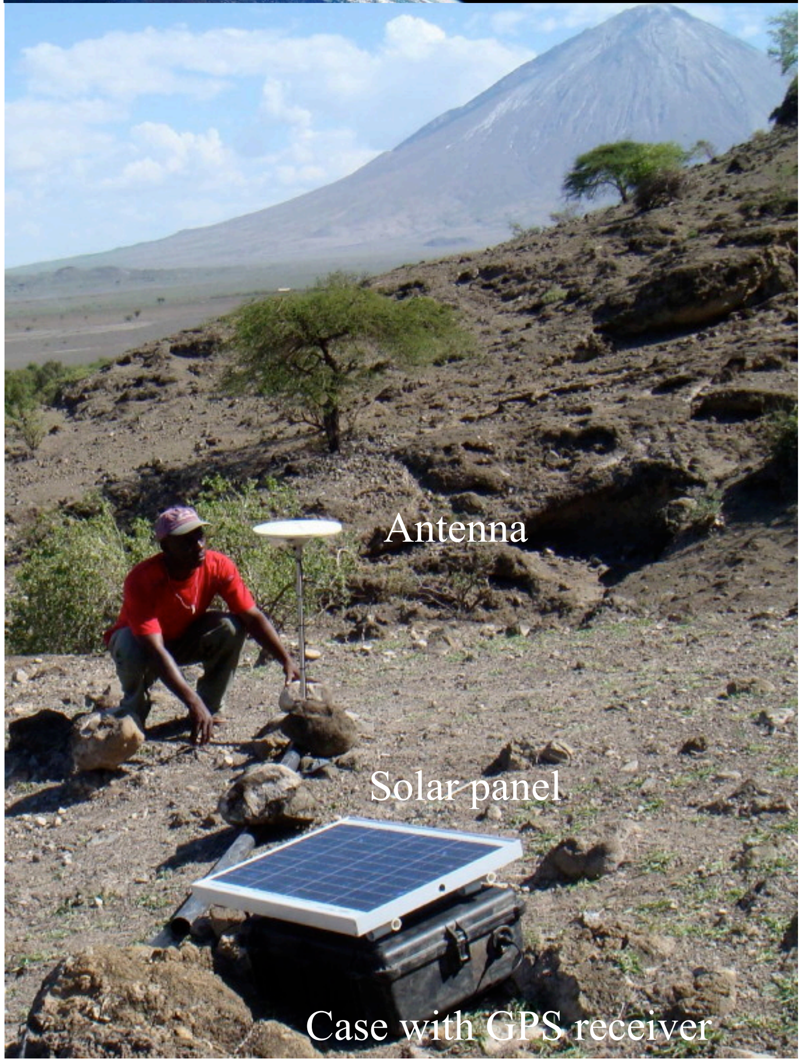
- Receiver clocks are:
  - Mediocre: stability ~10^{-5}-10^{-6} (~ crystal wrist watch)
  - Not synchronized with the satellite clocks.

- There is a time difference between the satellite clocks (t_s) and the receiver clock (t_r): $\delta t = t_r - t_s$
  - The receivers therefore measures: $t = t + \delta t$
  - In terms of distance: $(t + \delta t) \times c = r + \delta r = \rho$
  - r = true geometric range
  - Receiver actually measures ρ = pseudorange

- Practical consequences:
  - The time offset between satellite and receiver clocks is an additional unknown
  - We need 4 observations ⇒ 4 satellites visible at the same time
  - In order to compute a position, the receiver solves for δt ⇒ GPS receivers are very precise clocks! (Timing is a very important application of GPS)
  - δt is used by the receiver to synchronize its clock with the satellite clocks.

From the GPS signal to a position

- What do GPS satellites do?
  - Send a radio signal toward Earth at t_e
  - Radio signal contains:
    - Satellite number
    - Time of emission
    - Satellite position
- What do GPS receivers do?
  - Measure t_r
  - Decode the satellite signal:
    - Read t_e
    - Read satellite position from navigation message
  - Compute satellite receiver distances, or pseudorange ρ_r^s
  - Compute position from at least 4 simultaneous range measurements
- What do users do?
  - Set up the equipment…
  - Download the “GPS data” = range measurements
  - Postprocessing

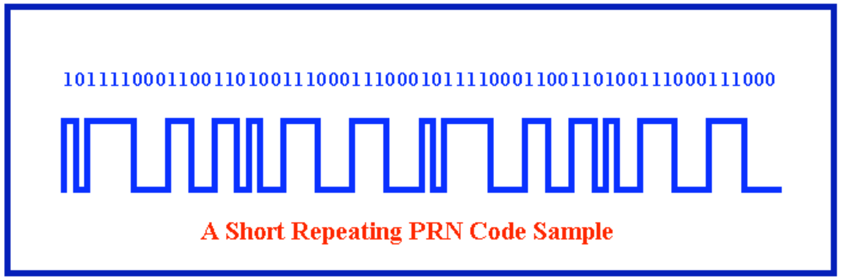


The GPS signal

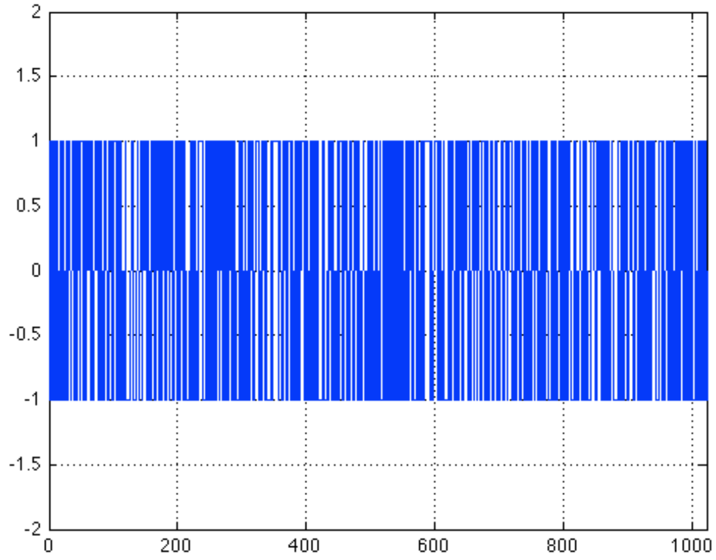
- Atomic clocks aboard the GPS satellites produce a fundamental frequency f_o = 10.23 Mhz
- Two frequencies are derived from it:
  - L1 *(fo x 154)* = 1.57542 GHz, wavelength 19.0 cm
  - L2 *(fo x 120)* = 1.22760 GHz, wavelength 24.4 cm
- L1 and L2 are the two carrier frequencies used to transmit timing information by the GPS satellites
- The information transmitted by the satellites is coded as a phase modulation of the carrier frequency

Pseudorandom codes

- Information transmitted in GPS signals:
  - Navigation message (incl. satellite ephemerides)
  - Pseudo random code (PRN – for pseudorandom noise):
    - Specific to each satellite
    - Known by receivers
    - Formulated as a series of zeros and ones (“chips”, or “bits”)
- Pseudorandom codes:
  - 1023 chips long ⇒ if random, then 2^{1023} possibilities
  - Only 37 are suitable = GOLD-codes (mathematician).
  - GOLD-codes = correlation among each other very weak ⇒ unequivocal identification of each satellite.



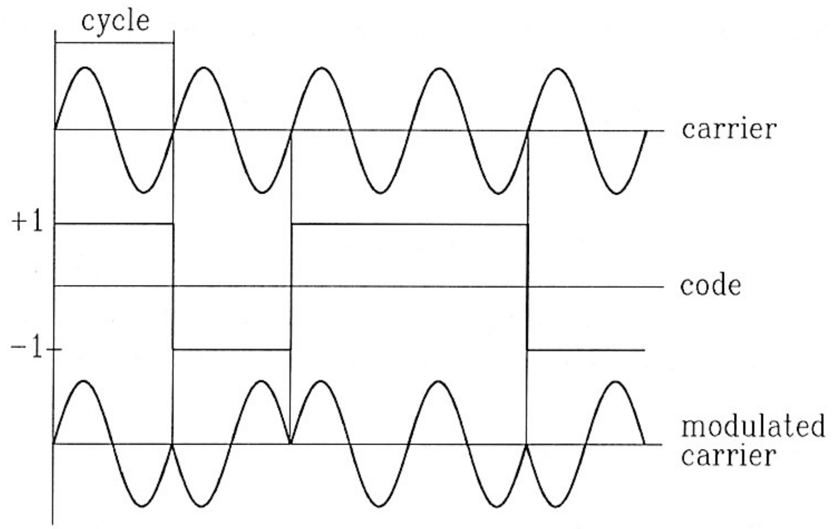
A Short Repeating PRN Code Sample



PRN 5 code sequence
(http://www.colorado.edu/ASEN/asen5090/)

Phase modulation

- Information transmitted in GPS signals:
  - Navigation message
  - Pseudo random code (PRN)
- Information is coded as a sequence of +1/-1 (binary values 0/1)
- Coding = π shift in carrier phase when code state changes = biphase modulation
- Rate at which the phase shift occurs = chip rate
- One chip ~ one bit

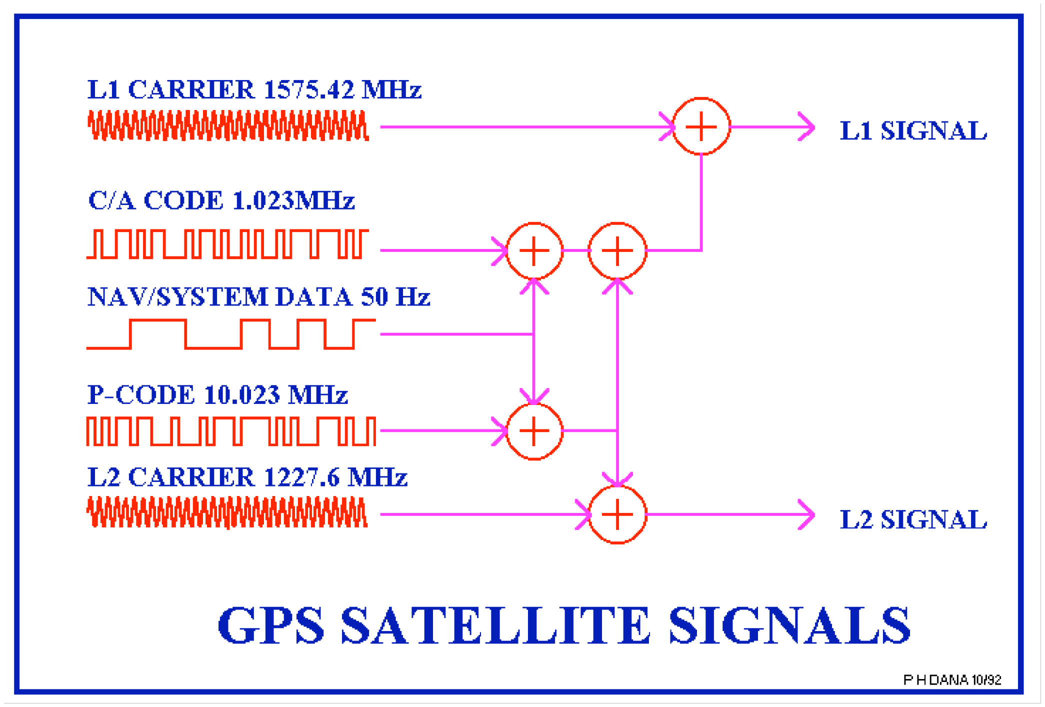


Biphase modulation of the GPS carrier phase

C/A and P codes

- Coarse acquisition code = C/A code
    - 1023 chips long, transmitted at frequency of 1.023 MHz ⇒ 293 meters chip length (at 3×10^8 m/s)
    - Transmitted on L1 only
    - Open to civilians

- Precise code = P code
    - 10.23 Mbps ⇒ 29.3 m chip length
    - Transmitted on both L1 and L2
    - Can be transmitted encrypted by a Y-code = “anti-spoofing” (A/S)
    - Y-code = precise (military) position determination
    - A/S continuously on since January 31, 1994.



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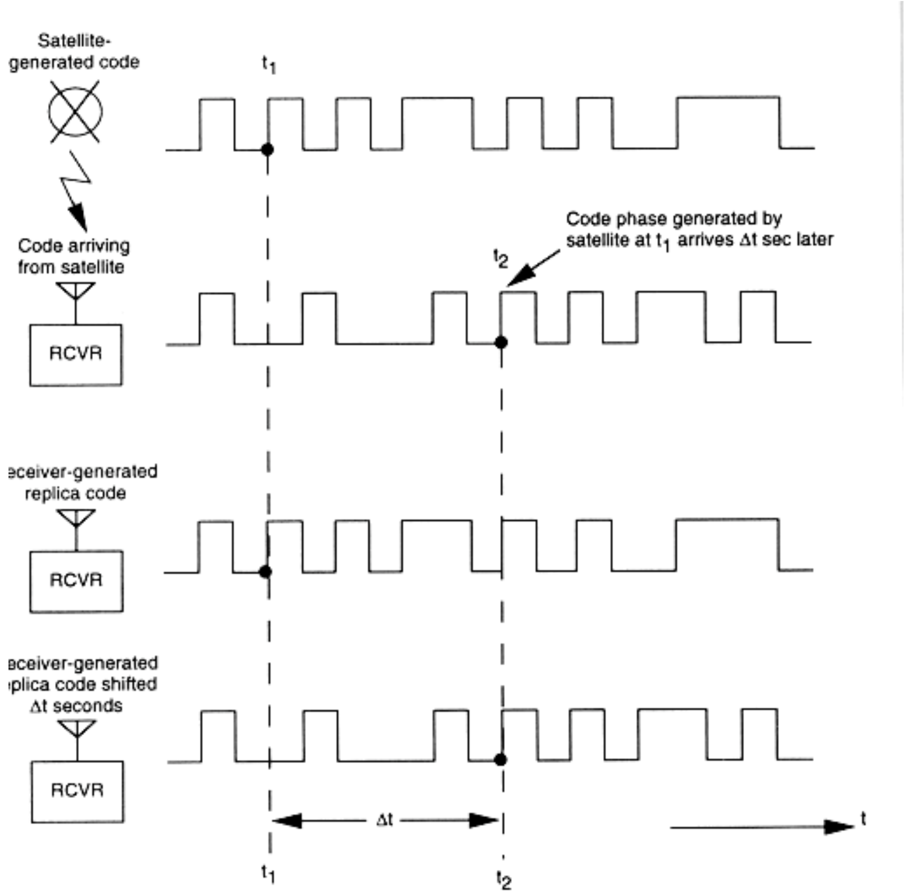
Decoding in the receiver

- Radio frequency (RF) part of the receiver processes incoming signals:
  - L1 only (single-frequency receivers)
  - L1 and L2 (dual-frequency receivers)

- RF unit:
  - Processes incoming signal from different satellites in different channels (multichannels receivers, 4 to 12 channels)
  - Generates internal replica of the GPS signal:
    - Contains an oscillator (= clock) that generates L1 and L2 frequencies
    - Knows each PRN code
  - Compares internally generated signal with incoming signal

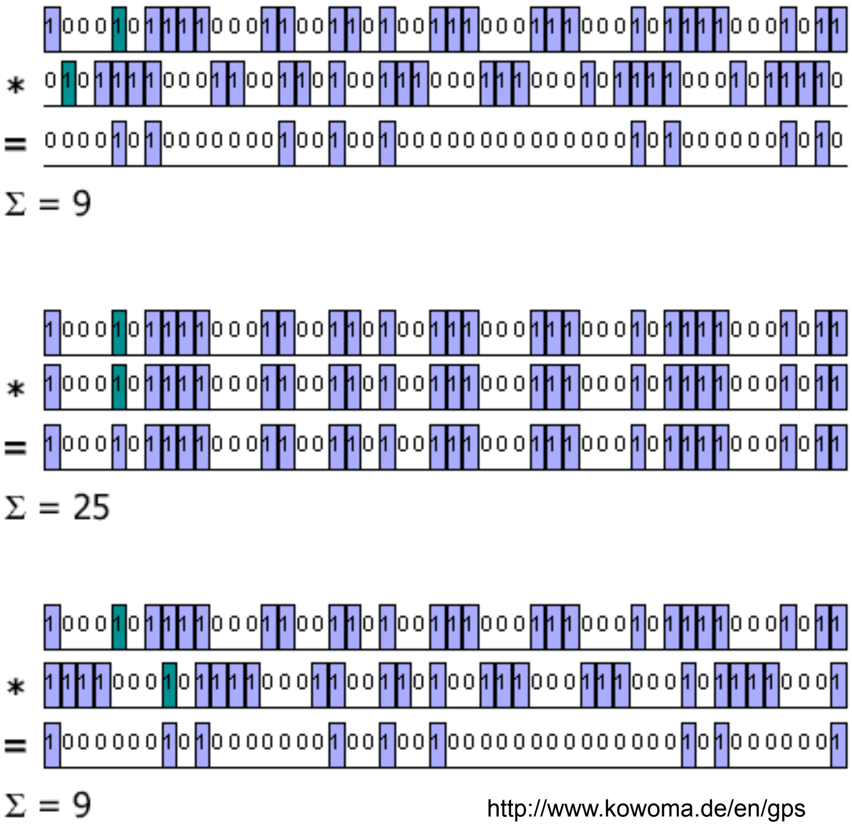
Decoding in the receiver

- Code-correlation:
  - Shift of the internally generated signal in time until it matches the incoming one (receiver “locked” on a satellite)
  - **Time shift needed = signal travel time from satellite to receiver**
- Other techniques to retrieve phase information, independent of PRN codes:
  - Squaring: autocorrelation of the incoming signal
  - Cross-correlation: correlation between L1 and L2 using Y-code (Y-code is identical on L1 and L2)
  - Z-tracking: correlation on L1 and L2 using the P-code to obtain W-code
  - All these techniques have a lower SNR than the code-correlation:
    - Squaring: -30 dB
    - Cross correlation: -27 dB
    - Z-tracking: -14 dB



Decoding in the receiver

- GPS signals very weak
- Algorithm:
  - Generate internal replica of PRN code in receiver
  - Multiply bits (0 or 1) with incoming signals
  - Sum resulting sequence to obtain correlation
  - Shift replica forward by one chip and repeat
- Search for the time lag corresponding to maximum correlation

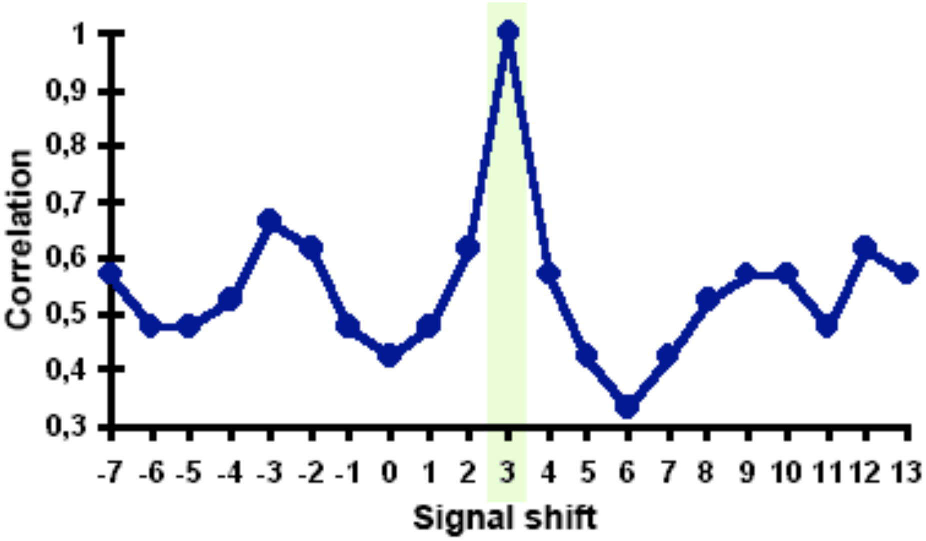


http://www.kowoma.de/en/gps

Top: Signal of the satellite
Middle: Signal of the receiver, delayed against the signal of the satellite.
Bottom: The two signals multiplied.

Correlation function

- Correlation function normalized between 0 and 1
- In this case, max. correlation for signal shift of 3 chip lengths
- C/A code:
  - One chip = 300 m
  - Measurement = 900 m
- P code:
  - One chip = 30 m
  - Measurement = 90 m



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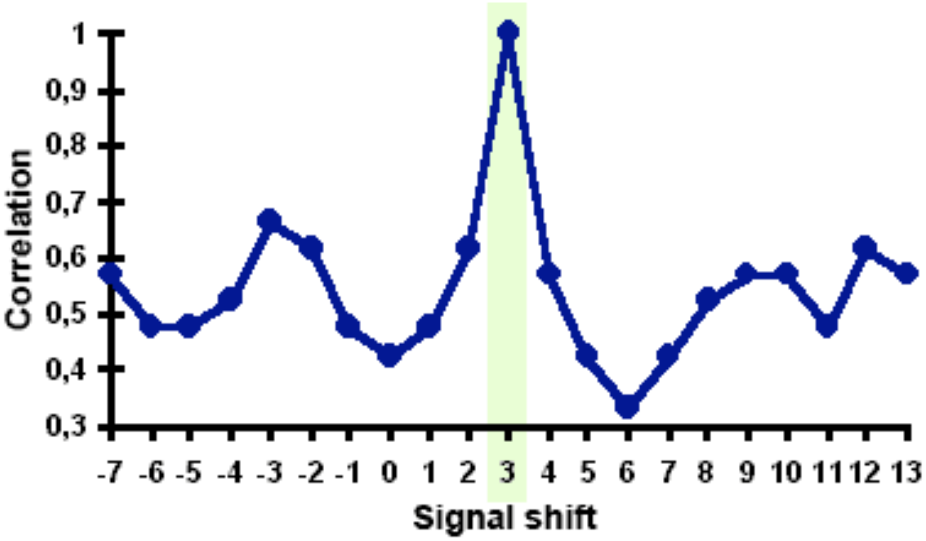
Correlation function

- Correlation function width: inversely proportional to the frequency of the signal
  - C/A code = 1 MHz frequency ⇒ correlation produces a peak that is 1 msec wide = 300 m
  - P code = 10 MHz frequency ⇒ correlation produces 0.1 msec peak = 30 m
- Modern GPS receivers can determine peak of correlation function can to 1% of width (with care):
  - Range accuracy = 3 m for C/A code
  - Range accuracy = 0.3 m for P code



http://www.kowoma.de/en/gps

Navigation message

- Ephemerides for all satellites, ionospheric correction parameters, system status, satellite clock offset and drift
- Chip rate = 50 bps
- 25 frames of 1500 bits each, divided into five 300 bits subframes
- 50 bps ⇒ 300/50 = 6 sec to transmit one subframe, 6x5x25 = 750 sec (=12.5 min) to transmit an entire navigation message

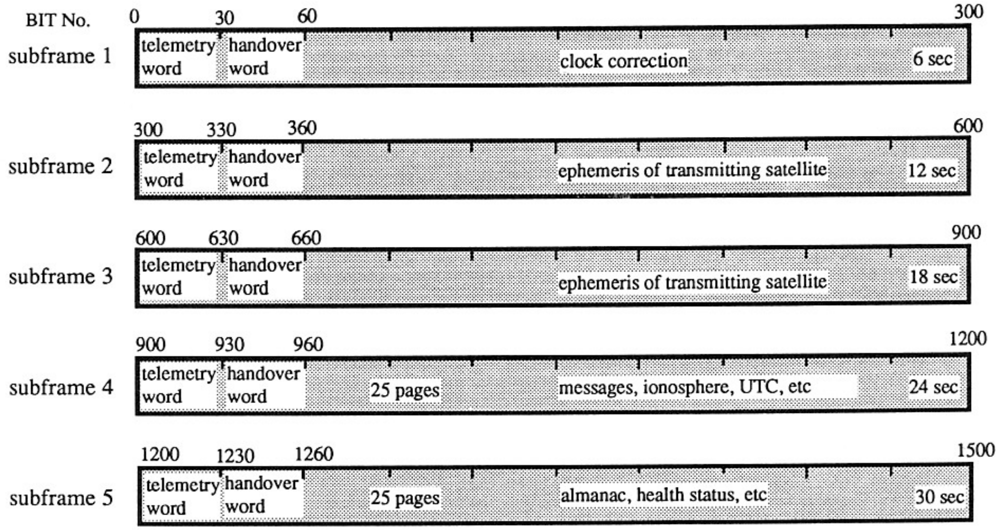


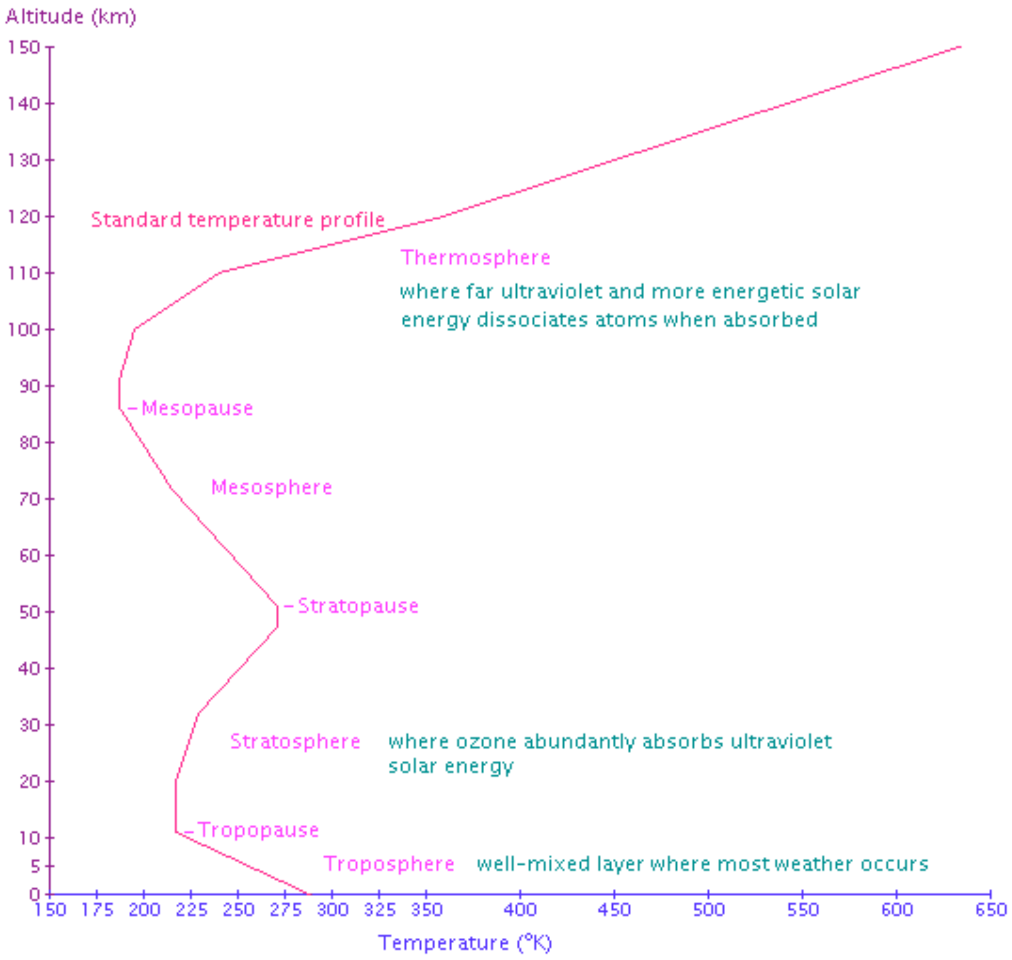
Figure 3.11 Structure of navigation message frame.

Receiver start-up

- General procedure:
  1. Acquire one satellite to get time and almanach
  2. Acquire 2 other satellites to get 2-D position
  3. Acquire 4th satellite to get 3-D position
  4. Acquire any other visible satellite
- Time needed to get good position:
  - Hot start: few secs (rcv was off for a few secs: almanach ok, time ok, position close to last one)
  - Warm start: few mins (rcv was off for less than a day: clock ~ok)
  - Cold start: 10s of minutes (rvc was off for several days: time off, almanach expired, last position off)

GPS signal propagation

- L1 and L2 frequencies are affected by **atmospheric refraction**:
  - Ray bending (negligible)
  - Propagation velocity decrease (w.r.t. vacuum) ⇒ propagation delay
- In the **troposphere**:
  - Delay is a function of (P, T, H), 1 to 5 m
  - Largest effect due to pressure
- In the **ionosphere**: delay function of the electron density, 0 to 50 m
- The **refractive delay** biases the satellite-receiver range measurements, and, consequently the estimated positions (effect more pronounced in the vertical).



GPS code model

- GPS receivers measure pseudoranges ${}^{j}R_i(t)$, that can be modeled as:

$$ {}^{j}R_i(t) = {}^{j}\rho_i(t) + c({}^{j}\delta(t) - \delta_i(t)) + \Delta I(t) + \Delta T(t) + MP(t) + \varepsilon $$

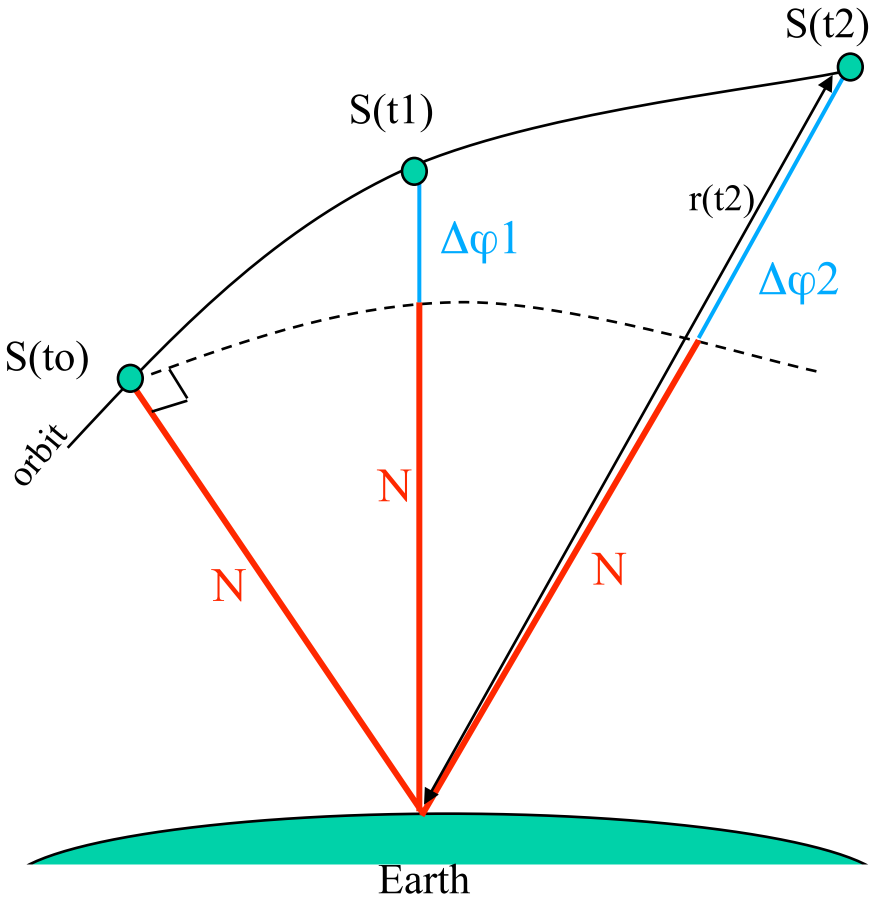
t = time of epoch
${}^{j}R_i$ = pseudorange measurement
${}^{j}\rho_i$ = satellite-receiver geometric distance
c = speed of light
${}^{j}\delta$ = satellite clock bias
δ_i = receiver clock bias
ΔI = ionospheric propagation error
ΔT = tropospheric propagation error
MP = multipath
ε = receiver noise
(ranges in meters, time in seconds)

- ΔI and ΔT are correction terms because GPS signal propagation is not in a vacuum (more later)

- MP = multipath noise, reflection of GPS signal off surfaces near antenna (more later)

Carrier phase measurements

- When a satellite is locked (at t_o), the GPS receiver starts tracking the incoming phase
- It counts the (real) number of phases as a function of time = $\Delta\varphi\ (t)$
- But the initial number of phases N at t_o is unknown…
- However, if no loss of lock, N is constant over an orbit arc.



Carrier phase model

- Geometrical interpretation:

$$\Delta\Phi = \frac{R}{\lambda} - N$$

$$R = \rho + c\delta t$$

$$\Rightarrow \Delta\Phi = \frac{\rho}{\lambda} + \frac{c}{\lambda}\delta t - N$$

$\Delta\Phi$ = phase measurement
R = pseudorange
c = speed of light
ρ = geometric range
λ = wavelength
δt = sat-rcv clock offset
N = phase ambiguity

- The phase equation (units of cycles):

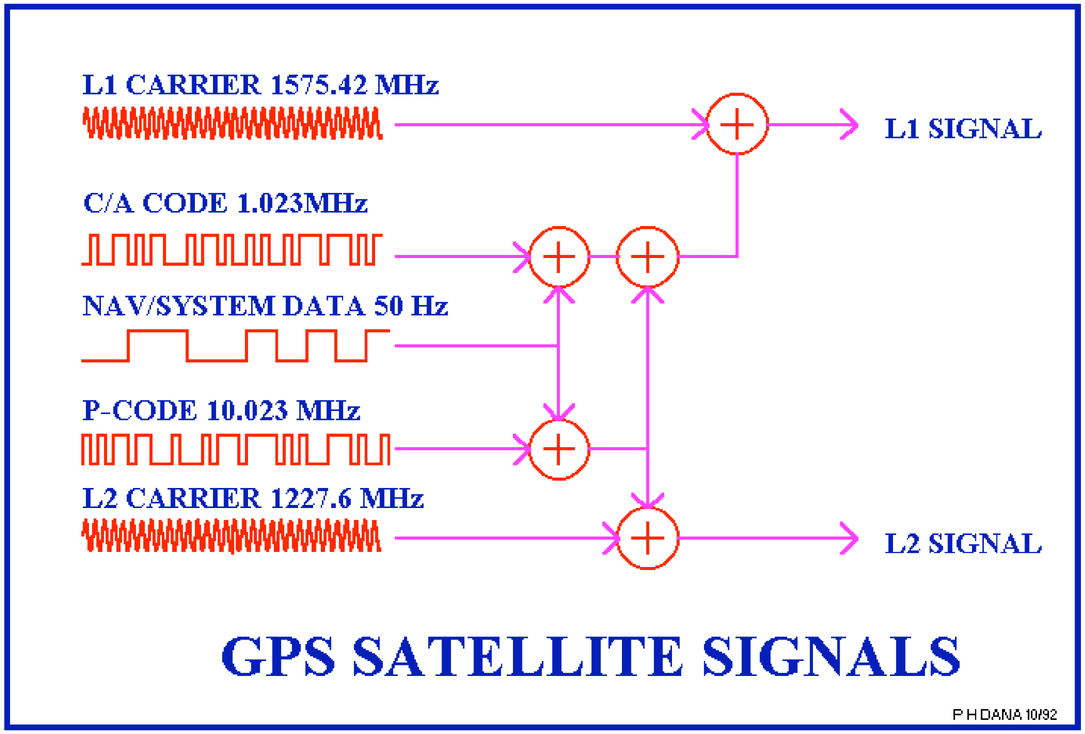
$$\Phi_i^k(t) = \rho_i^k(t) \times \frac{f}{c} + \left(h^k(t) - h_i(t)\right) \times f + ion_i^k(t) + trop_i^k(t) - N_i^k + \varepsilon$$

t = time of epoch
i = receiver, k = satellite
ρ_i^k = geometric range
h^k = satellite clock error, h_i = receiver clock error
ion_i^k = ionospheric delay, $trop_i^k$ = tropospheric delay
N_i^k = phase ambiguity, ε = phase noise

Carrier phase measurements

- Phase can be converted to distance by multiplying by the wavelength ⇒ phase measurements are another way for measuring the satellite-receiver distance
- Phase can be measured to ~1% of the wavelength ⇒ range accuracy 2 mm for L1, 2.4 mm for L2
- Phase measurements are very precise, but ambiguous
- To fully exploit phase measurements, one **<u>must</u>** correct for propagation effects (several meters)

GPS observables



- GPS receivers can record up to 5 observables:
  - $\varphi 1$ and $\varphi 2$: phase measurements on $L1$ and $L2$ frequencies, in cycles
  - C/A, $P1$, $P2$: pseudorange measurements, in meters
- Plus Doppler phase = $d\varphi/dt$

GPS observables

- **Pseudorange measurements (C/A, P1, P2):**
  - Geometric range + clock offset + noise:

    $$\rho = r + \Delta t \times c$$
  - Accuracy of pseudorange measurements by GPS receivers ~ 1% of correlation peak width:
    - 3 m with C/A code
    - 0.3 m with P code
  - Low accuracy but absolute measurements

- **Phase measurements (L1, L2):**
  - Geometric range + clock offset - initial phase ambiguity N:

    $$\varphi = r \times f/c + \Delta t \times f - N$$
  - Accuracy of phase measurements in GPS receivers ~ 0.005 cycle (0.005 x 20 cm = 0.2 mm) ⇒ millimeter accuracy theoretically possible
  - Very accurate measurements but ambiguous

GPS observables

- GPS observables stored in receivers in binary proprietary format
- Receiver Independent Exchange format (RINEX) = ASCII exchange format
- Format description: ftp://igscb.jpl.nasa.gov/igscb/data/format/rinex2.txt
- Conversion from binary proprietary to RINEX:
  - Proprietary software
  - Freewares: *e.g.* teqc (www.unavco.ucar.edu)

RINEX observation file

```
 2.00           OBSERVATION DATA    G (GPS)             RINEX VERSION / TYPE
teqc  1999Jul19     CNRS_UMRGA          20021201 12:04:20UTCPGM / RUN BY / DATE
Solaris 2.3|S-Sparc|cc SC3.0|=+|*Sparc                      COMMENT
BIT 2 OF LLI FLAGS DATA COLLECTED UNDER A/S CONDITION       COMMENT
SJDV                                                        MARKER NAME
10090M001                                                   MARKER NUMBER
___                 REGAL                                   OBSERVER / AGENCY
845                 ASHTECH Z-XII3      CD00                REC # / TYPE / VERS
317                 ASH700936A_M    NONE                    ANT # / TYPE
  4433469.9683   362672.6919  4556211.6229                  APPROX POSITION XYZ
        0.0000        0.0000        0.0000                  ANTENNA: DELTA H/E/N
     1     1                                                WAVELENGTH FACT L1/2
     5    L1    L2    C1    P1    P2                        # / TYPES OF OBSERV
    30.0000                                                 INTERVAL
Forced Modulo Decimation to 30 seconds                      COMMENT
 SNR is mapped to RINEX snr flag value [1-9]                COMMENT
  L1: 1 -> 1;  90 -> 5; 210 -> 9                            COMMENT
  L2: 1 -> 1; 150 -> 5; 250 -> 9                            COMMENT
  2002    11    30     0     0   30.0000000     GPS         TIME OF FIRST OBS
                                                            END OF HEADER
 02 11 30  0  0 30.0000000  0  8G14G 7G31G20G28G 1G25G11
  -7096034.24049  -5509904.97345  23971309.103    23971309.038    23971310.842
 -12570276.74149  -9768618.40046  23379169.469    23379168.448    23379172.496
  -4157689.84249  -3201324.38045  24195891.298    24195890.733    24195894.168
 -25480193.34249 -19826614.77248  20670858.774    20670857.983    20670861.191
  -5589280.20049  -4319738.39345  24553697.713    24553697.259    24553700.349
 -10252537.24449  -7918950.15946  23060092.127    23060091.841    23060095.687
  -4143445.15949  -2509987.53445  24581180.488    24581179.713    24581183.992
 -29659606.34049 -23089397.33548  20312382.965    20312382.530    20312384.719
 02 11 30  0  1  0.0000000  0  8G14G 7G31G20G28G 1G25G11
  -7004806.32949  -5438818.30145  23988669.195    23988668.970    23988671.466
 -12645245.09249  -9827035.30846  23364903.590    23364902.944    23364907.274
  -4043324.79449  -3112208.77545  24217654.165    24217653.747    24217658.209
 -25518762.53849 -19856668.69248  20663519.280    20663518.524    20663521.550
  -5521754.77149  -4267121.22845  24566547.413    24566547.593    24566550.660
 -10357839.61649  -8001003.94446  23040053.767    23040053.443    23040058.358
  -4207531.87749  -2559925.21345  24568984.944    24568985.325    24568989.371
 -29640011.07349 -23074128.30548  20316111.836    20316111.559    20316113.648
```

Header

Data blocks:

- Range in meters
- Phase in cycles

Observation models

Code (meters):

$$R_i^k(t) = \rho_i^k(t) + c(h^k(t) - h_i(t)) + I_i^k(t) + T_i^k(t) + MP_i^k(t) + \varepsilon$$

Carrier phase (cycles):

$$\Phi_i^k(t) = \rho_i^k(t) \times \frac{f}{c} + \left(h^k(t) - h_i(t)\right) \times f + I_i^k(t) + T_i^k(t) + MP_i^k(t) - N_i^k + \varepsilon$$

t = time of epoch
R = pseudorange measurement
Φ = carrier phase measurement
ρ = satellite-receiver geometric distance
c = speed of light
f = carrier frequency
h^k = satellite clock bias, h_i = receiver clock bias

I = ionospheric propagation error
T = tropospheric propagation error
MP = multipath
N = phase ambiguity
ε = other small errors, including receiver noise
(ranges in meters, time in seconds, phase in cycles)

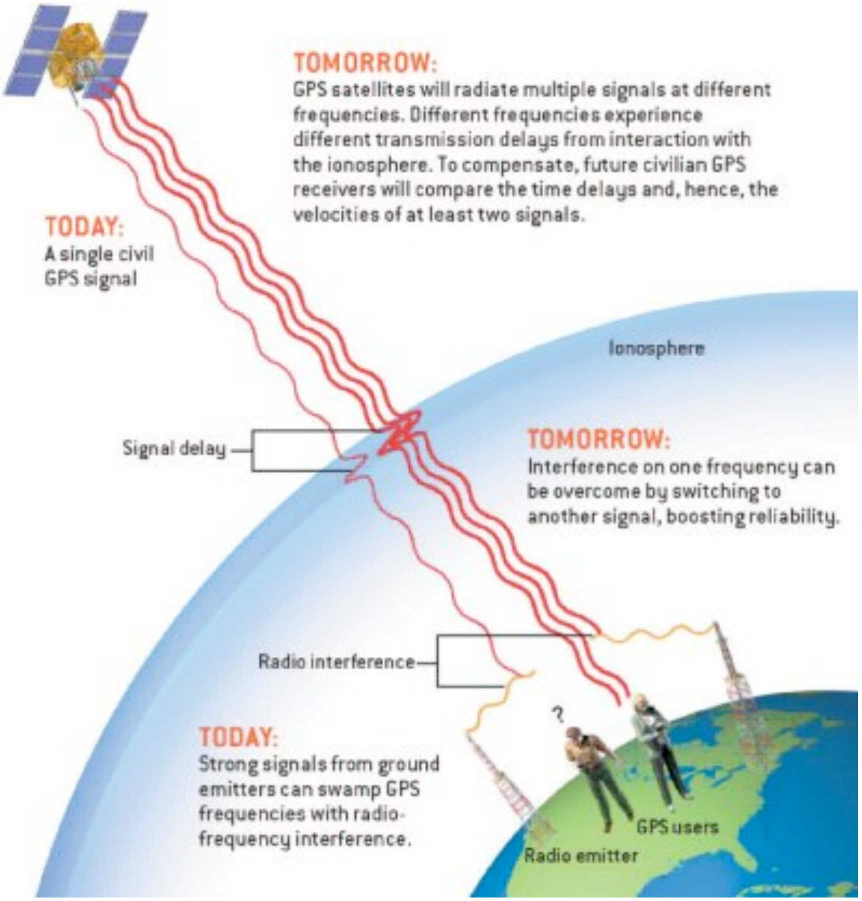
With:

$$\rho_i^k = \sqrt{(X^k - X_i)^2 + (Y^k - Y_i)^2 + (Z^k - Z_i)^2}$$

X^k, Y^k, Z^k = satellite position
X_i, Z_i, Z_i = site position

GPS modernization

- Add new signals for:
  - Military users: increase signal power (L1 and L2 very low power).
  - Civilian users: improve accuracy, availability and signal redundancy.
  - Higher chip rate, longer codes, more power
- L2C
  - Civilian code on L2
  - IIR-M satellite series (8 svs as of 9/28/09, cf. ftp://tycho.usno.navy.mil/pub/gps/gpsb2.txt)
  - More powerful L2
- L5
  - Third civil signal at 1,176 MHz (L5)
  - "Safety of life" signal
  - IIF satellite series (currently being tested – as of 09/2009)



TOMORROW: GPS satellites will radiate multiple signals at different frequencies. Different frequencies experience different transmission delays from interaction with the ionosphere. To compensate, future civilian GPS receivers will compare the time delays and, hence, the velocities of at least two signals.

TODAY: A single civil GPS signal

TOMORROW: Interference on one frequency can be overcome by switching to another signal, boosting reliability.

TODAY: Strong signals from ground emitters can swamp GPS frequencies with radio-frequency interference.

http://scpnt.stanford.edu/