Geodesy and Geodynamics

By

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Addressed topics

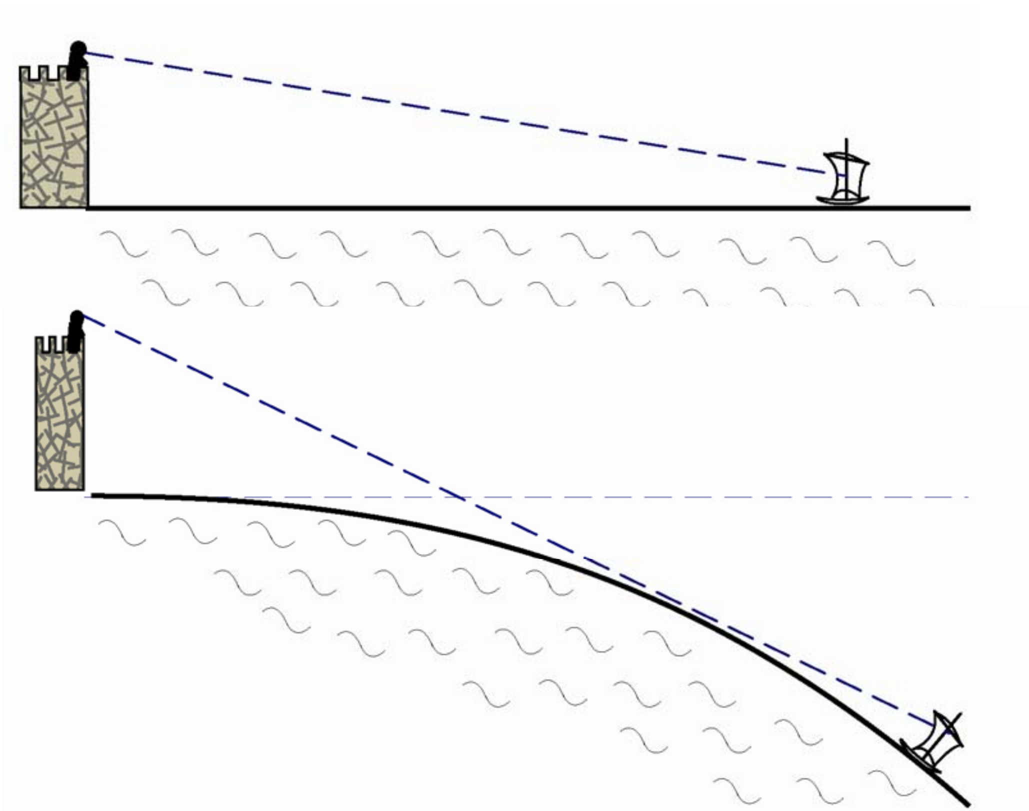
- Geodesy and Earth remote sensing : general introduction
- Global properties of the Earth : Topography, gravity and Geoïd, Global surface deformation and plate tectonics
- Measuring the Earth deformation : terrestrial and spatial geodesy : a review of present day tools (leveling, triangulation, SLR, VLBI, DORIS, GPS, Insar, etc…)
- GPS : How and what for ? Technical and detailed explanations on this tool : basics of the theory and from network design to data acquisition, processing and modeling
- Measuring plate tectonics, monitoring faults, and surveying earthquakes with GPS

Text books and related litterature

- Applications of continuum physics to geological problems, **D.L. Turcotte and G. Schubert**, *John Wiley & sons Inc., 1982.* ISBN 0-471-06018-6
- Plate Tectonics: How it Works, **A. Cox and R.B. Hart**, *Blackwell scientific publications, 1986.* ISBN 0-86542-313-X
- Inside the Earth, **B.A. Bolt**, *W.H. Freeman and company, 1982.* ISBN 0-7167-1359-4
- Geophysical geodesy, **K. Lambeck**, *Oxford University Press, 1988.* ISBN 0-19-854438-3
- Geodesy : the concepts, **P. Vanicek and E. Krakiwsky**, *Elsevier Science Publisher, 1982.* ISBN 0-444-87777-0
- GPS for Geodesy, **A. Kleusber and P.J.G. Teunissen** Editors, *Springer-Verlag, 1996.* ISBN 3-540-60785-4

Ancient times Geodesy (6 century bc)

- Geodesy is a very old science. It comes from the first question mankind ask themselves : **what is the shape and the size of the earth ?**



If the Earth were flat, then one could see very far

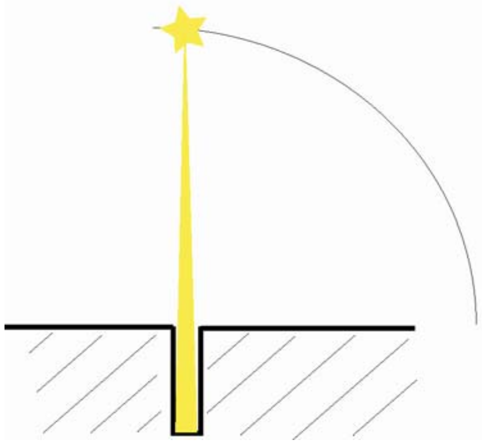
=> no horizon

Because there is an horizon (i.e. objects disappear below the horizon)

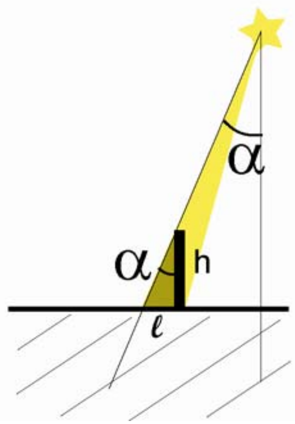
⇒ **Earth is spherical**

Ancient times Geodesy (Eratosthene, 300 bc)

Size of the Earth : **circ = 360°/α * d_{12}** = **40000 km**

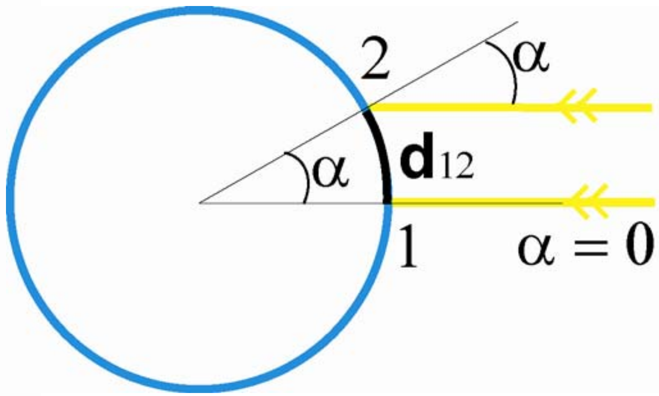


At one place on Earth, the Sun is vertical (lights the bottom of a well) only once a year



At the same time, at a different place, the Sun is not vertical

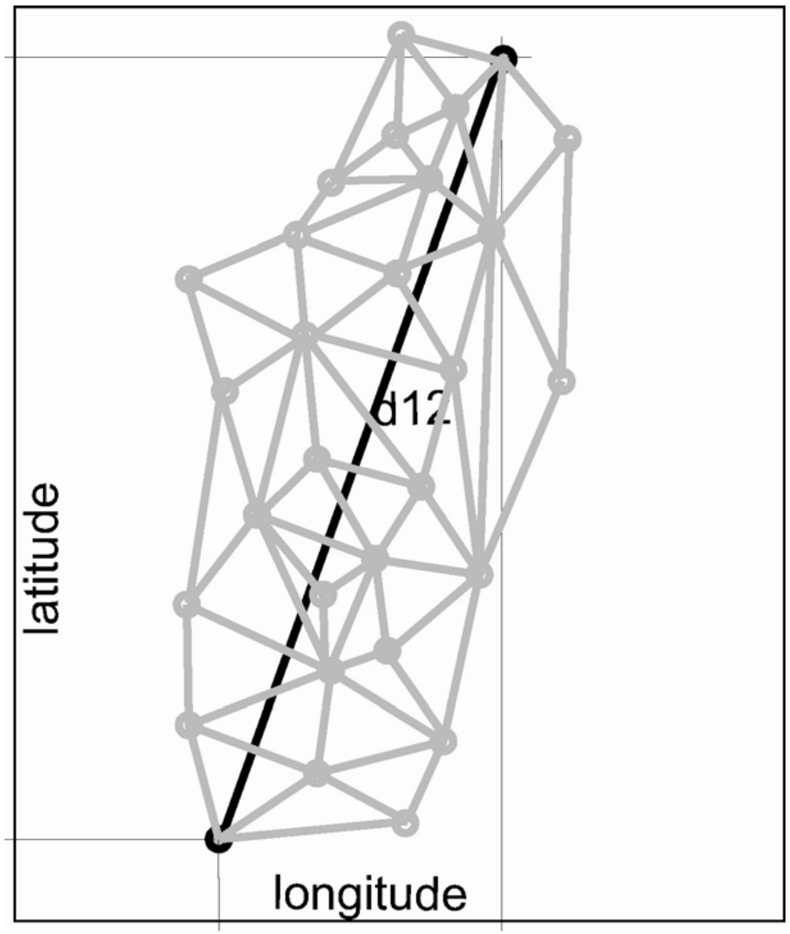
The angle can be measured from the length of the shadow of a vertical pole



The angle α of the sun light direction depends on the **local vertical** direction

=> Depends on the **latitude** of the site

«Modern» Geodesy (17th century)



A correction has to be made if distance is **not aligned** with longitude

d_{12} can be computed from the **sum** (oriented) of many smaller distances

Measuring many (if not all) **distances** and **angles** within a network of points give the more accurate solution for d_{12}

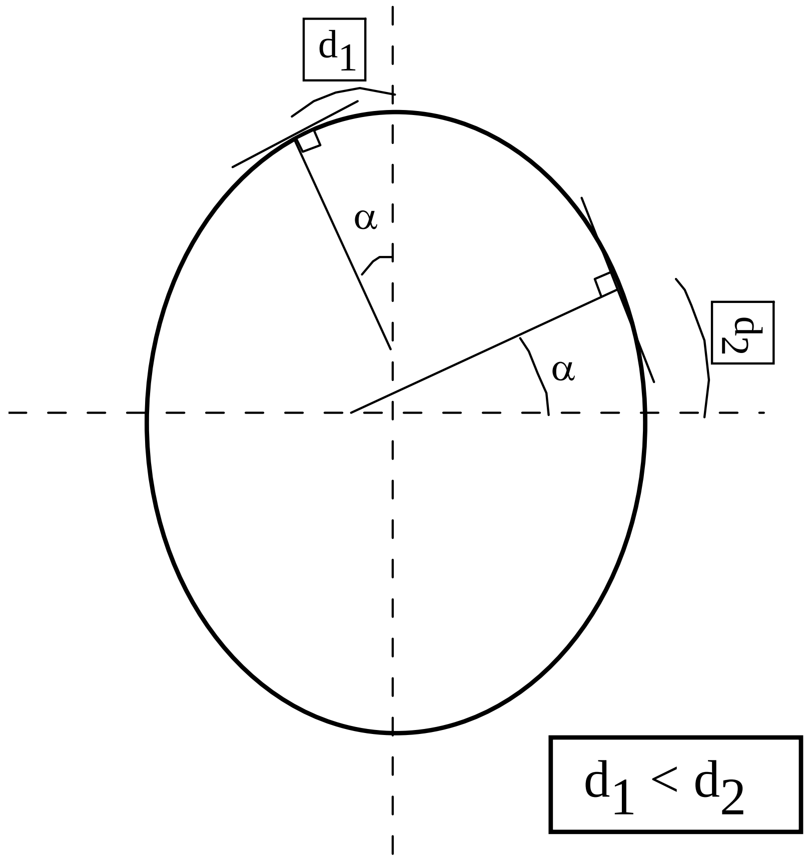
The shape of the Earth (18th century)

Making those measurements, different people find different values for the length of an arc of **1°** at different places in Europe

- **Snellius (1617) : 104 km**
- **Norwood (1635) : 109 km**
- **Riccioli (1661) : 119 km**

In France, **Picard** finds :

- **108 km** in the **north** of France
- **110 km** in the **south** of France

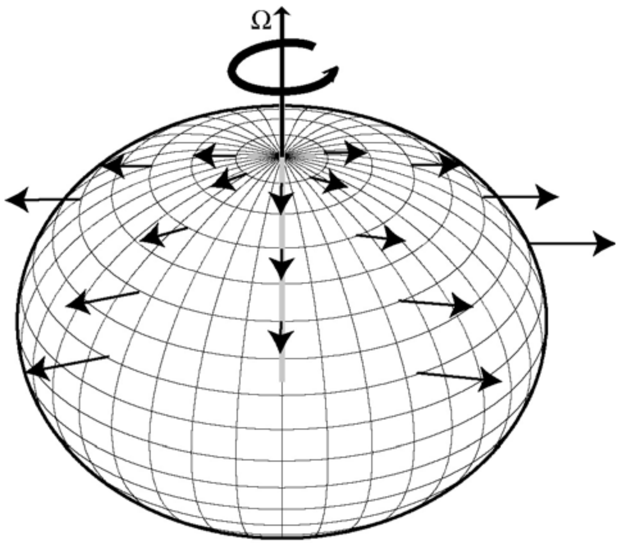


Other ways of doing geodesy

Newton and **Huygens** introduce gravity and acceleration forces :

$$g = {}^{GM}/_{R^2} \quad \text{and} \quad F = R\,\Omega^2$$

=> A rotating body has to inflate at the equator and flattened at the pole



The body will deform until an equilibrium is reached, i.e. where :

$$\Delta g = GM\left[{}^{1}/_{R_{eq}^{2}} - {}^{1}/_{R_{pol}^{2}}\right] = \Delta F = R_{eq}\,\Omega^2$$

For the Earth (R = 6378 km, Ω = 1 rotation/day) this is obtained with $R_{pole} = R_{equator}$ – 20 km or with a flattening of around $^{1}/_{300}$

Potential of gravity field

If the Earth were an homogeneous sphere in rotation, then its shape would be an ellipsoid :

At first order, the gravity field is then essentialy vertical :

$$\mathbf{g} = \begin{vmatrix} -GM/R^2 \,.\, \mathbf{e}_r \\ 0 \,.\, \mathbf{e}_\theta \\ 0 \,.\, \mathbf{e}_\phi \end{vmatrix}$$

In that case, it is easy to demonstrate that there always exist a field **V**, function of **r** so that **g** is the gradient of **V** : $\nabla(\mathbf{v}) = \mathbf{g}$

V is called the **potential field** of **g** and obeys equation $\Delta(\mathbf{v}) = 0$

*Note there are an infinite number of **V** (any additional constant would not change the gradient)*

It can be shown that **V** can be written as an infinite sum of specific function defined on a sphere : the **spherical harmonics** :

$$V(r,\theta,\phi) = \sum K_{\ell,m} \,.\, f(r) \,.\, P_{\ell,m}(\cos\theta) \,.\, e^{im\phi}$$

$$V(r,\theta,\phi) = \sum K_{\ell,m} . f(r) . P_{\ell,m}(\cos\theta) . e^{im\phi}$$

Potential of gravity field

This equation can be solved to write explicitly the dependence with the distance to center of mass (r) and the surface variation with latitude and longitude (θ,ϕ) at a given r :

We then find : $$V(r,\theta,\phi) = \sum_{\ell=0}^{+\infty} \left({}^{R}/_{r}\right)^{\ell+1} \sum_{m=-\ell}^{\ell} K_{\ell,m} . P_{\ell,m}(\cos\theta) . e^{im\phi}$$

or : $$V(r,\theta,\phi) = {}^{GM}/_{r} \left(1 + \sum_{\ell=1}^{+\infty} \left({}^{R}/_{r}\right)^{\ell} \sum_{m=-\ell}^{\ell} K_{\ell,m} . \underline{\underline{P_{\ell,m}(\cos\theta)} . e^{im\phi}}\right)$$

***Legendre** polynomial*

Spherical harmonic

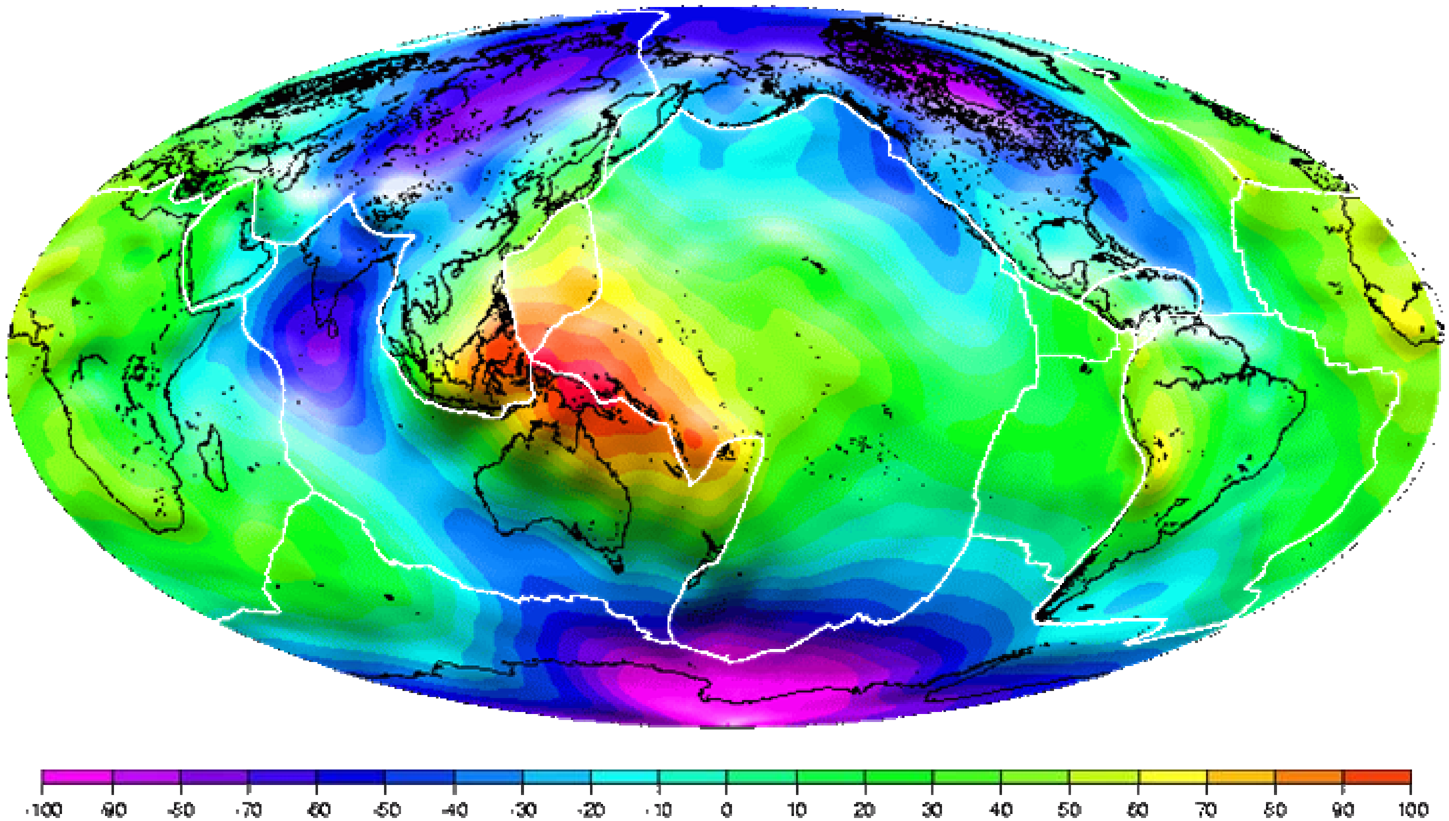
- It is easy to check that **at first order** ($\ell = 0$), this equation gives back the potential for a spherical earth (**V = GM/R**)
- We also get the same spherical potential far away in space (**r >>R**). In other words: *we don't see the details from far away*

$$V(r,\theta,\phi) = \sum K_{\ell,m} . f(r) . P_{\ell,m}(\cos\theta) . e^{im\phi}$$

Potential of gravity field -> Geoid

- If the Earth were an homogeneous sphere in rotation, then its shape would be an ellipsoid, **and so for its gravity field potential.** The terms of degree ℓ higher than 2 are **perturbations** to this ellipsoid**.**
- The potential generated by an ellipsoid in rotation is called: **hydrostatic potential.**
- *This is because if the planet were covered with water, then the surface would follow exactly this ellipsoid***.**
- Thus, at the ocean surface, water exactly follows the potential. This particular potential surface (the one that coincides with ocean level) is called the **Geoid**
- The **Hydrostatic Geoid** is the surface the ocean would have if the Eart were nothing else but an homogeneous sphere in rotation
- The **Non-hydrostatic Geoid** is what remains after this effect is taken into account.
 Again, it would be zero if the earth were a simple rotating sphere

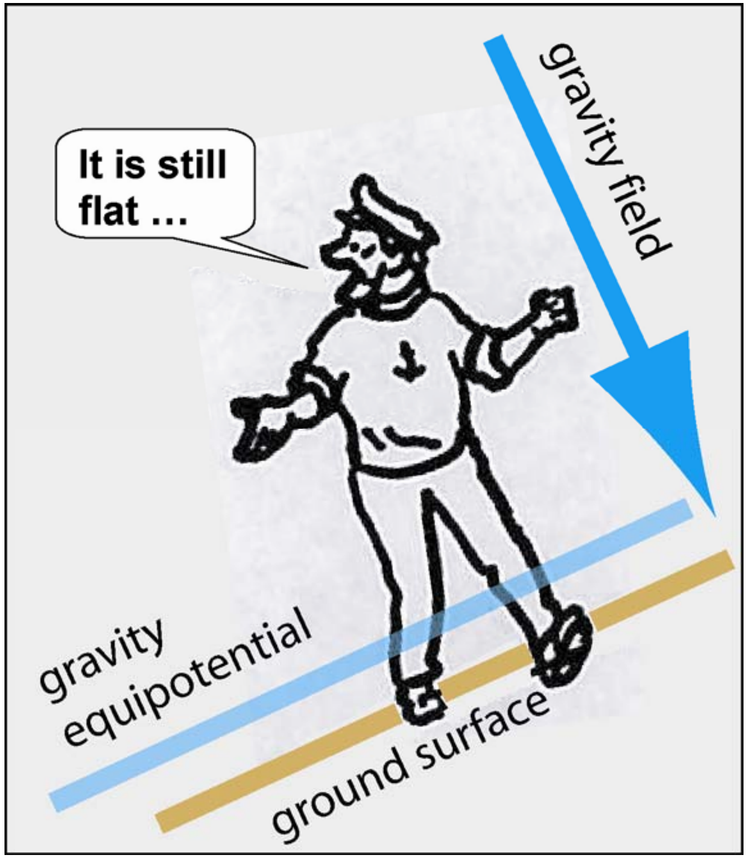
Geoid : Non hydrostatic Geoid, model GEM-T1 from GFZ

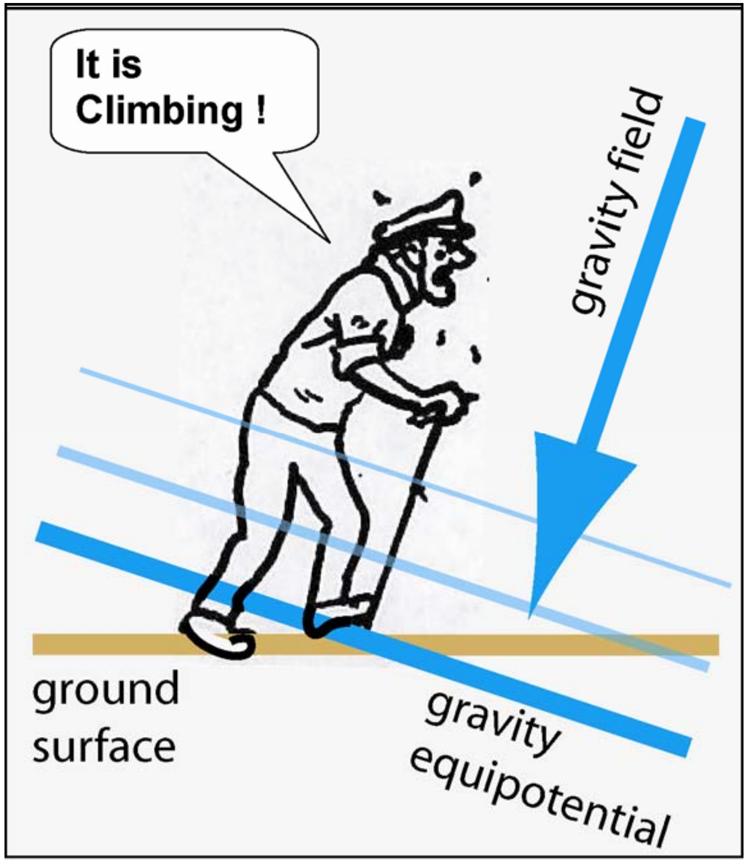


Blue indicates Geoid lows and less intense gravity, red indicates Geoid highs and more intense gravity. The surface shows lows and highs of +/- 100 m. it is shaped like a tennis ball.

Geoid : Definition of altitude

Altitude (height) **is not** a purely geometrical concept (i.e. distance from one point to the other) it is **defined with respect to the gravity potential**.





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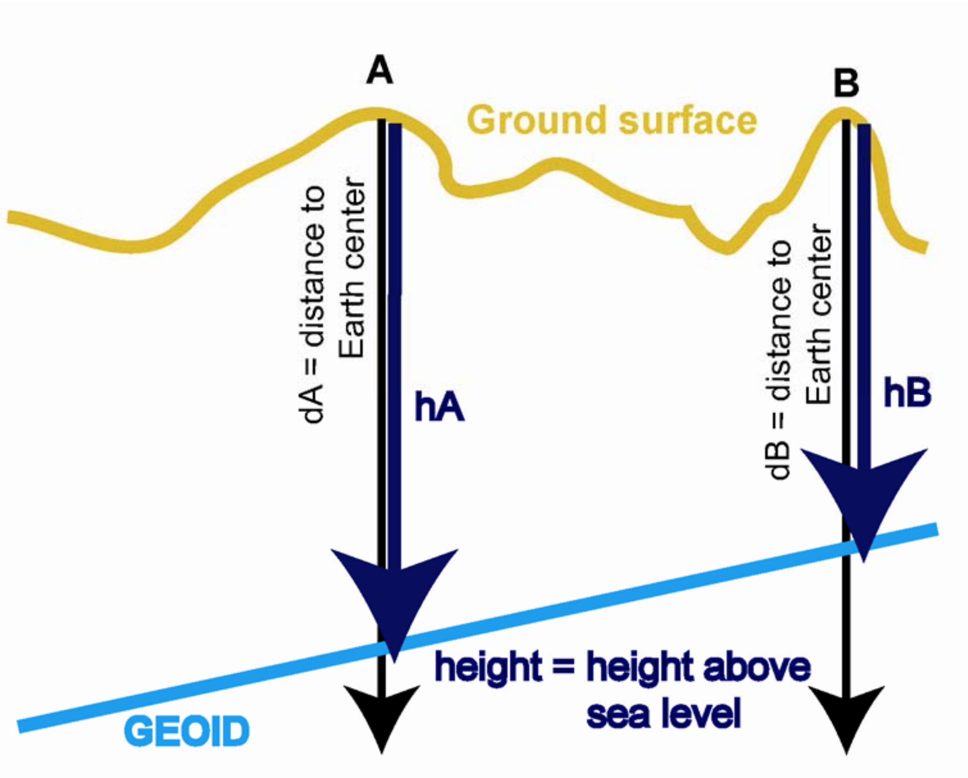


2 points **A** and **B** at ground level

One might think their altitude is d_A and d_B

But it is not !!!!

The altitude is the distance to the **geoid (**i.e. the **sea level) : h_A** and **h_B**

If the Geoid is not flat (i.e. at the same distance from the center of the Earth at A and B), **then the altitude changes**

$$G(\theta,\phi) = \sum_{\ell=0}^{+\infty} \sum_{m=-\ell}^{\ell} K_{\ell,m} \, . \, P_{\ell,m}(\cos\theta) \, . \, e^{im\phi}$$

Geoid : Spectral contains

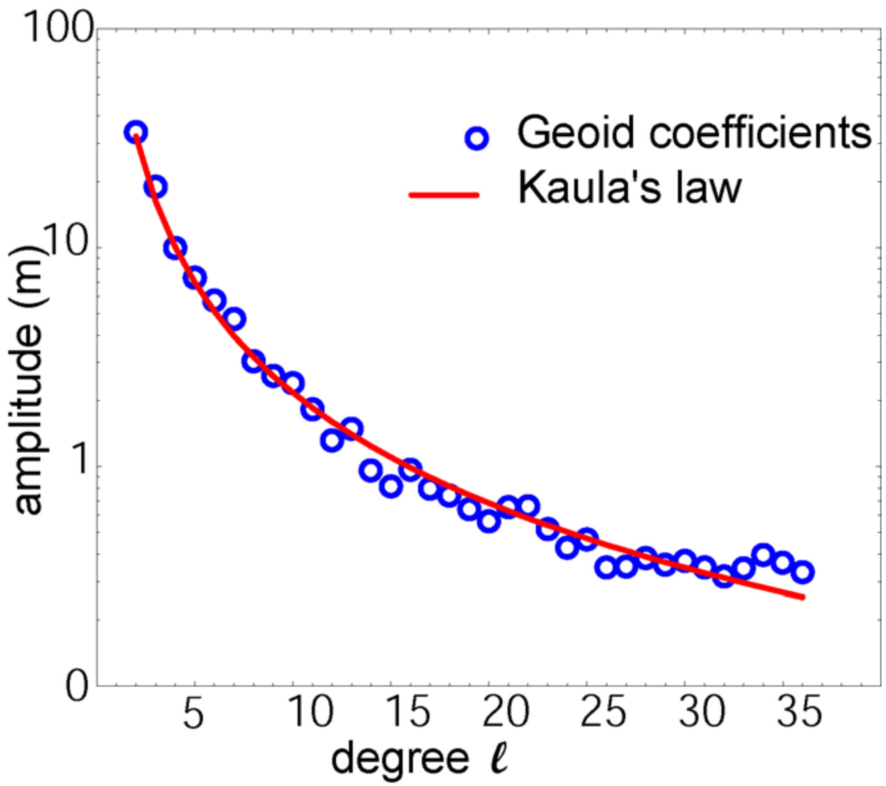
Spectrum of a field = amplitude of coefficients at a given degree ℓ of the spherical harmonic decomposition

$$S_\ell = \sqrt{\sum_{m=-\ell}^{\ell} (K_{\ell,m})^2}$$

The spectrum of the Geoid obeys a power law :

$$S_\ell \sim 1/\ell^2$$

It is called **Kaula's law**



$$V(\theta,\phi) = \sum_{\ell=0}^{+\infty} \left(\frac{R}{r}\right)^{\ell} \sum_{m=-\ell}^{\ell} K_{\ell,m} \,.\, P_{\ell,m}(\cos\theta) \,.\, e^{im\phi}$$

Altitude dependent spectral contains :

The potential **V** at altitude r-R is attenuated by a coefficient :

$$\left({}^{R}/_{r}\right)^{\ell}$$

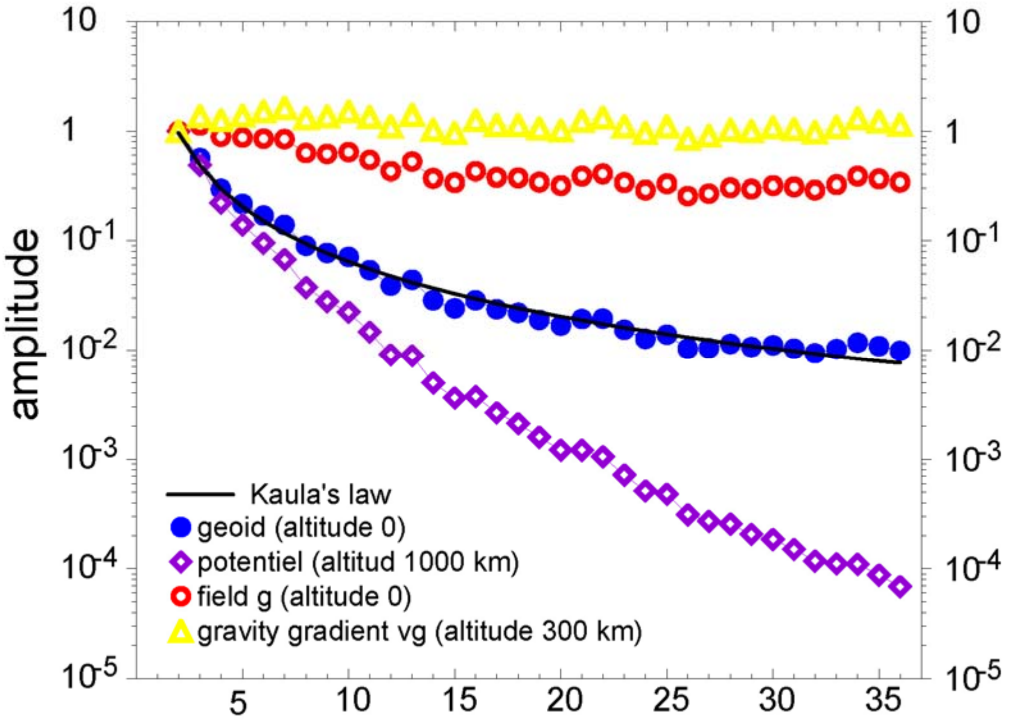
The gravity field **g** is the derivative of the gravity potential :

$$\mathbf{g} = {}^{\partial \mathbf{V}}/_{\partial\, \mathbf{r}}$$

$$\Rightarrow \quad \mathbf{g}_{\ell} \sim \mathbf{V}_{\ell} \times \ell$$

The gradient of the gravity field $\nabla\mathbf{g}$ is the 2nd derivative of the gravity potential :

$$\Rightarrow \quad \nabla\mathbf{g}_{\ell} \sim \mathbf{V}_{\ell} \times \ell^{2}$$



Correlation coefficients

Correlation between 2 fields (A and B) =

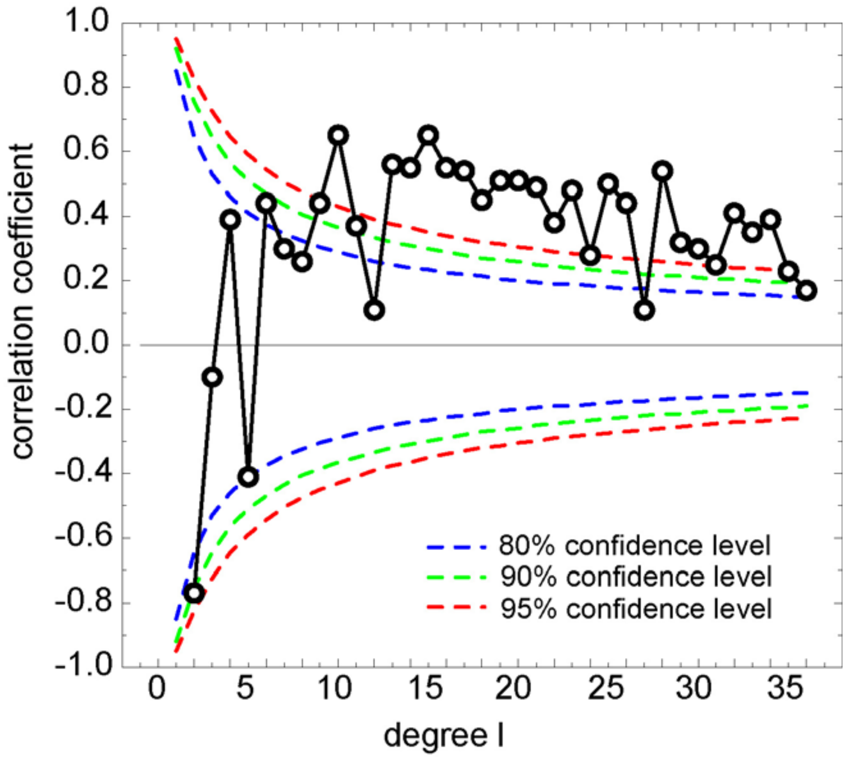
amplitude of cross products of coefficients at a given degree ℓ of the spherical harmonic decomposition

$$C_\ell = \frac{\sum_{m=-\ell}^{\ell} (A_{\ell,m} \cdot B_{\ell,m})}{\sqrt{\sum_{m=-\ell}^{\ell} (A_{\ell,m})^2 \sum_{m=-\ell}^{\ell} (B_{\ell,m})^2}}$$

The value of C represent the probability that the field are correlated :

- **0** means no correlation
- **1** means high correlation

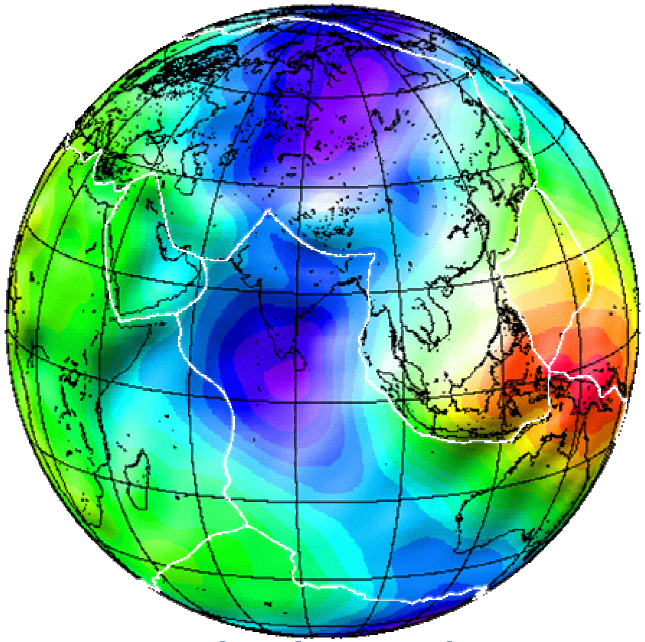
Correl. Geoid / surface topography



- No correlation for long wavelength
- High correlation for short wavelength

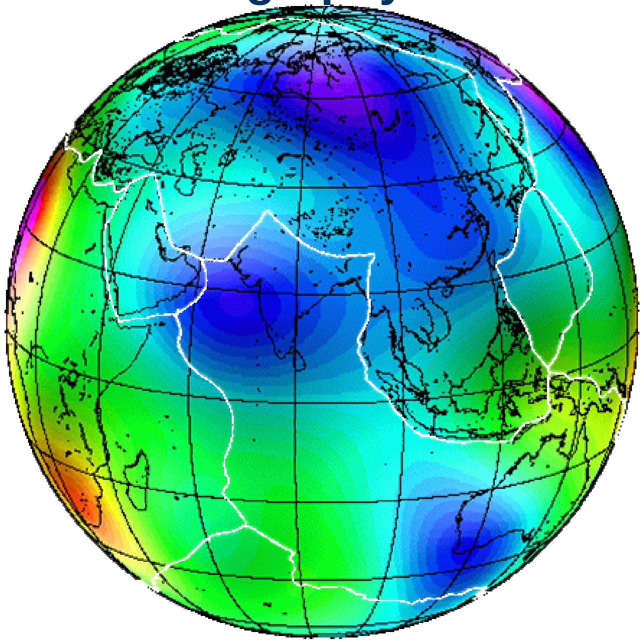
Origin of the Geoid : density anomalies

Geoid over India



Blue=low gravity
Red = high gravity

Seismic tomography in the mantle



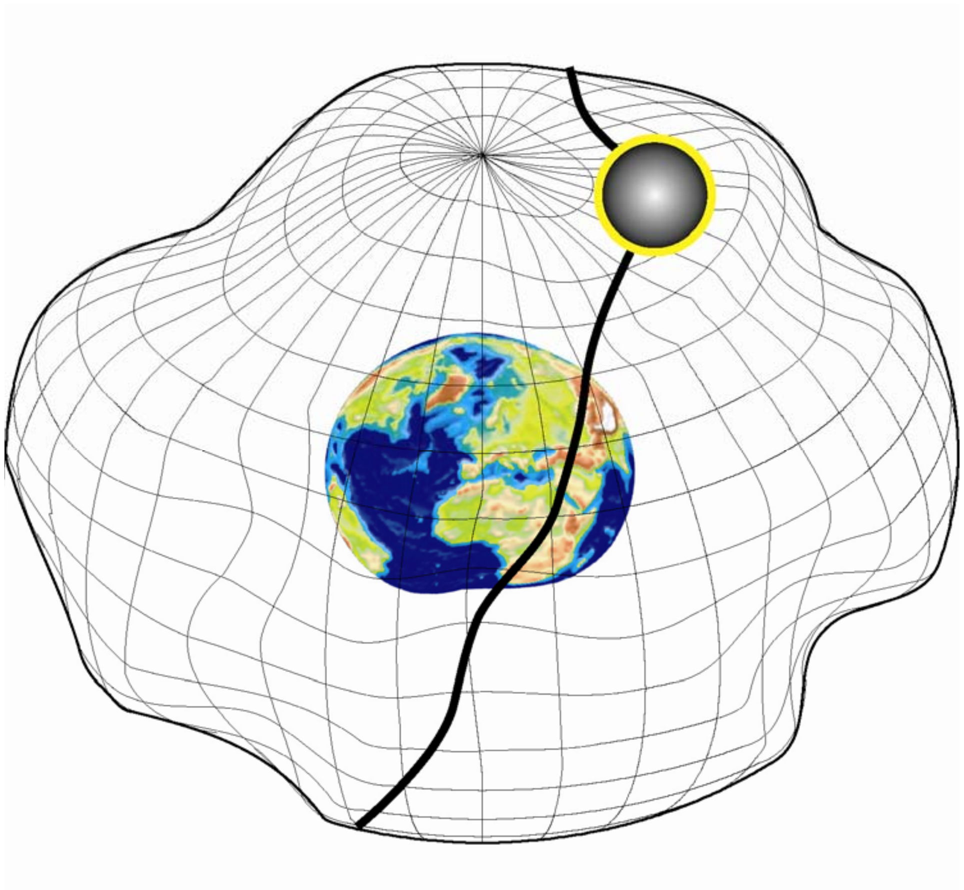
Blue="cold"="more dense"
Red = "hot" = "less dense"

It is very clear that long wavelength Geoid lows are associated to cold and dense material in the mantle. Therefore :

Long wavelength Geoid = density anomalies in the mantle

short wavelength Geoid = surface topography (i.e. mountains)

Measurement of the Geoid : spatial geodesy



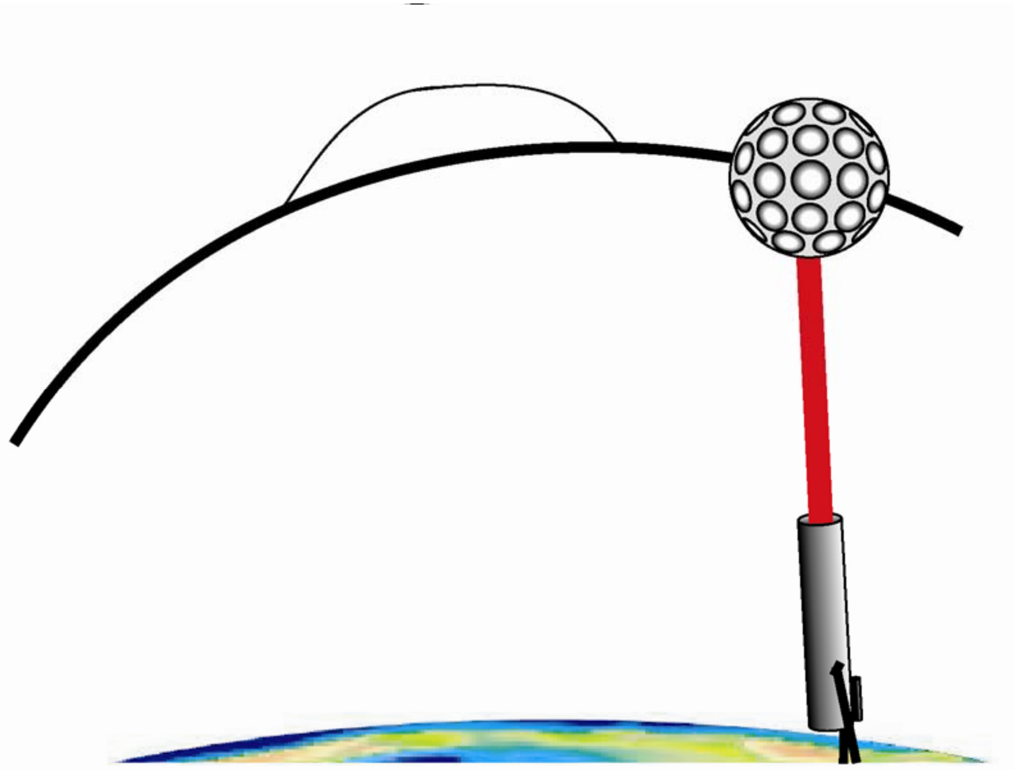
A satellite orbiting around the Earth will be sensitive to gravity: Its motion is such that the rotation force exactly equilibrates the gravity forces.

If the gravity is stronger (i.e. the gravity potential higher), then the satellite will have to orbit a little bit farther away from the earth (to increase the rotation force, and remain in equilibrium)

Conclusion : an orbiting satellite will follow <u>exactly</u> the Gravity potential !

measuring the satellite orbit will give us the gravity potential (i.e. the Geoid)

SLR : Satellite Laser Ranging



A High power laser fires on the satellite

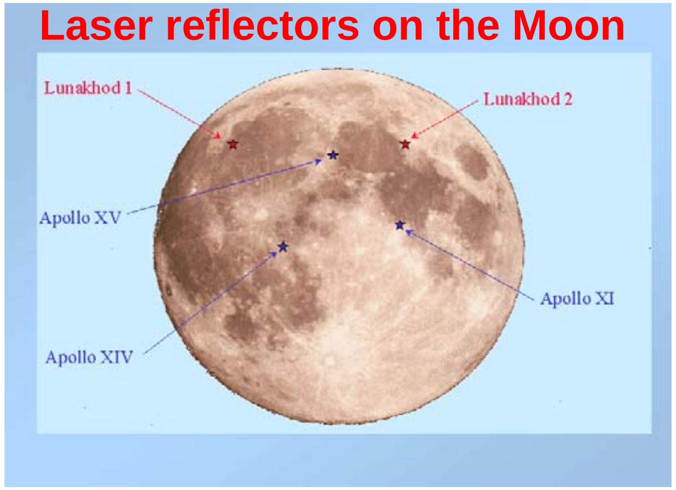
The impulse comes back, so the travel time is measured.

Given the speed of light (**C**), one can compute the distance from laser station to satellite :

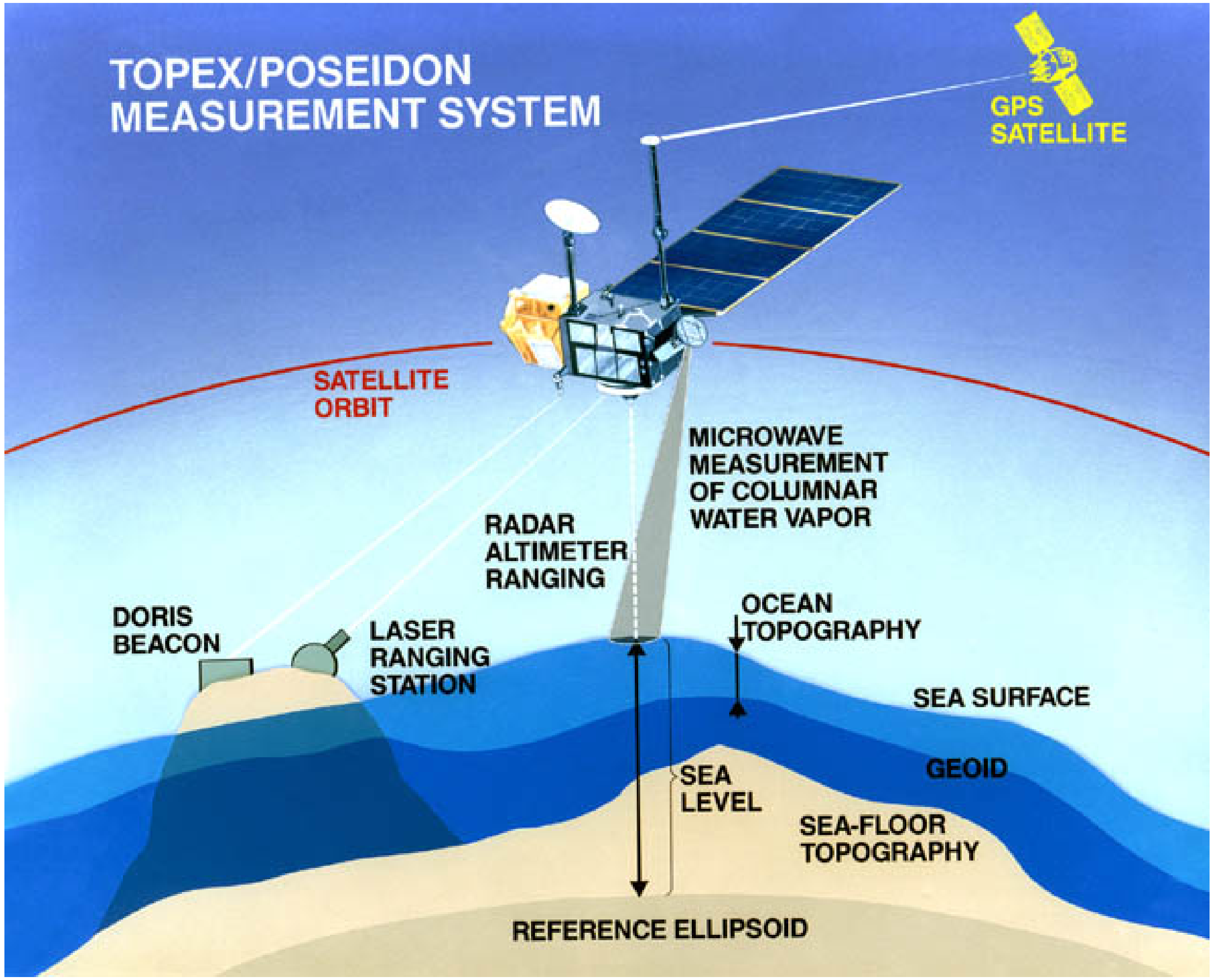
$$\mathbf{L = \Delta t \times C}$$

Measuring distances along the orbit give the shape of this orbit, I.E., the shape of the gravity potential

SLR : Satellite Laser Ranging



Satellite altimetry : principle



A satellite radar measures the distance between the satellite and the surface of the sea

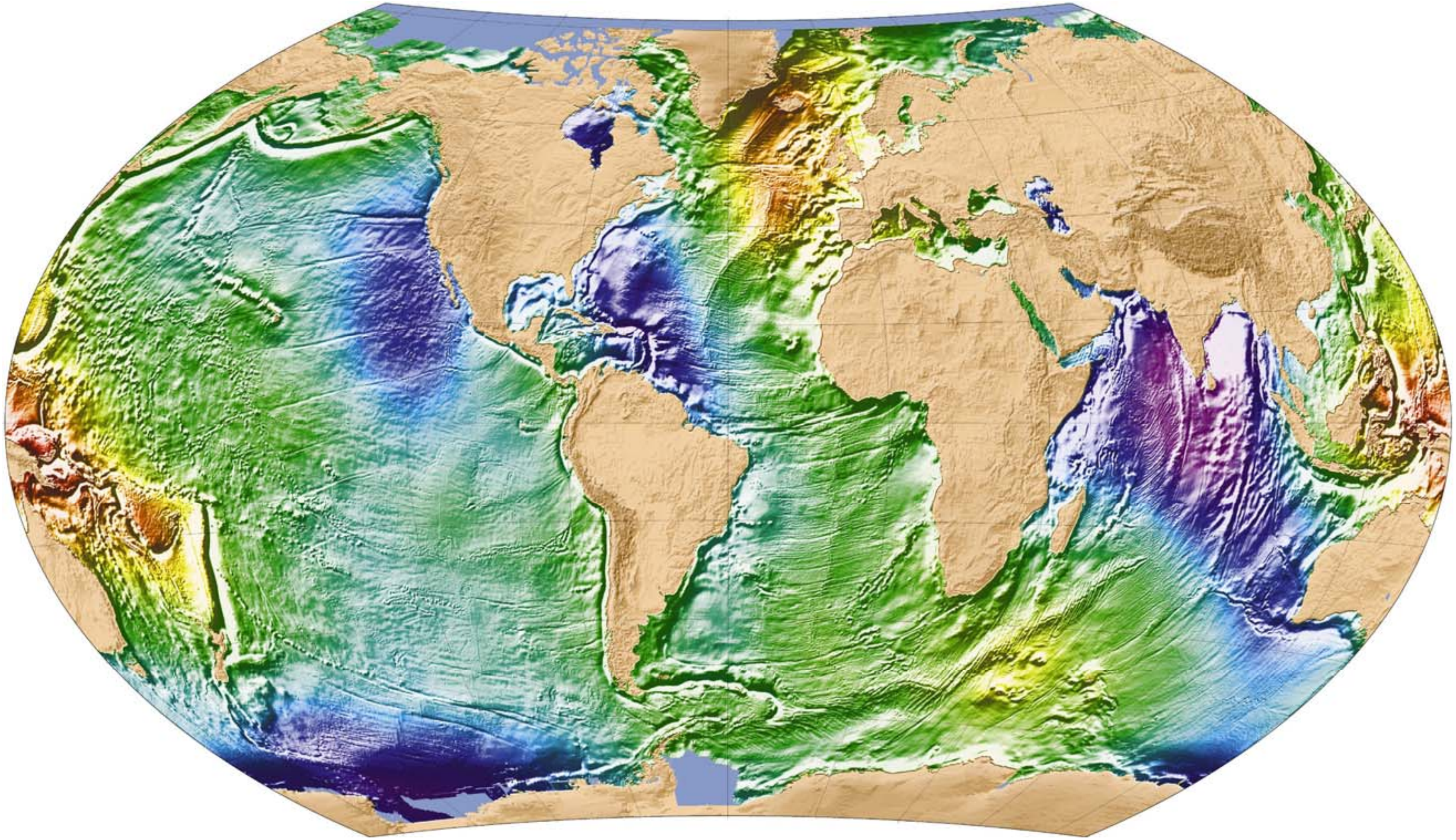
In average (not considering waves, tides and oceanic currents) **the sea surface is the Geoid**

ds = distance satellite to center of Earth

h = distance satellite to sea surface (measured)

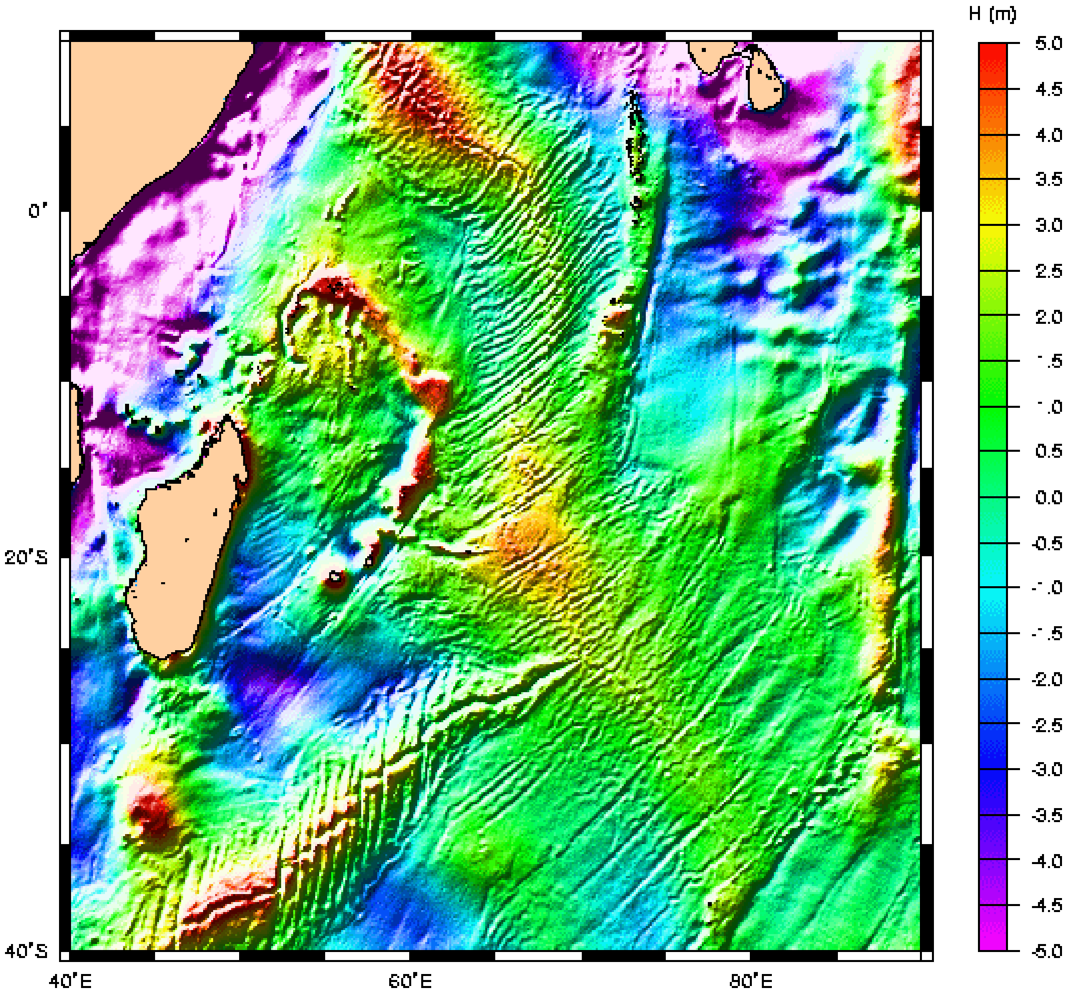
Geoid = ds-h

Satellite altimetry



The result is a high resolution map of the Geoid on 70% of the earth surface

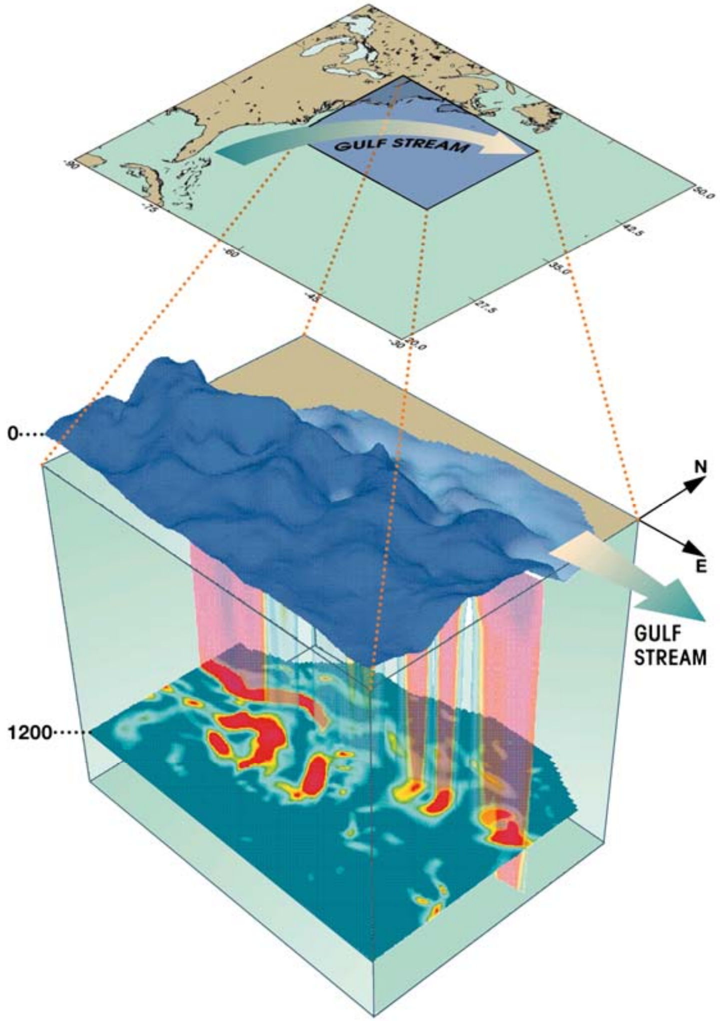
Satellite altimetry



A zoom of the oceanic Geoid shows that we see in detail **short wavelength** gravity anomalies

These come from density anomalies at the surface of the sea bottom. They are **ridges**, **sea mounts**, **transform faults**, etc…

Satellite altimetry

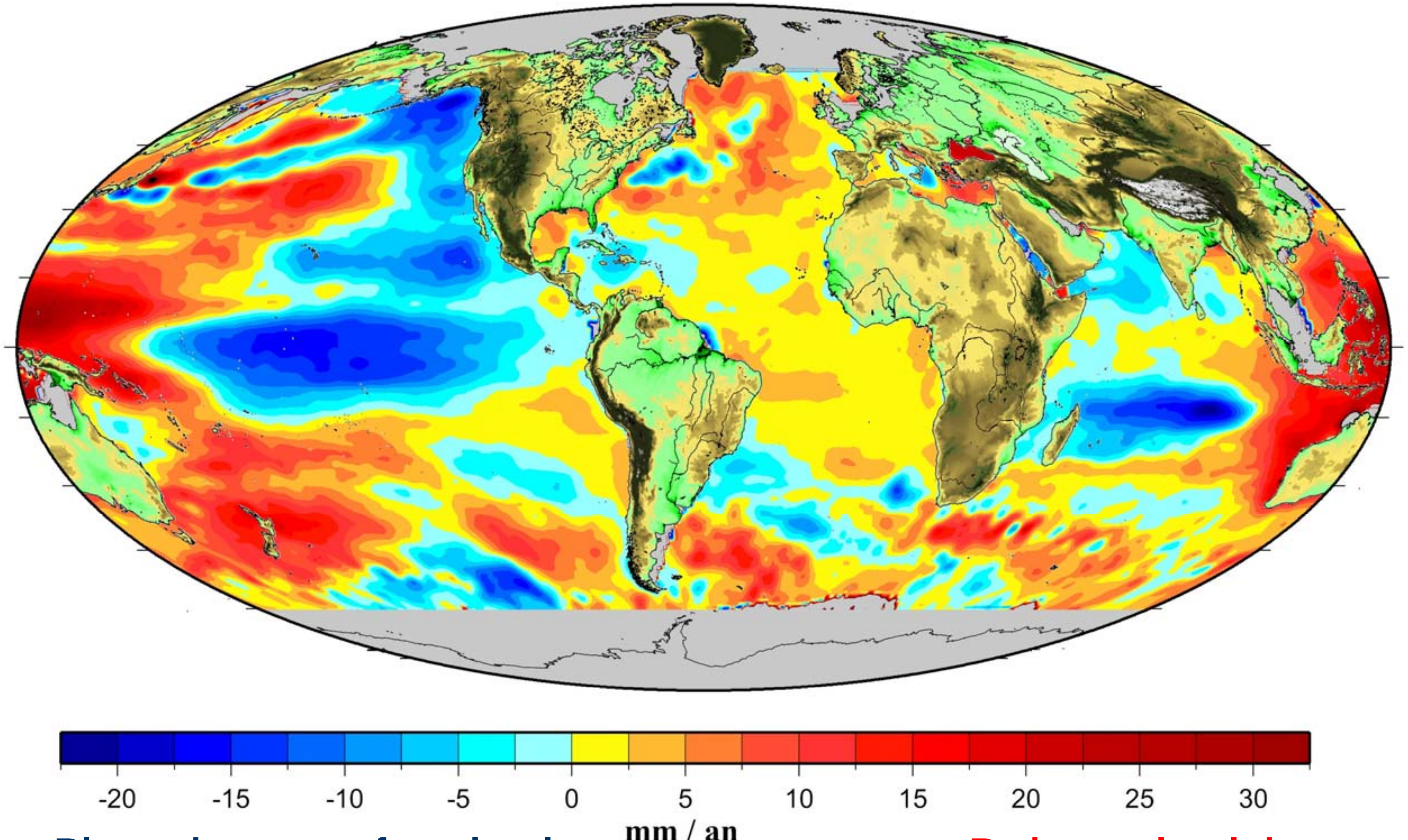


An anomaly of the sea surface can also be related to **water anomaly**

The precision of current altimeter allow to map swells of no more than **10 cm.**

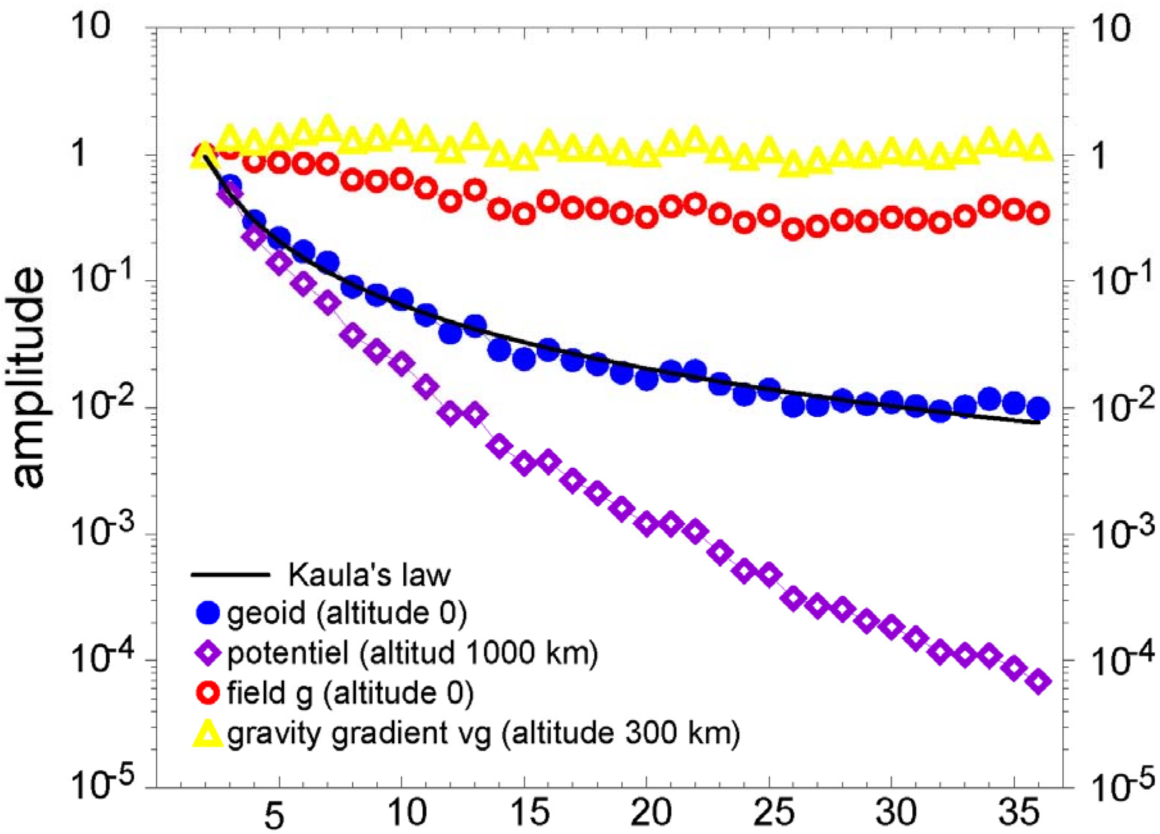
Doing this, we can trace oceanic currents like the **Gulf Stream**

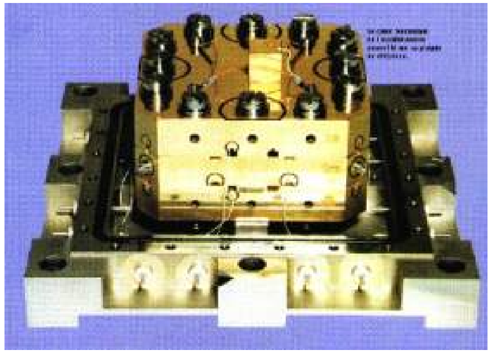
Satellite altimetry: Sea level variation

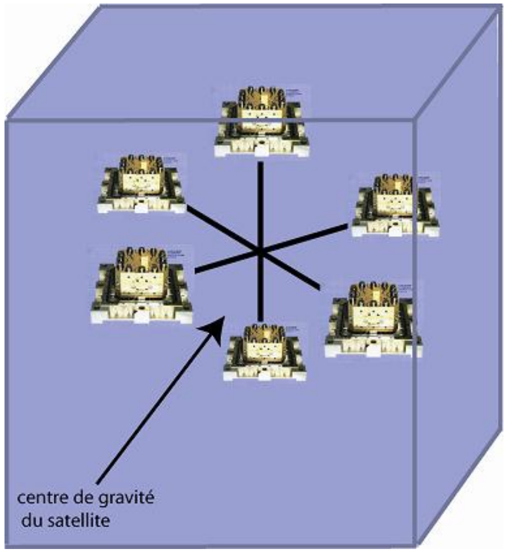


Blue = decrease of sea level **Red = sea level rise**

Satellite gradiometry







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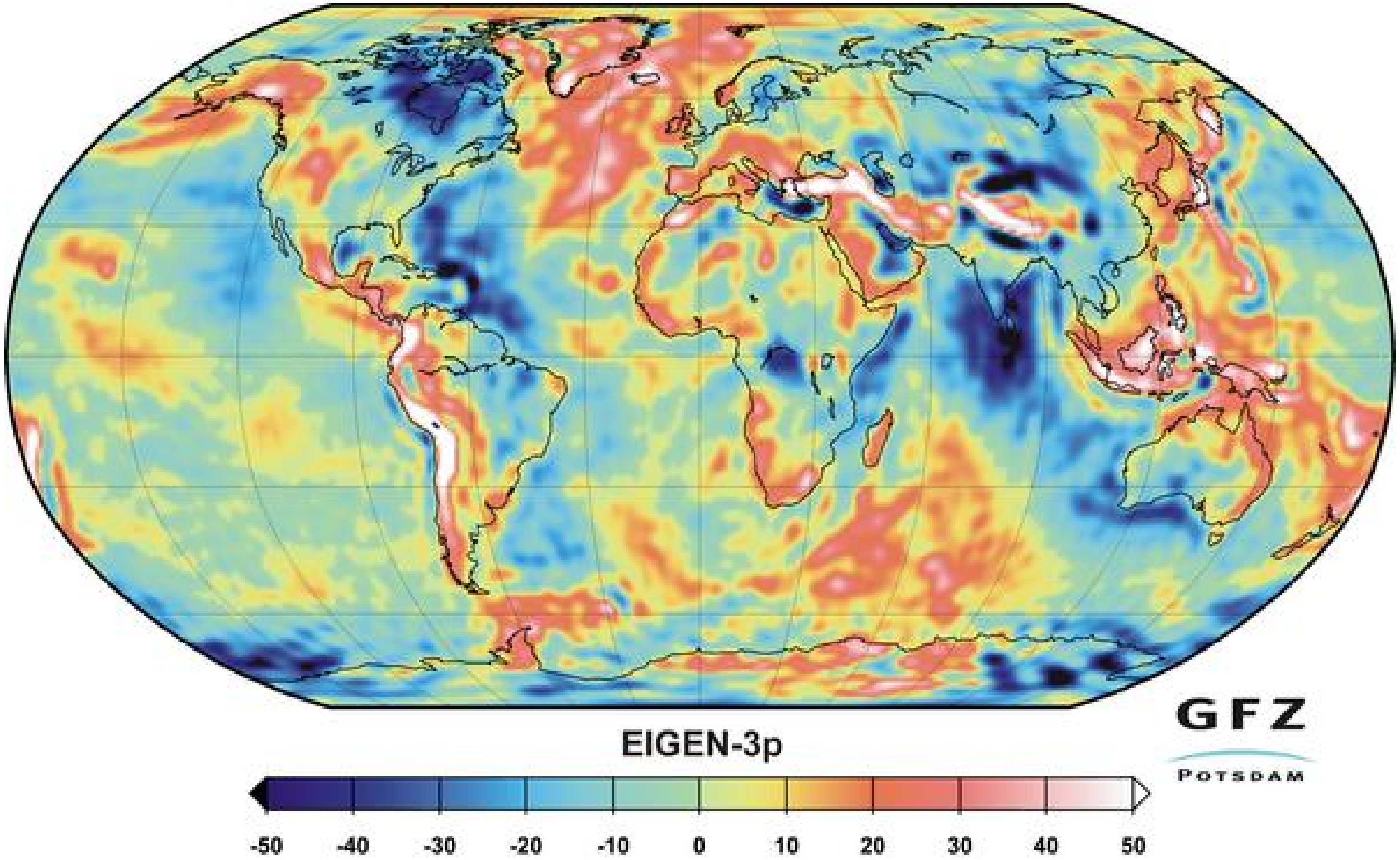

GFZ POTSDAM