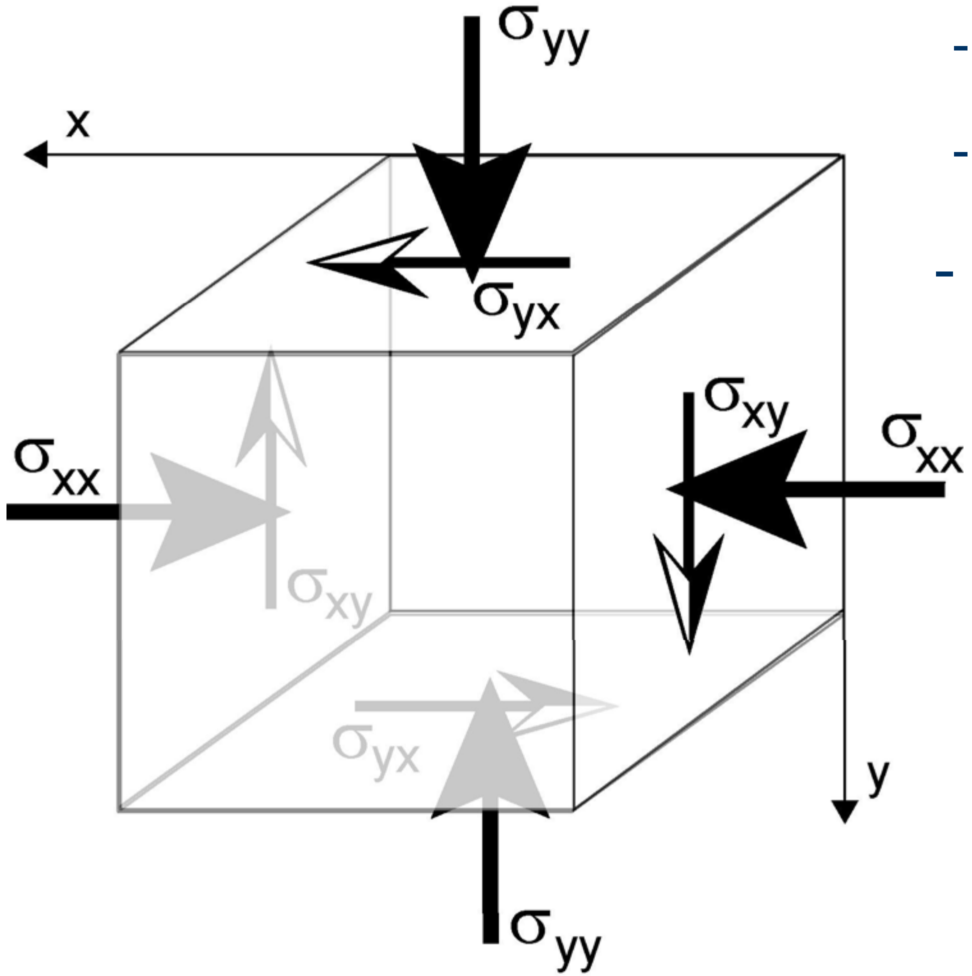
DEFORMATION PATTERN IN ELASTIC CRUST

- Stress and force in 2D
- Strain : normal and shear
- Elastic medium equations
- Vertical fault in elastic medium => arctangent
- General elastic dislocation (Okada's formulas)
- Fault examples

Stress (σ) in 2D



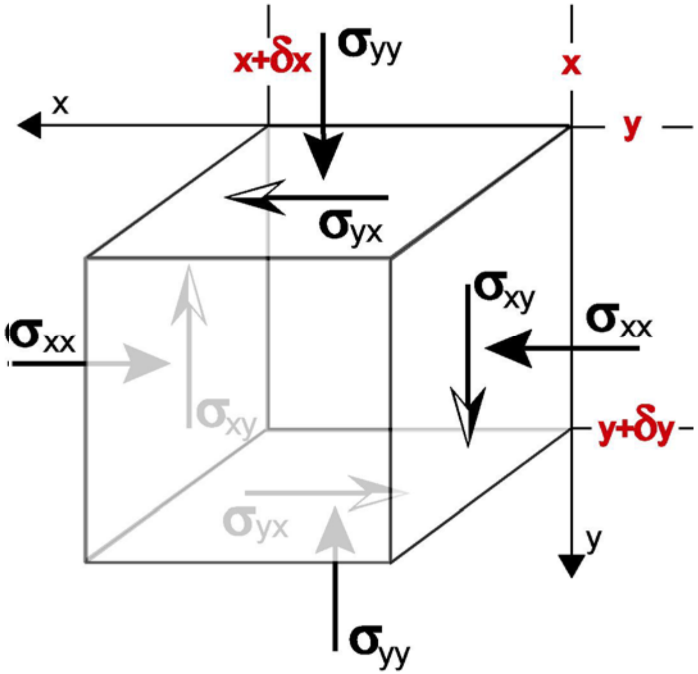
- Normal stress = σ_{ii}
- Shear stress = σ_{ij}
- Force = σ x surface
  - no rotation =>

$$\sigma_{xy} = \sigma_{yx}$$

- only 3 independent components :

$$\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$$

Applied forces



Normal forces on x axis :

$$= \sigma_{xx}(x).\ \delta y - \sigma_{xx}(x+\delta x).\ \delta y$$

$$= \delta y \left[\sigma_{xx}(x). - \sigma_{xx}(x+\delta x)\right]$$

$$= - \delta y\ {}^{d\sigma_{xx}}/_{dx}\ .\ \delta x \qquad (1)$$

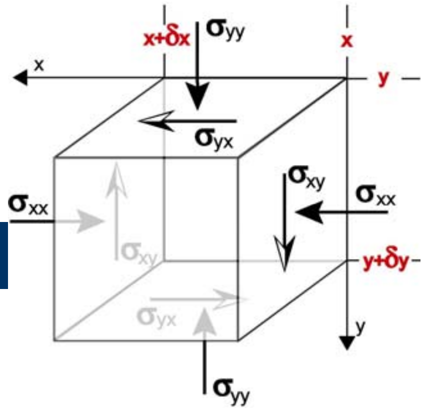
Shear forces on x axis :

$$= \sigma_{yx}(y).\ \delta x - \sigma_{yx}(y+\delta y).\ \delta x$$

$$= - \delta x\ {}^{d\sigma_{yx}}/_{dy}\ .\ \delta y \qquad (2)$$

Total on x axis = (1)+(2) $= \boxed{\left[{}^{d\sigma_{xx}}/_{dx} + {}^{d\sigma_{yx}}/_{dy}\right] \delta x\ \delta y}$

Forces Equilibrium



Total on x axis = $\left[\frac{d\sigma_{xx}}{dx} + \frac{d\sigma_{yx}}{dy} \right] \delta x \, \delta y$

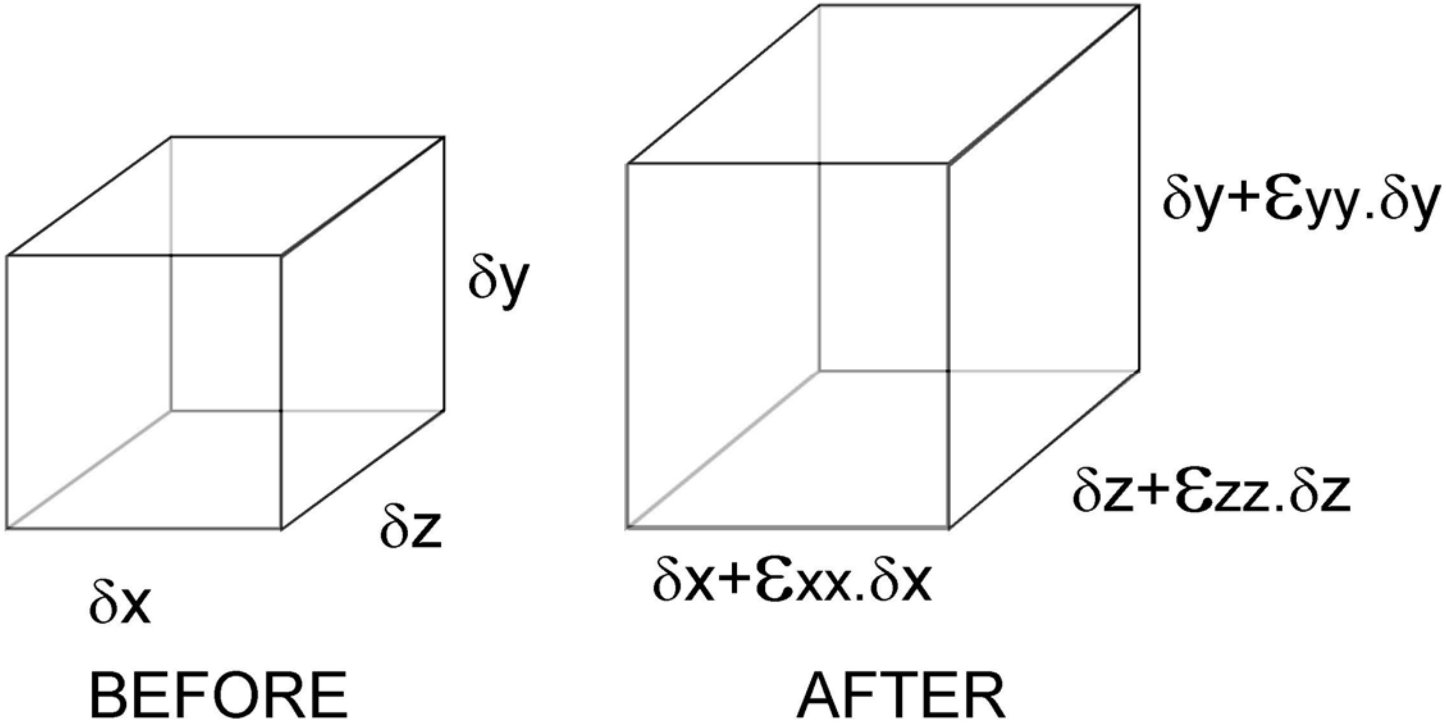
Total on y axis = $\left[\frac{d\sigma_{yy}}{dy} + \frac{d\sigma_{yx}}{dx} \right] \delta y \, \delta x$

Equilibrium =>

$$\left[\frac{d\sigma_{yy}}{dy} + \frac{d\sigma_{yx}}{dx} \right] = 0$$

$$\left[\frac{d\sigma_{xx}}{dx} + \frac{d\sigma_{yx}}{dy} \right] = 0$$

Normal strain : change length (not angles)

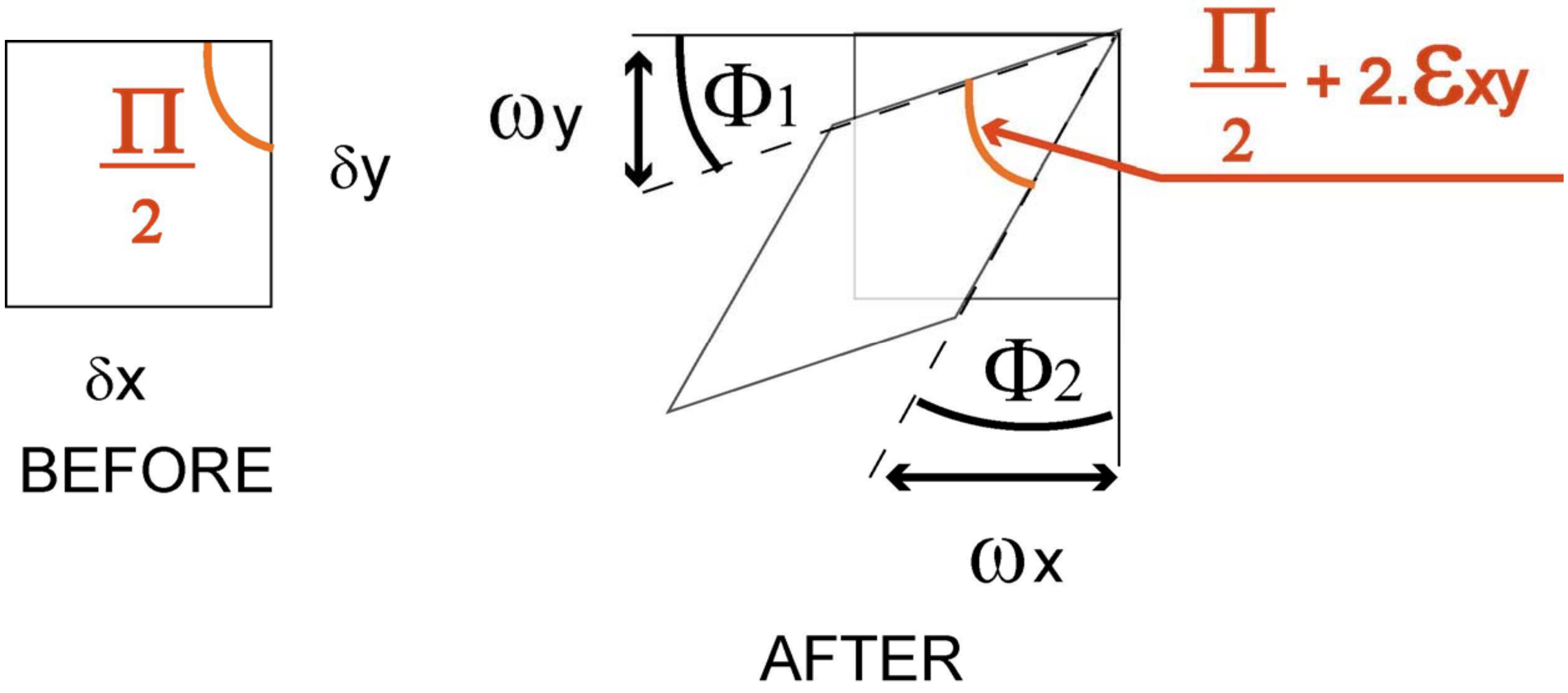


- Change of length proportional to length

- ε_{xx}, ε_{yy}, ε_{zz} are normal component of **strain**

nb : If deformation is small, change of volume is $\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$ *(neglecting quadratic terms)*

Shear strain : change angles



$$\varepsilon_{xy} = -\tfrac{1}{2}\left(\Phi_1 + \Phi_2\right) = \tfrac{1}{2}\left(\frac{d\omega_y}{dx} + \frac{d\omega_x}{dy}\right)$$

$\varepsilon_{xy} = \varepsilon_{yx}$ (obvious)

Solid elastic deformation (1)

- Stresses are proportional to strains
- No preferred orientations

$$\sigma_{xx} = (\lambda+2G)\,\varepsilon_{xx} + \lambda\,\varepsilon_{yy} + \lambda\,\varepsilon_{zz}$$

$$\sigma_{yy} = \lambda\,\varepsilon_{xx} + (\lambda+2G)\,\varepsilon_{yy} + \lambda\,\varepsilon_{zz}$$

$$\sigma_{zz} = \lambda\,\varepsilon_{xx} + \lambda\,\varepsilon_{yy} + (\lambda+2G)\,\varepsilon_{zz}$$

- λ and G are *Lamé* parameters

The material properties are such that a principal strain component ε produces a stress $(\lambda+2G)\varepsilon$ in the same direction and stresses $\lambda\varepsilon$ in mutually perpendicular directions

Solid elastic deformation (2)

Inversing stresses and strains give :

$$\varepsilon_{xx} = {}^1/_E\, \sigma_{xx} - {}^\nu/_E\, \sigma_{yy} - {}^\nu/_E\, \sigma_{zz}$$

$$\varepsilon_{yy} = -{}^\nu/_E\, \sigma_{xx} + {}^1/_E\, \sigma_{yy} - {}^\nu/_E\, \sigma_{zz}$$

$$\varepsilon_{zz} = -{}^\nu/_E\, \sigma_{xx} - {}^\nu/_E\, \sigma_{yy} + {}^1/_E\, \sigma_{zz}$$

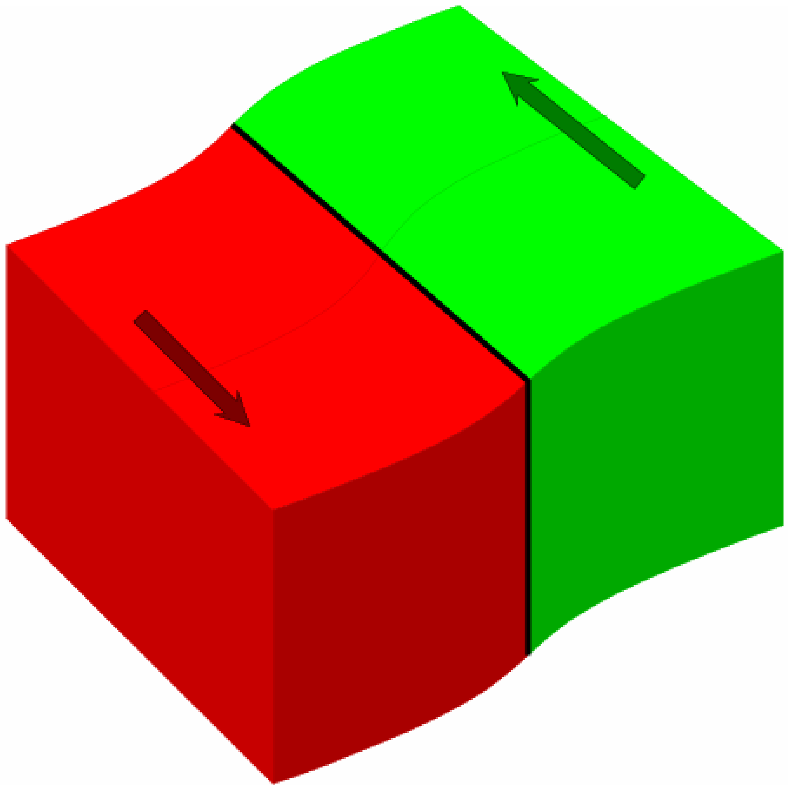
- E and ν are *Young*'s modulus and *Poisson*'s ratio

a principal stress component σ produces

a strain ${}^1/_E\, \sigma$ in the same direction and

strains ${}^\nu/_E\, \sigma$ in mutually perpendicular directions

Elastic deformation across a locked fault

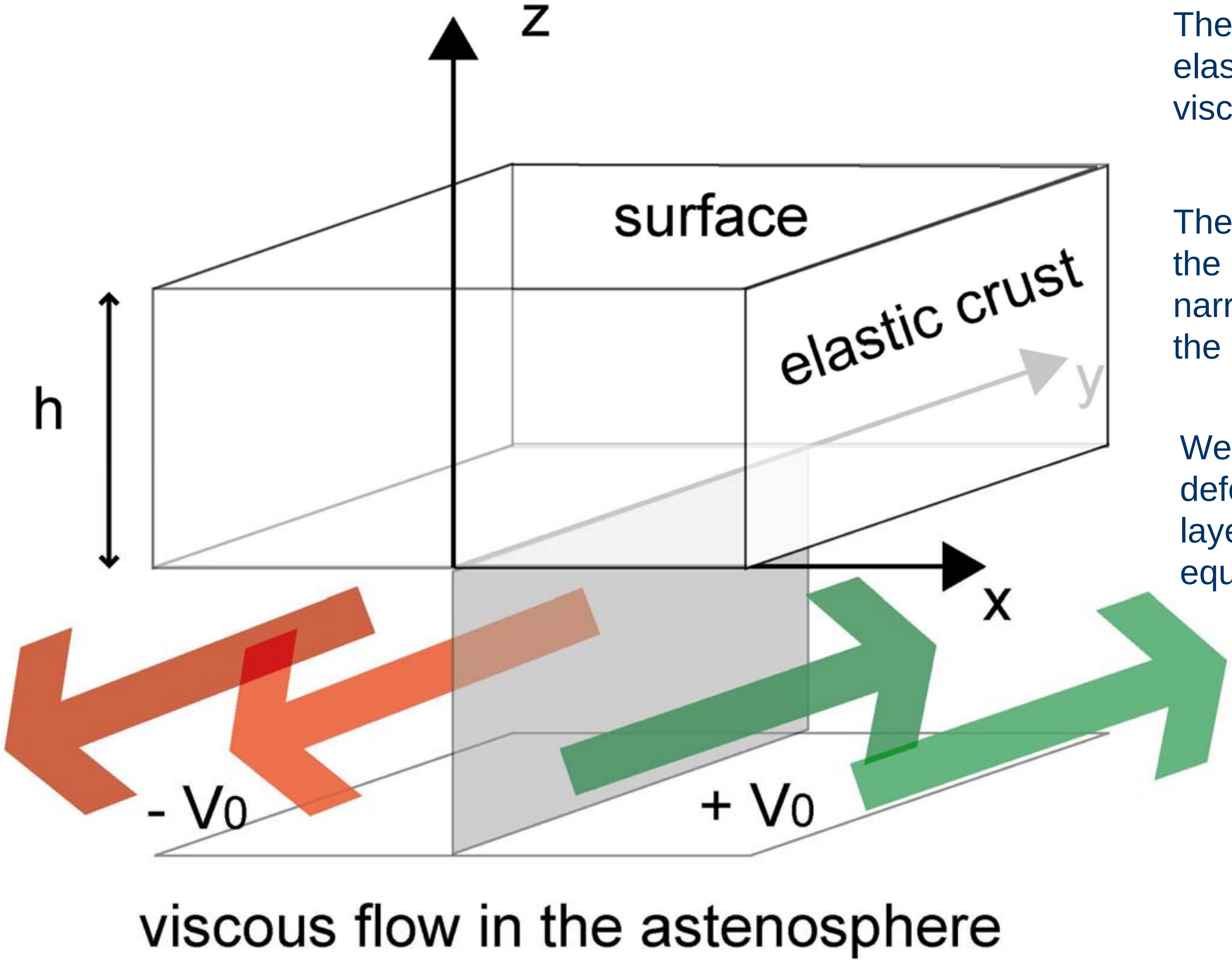


Two plates (red and green) are separated by a vertical strike-slip fault.

If the fault was slick then the two plates would slide freely along each other with no deformation. But because its surface is rough and there is some friction, the fault is locked. So because the plates keep moving far away from the fault, and don't move on the fault, they have to deform.

What is the shape of the accumulated deformation ?

Mathematical formulation

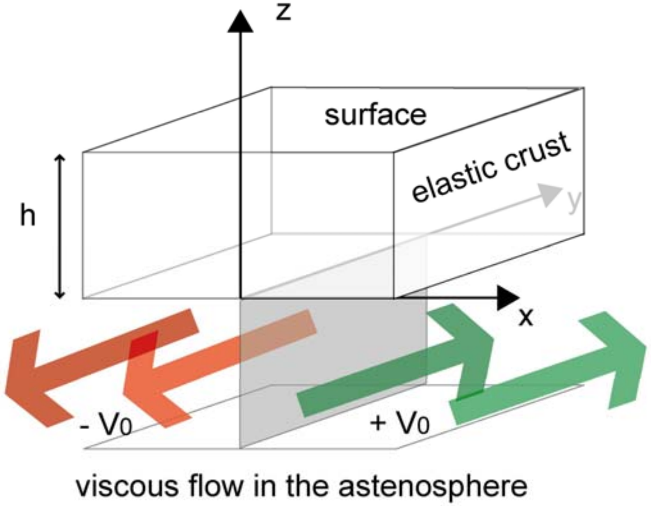


The plates are made of elastic crust above a viscous mantle.

The viscous flow localizes the deformation in a narrow band just beneath the elastic layer

We can compute the deformation of the elastic layer using the elastic equations detailed before

Mathematical formulation



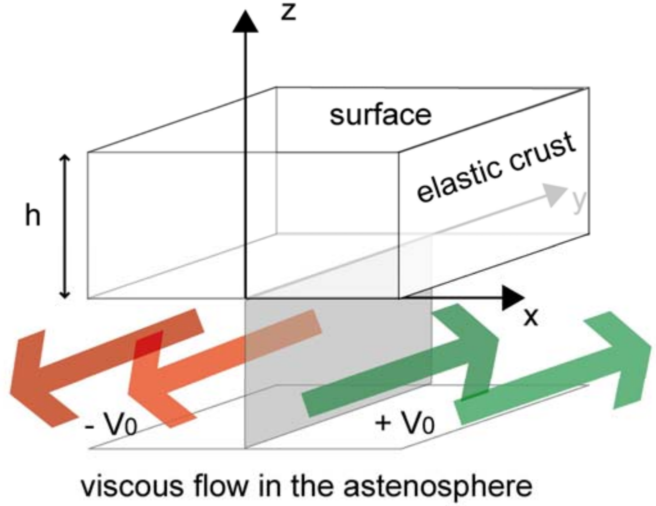
- Symetry => all derivative with y = 0

  $\varepsilon_{yy} = 0$

- No gravity => $\sigma_{zz} = 0$

- What is the displacement field U in the elastic layer ?

Mathematical formulation



• Elastic equations :

(1) $\sigma_{xx} = (\lambda+2G)\,\varepsilon_{xx} + \lambda\,\varepsilon_{zz}$

(2) $\sigma_{yy} = \lambda\,\varepsilon_{xx} + \lambda\,\varepsilon_{zz}$ $\sigma_{xy} = 2G\,\varepsilon_{xy}$ $\sigma_{xz} = 2G\,\varepsilon_{xz}$

(3) $\sigma_{zz} = \lambda\,\varepsilon_{xx} + (\lambda+2G)\,\varepsilon_{zz}$ $\sigma_{yz} = 2G\,\varepsilon_{yz}$

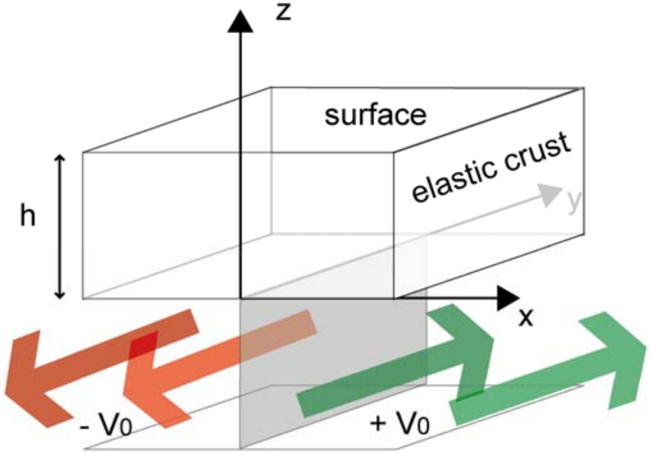
(3) + $\sigma_{zz} = 0$ => $\lambda\,\varepsilon_{xx} + \lambda\,\varepsilon_{zz} = -2G\,\varepsilon_{zz}$

↳ and (2) => $\sigma_{yy} = \lambda\,\varepsilon_{xx} + \lambda\,\varepsilon_{zz} = -2\,G\,\varepsilon_{zz}$

=> $\varepsilon_{xx} = -\,(2G+\lambda)/\,\lambda\,\varepsilon_{zz}$

↳ and (1) => $\sigma_{xx} = \left[-\,{}^{(\lambda+2G)^2}/_{\lambda} + \lambda\right]\varepsilon_{zz}$

Mathematical formulation



• Force equilibrium along the 3 axis

(x) $d\sigma_{xx}/dx + d\sigma_{yx}/dy + d\sigma_{xz}/dz = 0$

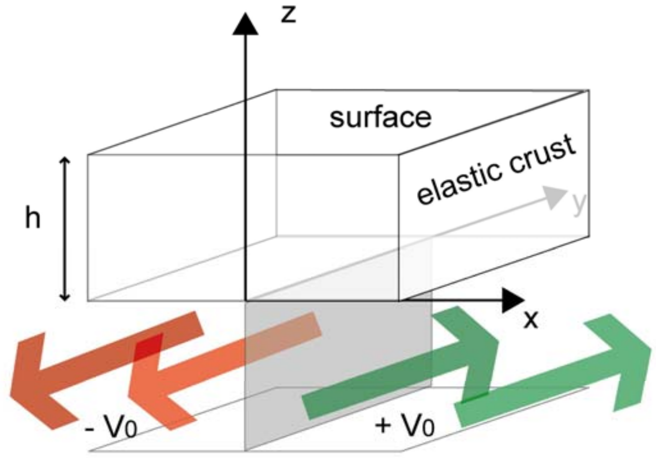
(y) $d\sigma_{xy}/dx + d\sigma_{yy}/dy + d\sigma_{yz}/dz = 0$

(z) $d\sigma_{xz}/dx + d\sigma_{yz}/dy + d\sigma_{zz}/dz = 0$

• Derivation of eq. 1 with x and eq. 3 give : $d^2\sigma_{xx}/dx^2 = 0$

• equation 2 becomes : $d\sigma_{xy}/dx + d\sigma_{yz}/dz = 0$

Mathematical formulation



viscous flow in the astenosphere

relations between

stress (σ) and displacement vector (U)

$$\sigma_{xy} = 2G\,\varepsilon_{xy} = 2G\left[\partial U_x/\partial y + \partial U_y/\partial x\right].{}^1/_2$$

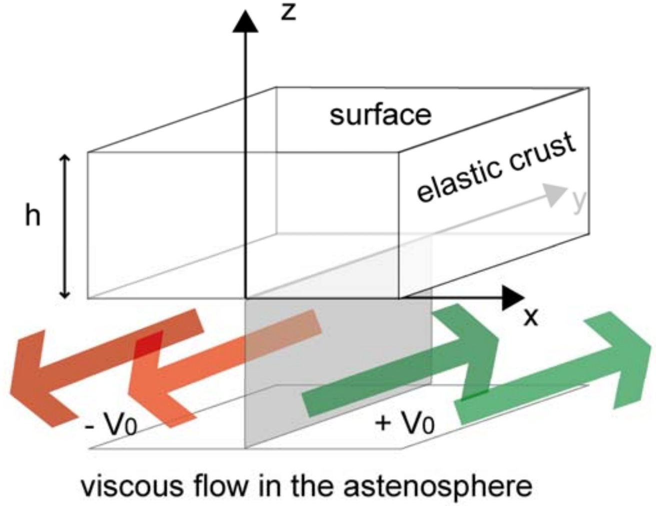
$$\sigma_{yz} = 2G\,\varepsilon_{yz} = 2G\left[\partial U_z/\partial y + \partial U_y/\partial z\right].{}^1/_2$$

Using $\partial\sigma_{xy}/\partial x + \partial\sigma_{yz}/\partial z = 0$ we obtain :

$$\partial/\partial x\left[\cancel{\partial U_x/\partial y} + \partial U_y/\partial x\right] + \partial/\partial z\left[\cancel{\partial U_z/\partial y} + \partial U_y/\partial z\right] = 0$$

↳ $$\partial^2 U_y/\partial x^2 + \partial^2 U_y/\partial z^2 = 0$$

Mathematical formulation



$$d^2U_y/dx^2 + d^2U_y/dz^2 = 0$$

What is U_y, function of x and z, solution of this equation ?

Guess : **U_y = K arctang ($^x/_z$)** works fine !

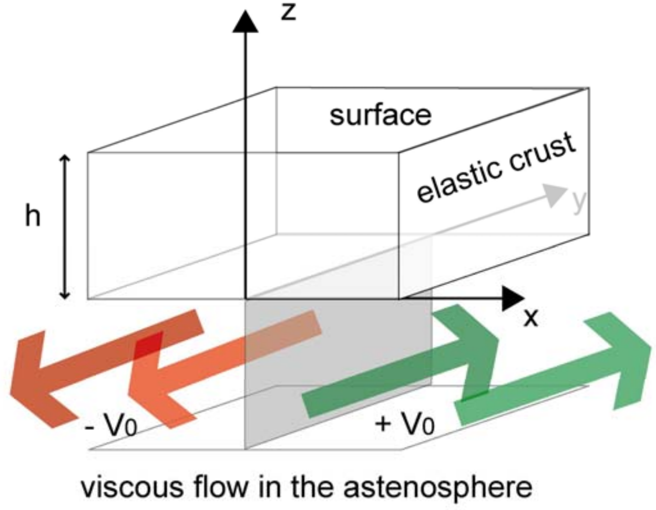
Nb. $\mathrm{datan}(\alpha)/_{d\alpha} = {}^1/_{(1+\alpha 2)}$

$dU_y/dx = {}^K/_{z(1+x^2/z^2)} \Rightarrow d^2U_y/dx^2 = -2Kxz/(z^2+x^2)$

$dU_y/dz = {}^{-Kx}/_{z^2(1+x^2/z^2)} \Rightarrow d^2U_y/dz^2 = 2Kxz/(x^2+z^2)$

Mathematical formulation

$$U_y = K \arctan g\,(x/z)$$



Boundary condition at the base of the crust (z=0)

$U_y \quad = K \,.\, \Pi/2 \quad \text{if } x > 0 \qquad = K \,.\, -\Pi/2 \quad \text{if } x < 0$

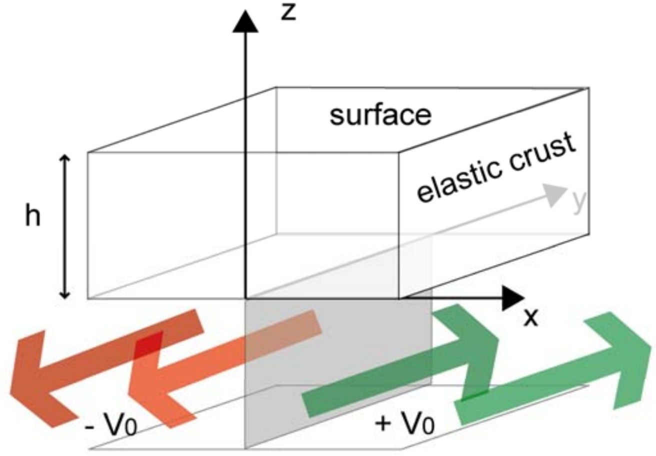
And also :

$U_y \quad = +V_0 \quad \text{if } x > 0 \qquad = -V_0 \quad \text{if } x < 0$

$$\Rightarrow \quad K = 2 \,.\, V_0/\Pi$$

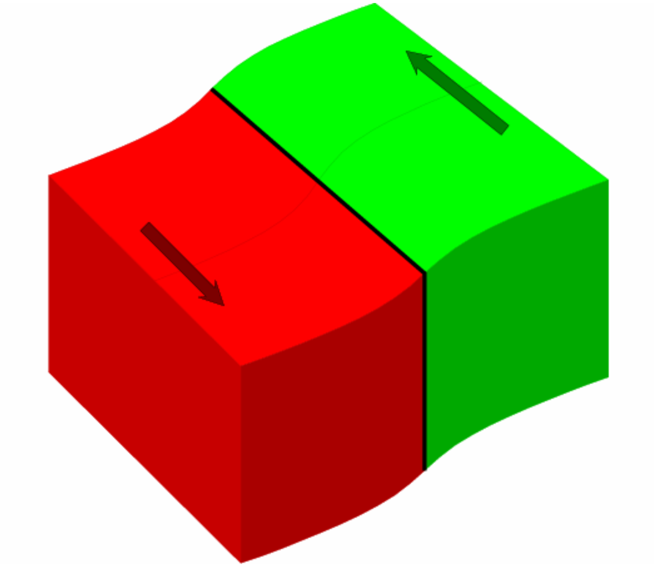
Mathematical formulation

$$U_y = K \arctan g\,(^x/_z)$$



at the surface (z=h)

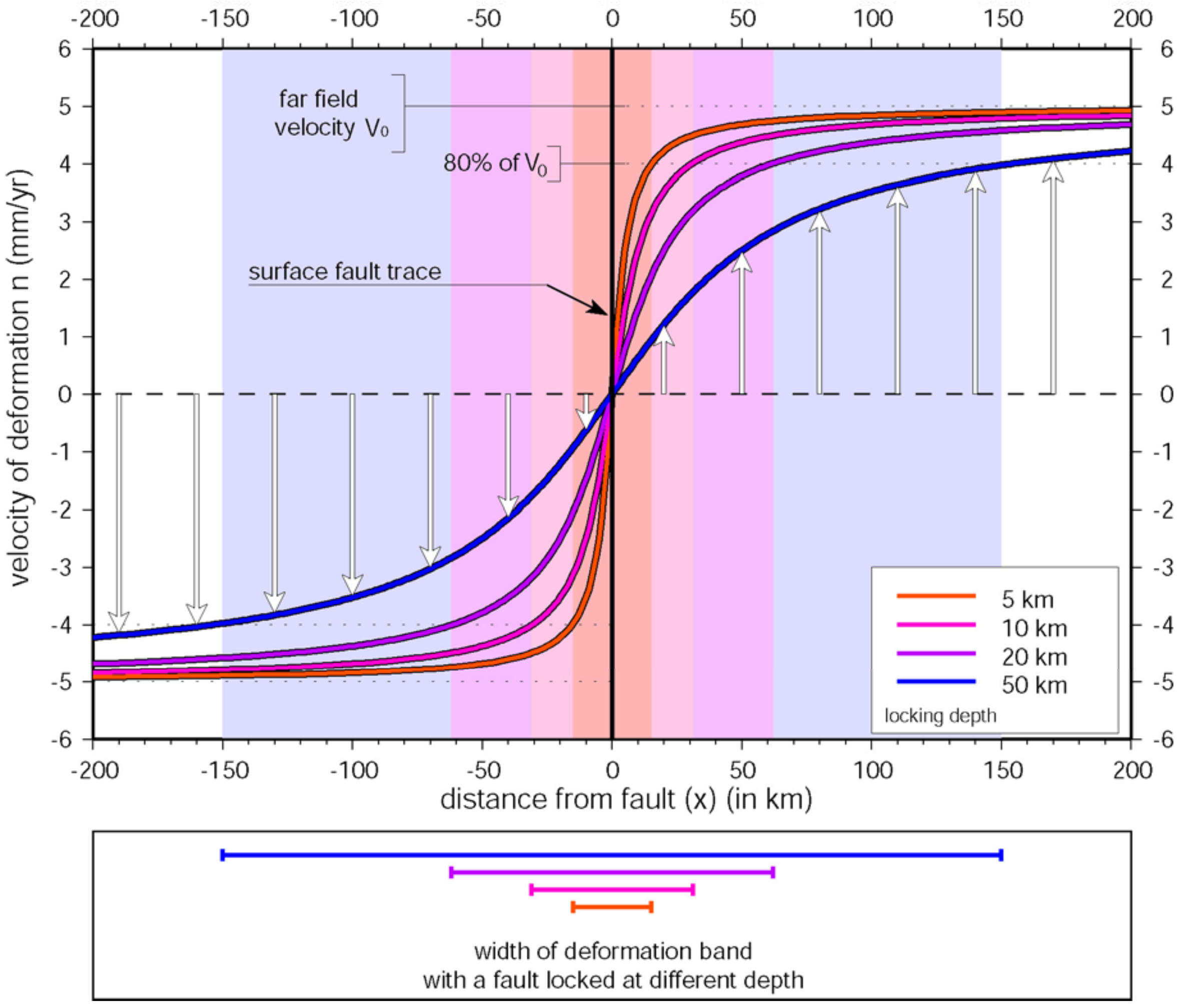
$$U_y = 2\,.\,^{V_0}/_{\Pi} \arctan g\,(^x/_h)$$



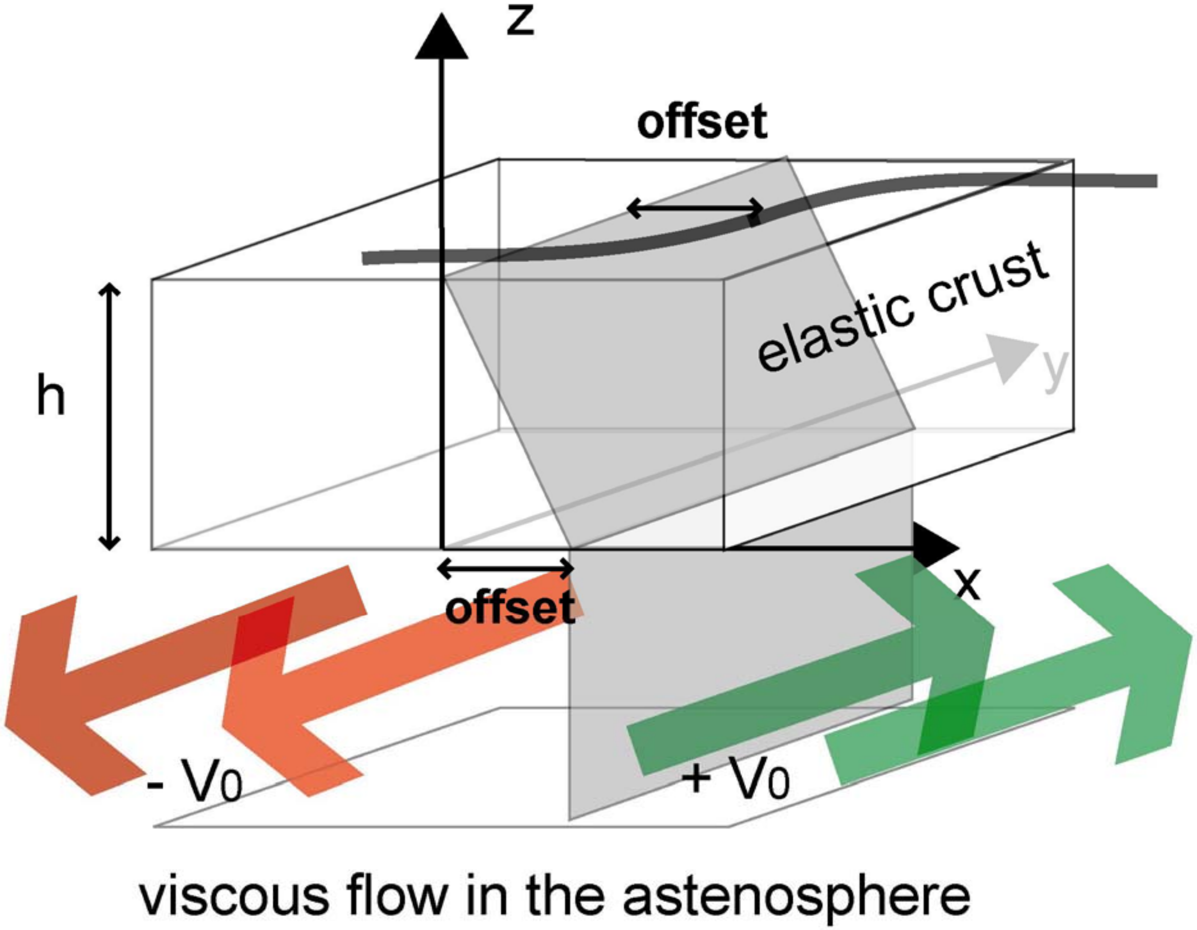
The expected profile of deformation across a strike slip fault we should see at the surface of the earth (if the crust is elastic) is shape like an **arctangant** function. The exact shape depends on the thickness of the elastic crust, also called the **locking depth**.

Arctang profiles

$U_y = 2 \cdot V_0 / \Pi \; \mathrm{arctang}\,(x/h)$



dipping fault



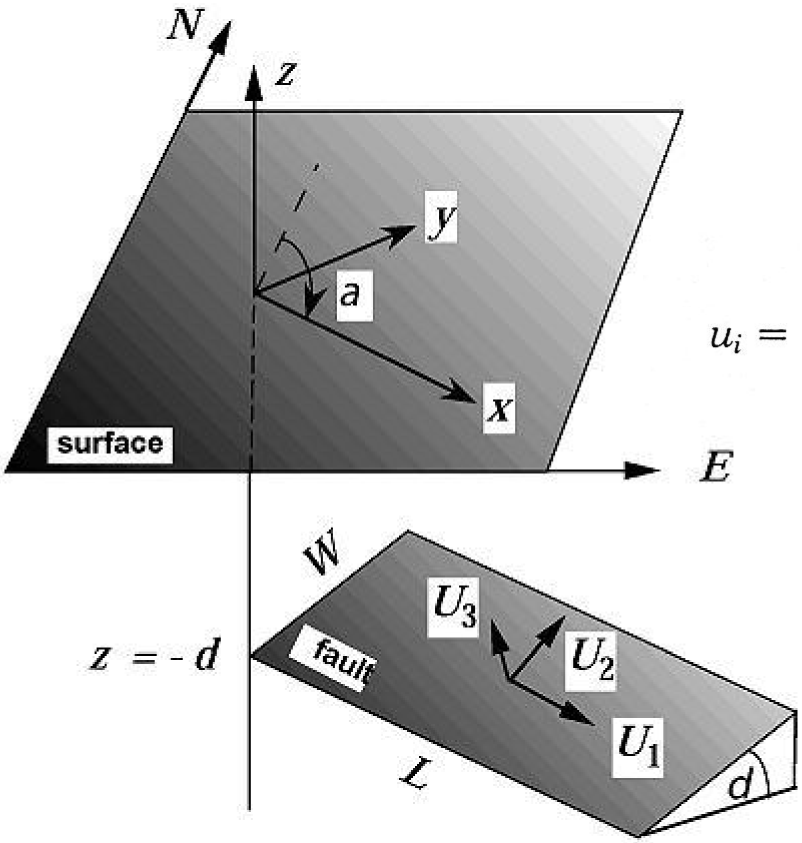
If the fault is not vertical but dipping with a given angle, then the profile at the surface is just the same.

Only the center of the profile is shifted to be at the vertical of the shear flow in the viscous layer.

Then, the geological trace of the fault is not the place where the shear gradient is maximum

Elastic dislocation (Okada, 1985)

Surface deformation due to shear and tensile faults in a half space, BSSA vol75, n°4, 1135-1154, 1985.



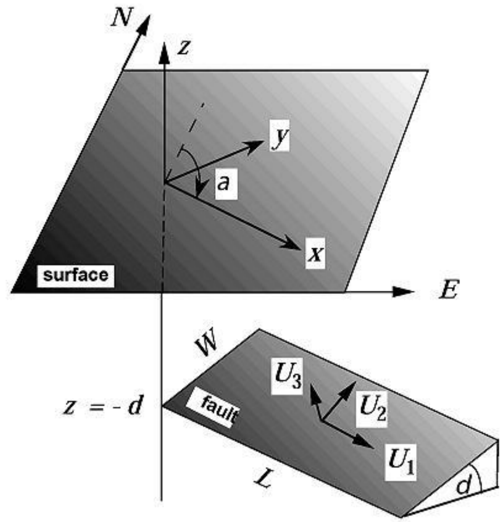
The displacement field $u_i(x_1,x_2,x_3)$ due to a dislocation $\Delta\ u_j\ (\xi_1,\xi_2,\xi_3)$ across a surface Σ in an isotropic medium is given by :

$$u_i = \frac{1}{F}\int\int_\Sigma \Delta u_j \left[\lambda\delta_{jk}\frac{\partial u_i^n}{\partial\xi_n} + \mu\left(\frac{\partial u_i^j}{\partial\xi_k} + \frac{\partial u_i^k}{\partial\xi_j}\right)\right]\nu_k\, d\Sigma$$

Where δ_{jk} is the Kronecker delta, λ and μ are Lamé's parameters, ν_k is the direction cosine of the normal to the surface element $d\Sigma$.

u_i^j is the ith component of the displacement at (x_1,x_2,x_3) due to the jth direction point force of magnitude F at (ξ_1,ξ_2,ξ_3)

Elastic dislocation (Okada, 1985)



(1) displacements

For strike-slip

$$\begin{cases} u_x^0 = -\dfrac{U_1}{2\pi}\left[\dfrac{3x^2q}{R^5} + I_1^0 \sin\delta\right]\Delta\Sigma \\ u_y^0 = -\dfrac{U_1}{2\pi}\left[\dfrac{3xyq}{R^5} + I_2^0 \sin\delta\right]\Delta\Sigma \\ u_z^0 = -\dfrac{U_1}{2\pi}\left[\dfrac{3xdq}{R^5} + I_4^0 \sin\delta\right]\Delta\Sigma. \end{cases}$$

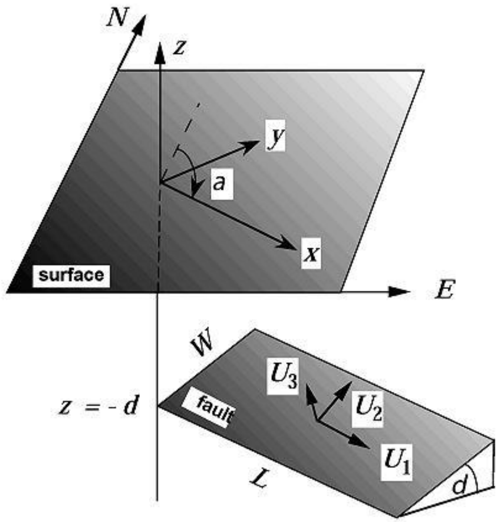
For dip-slip

$$\begin{cases} u_x^0 = -\dfrac{U_2}{2\pi}\left[\dfrac{3xpq}{R^5} - I_3^0 \sin\delta\cos\delta\right]\Delta\Sigma \\ u_y^0 = -\dfrac{U_2}{2\pi}\left[\dfrac{3ypq}{R^5} - I_1^0 \sin\delta\cos\delta\right]\Delta\Sigma \\ u_z^0 = -\dfrac{U_2}{2\pi}\left[\dfrac{3dpq}{R^5} - I_5^0 \sin\delta\cos\delta\right]\Delta\Sigma. \end{cases}$$

For tensile fault

$$\begin{cases} u_x^0 = \dfrac{U_3}{2\pi}\left[\dfrac{3xq^2}{R^5} - I_3^0 \sin^2\delta\right]\Delta\Sigma \\ u_y^0 = \dfrac{U_3}{2\pi}\left[\dfrac{3yq^2}{R^5} - I_1^0 \sin^2\delta\right]\Delta\Sigma \\ u_z^0 = \dfrac{U_3}{2\pi}\left[\dfrac{3dq^2}{R^5} - I_5^0 \sin^2\delta\right]\Delta\Sigma \end{cases}$$

Elastic dislocation (Okada, 1985)

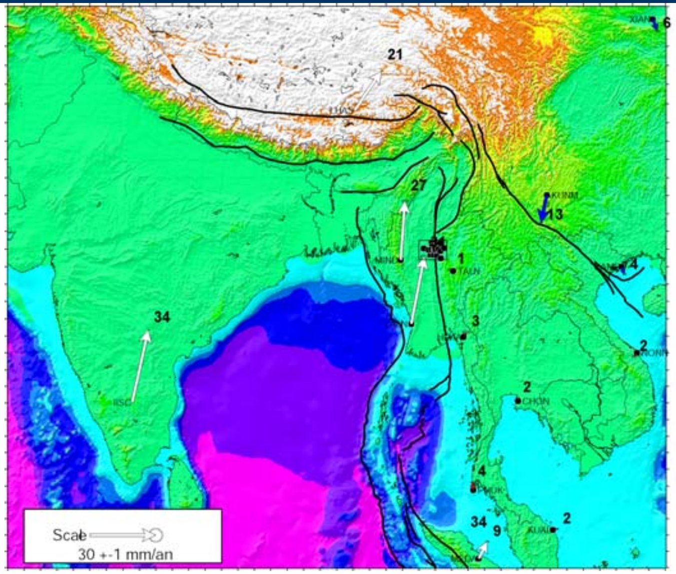


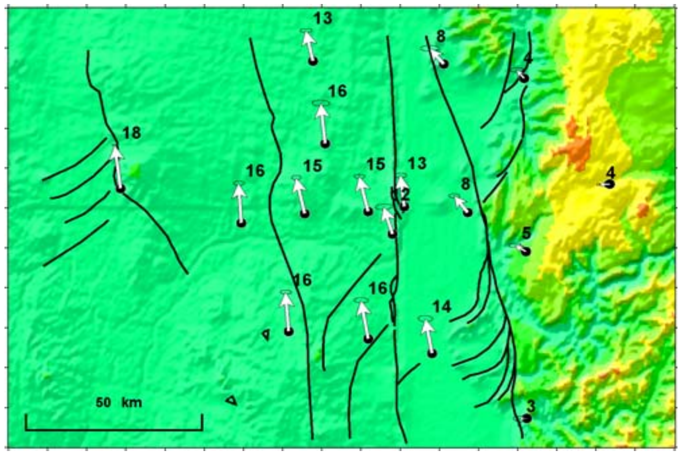
Where :

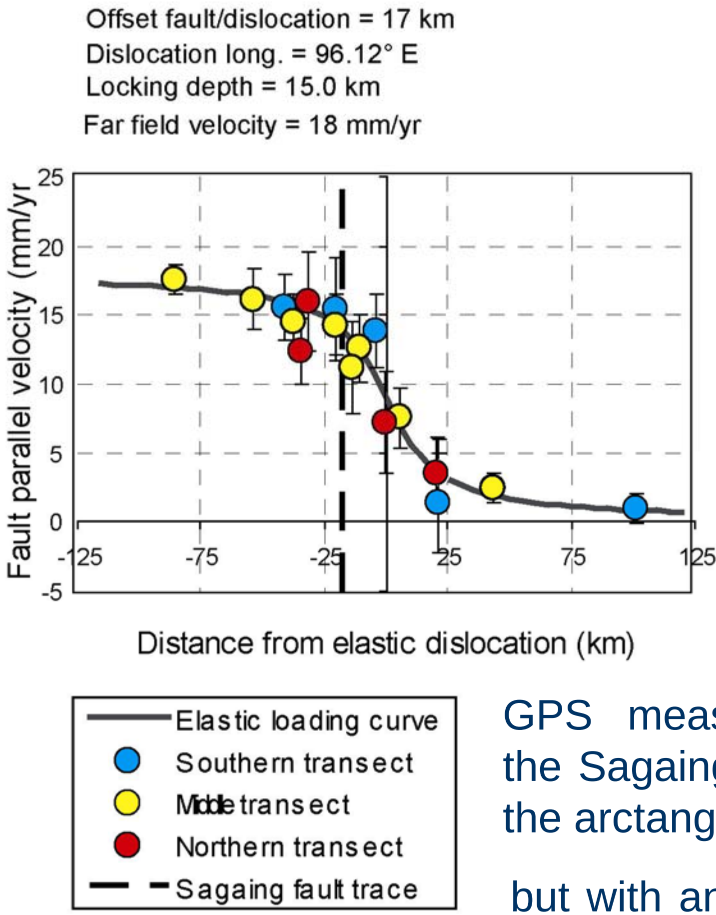
$$
\begin{cases}
I_1^{\,0} = \dfrac{\mu}{\lambda + \mu}\, y\left[\dfrac{1}{R(R+d)^2} - x^2 \dfrac{3R+d}{R^3(R+d)^3}\right] \\[2ex]
I_2^{\,0} = \dfrac{\mu}{\lambda + \mu}\, x\left[\dfrac{1}{R(R+d)^2} - y^2 \dfrac{3R+d}{R^3(R+d)^3}\right] \\[2ex]
I_3^{\,0} = \dfrac{\mu}{\lambda + \mu}\left[\dfrac{x}{R^3}\right] - I_2^{\,0} \\[2ex]
I_4^{\,0} = \dfrac{\mu}{\lambda + \mu}\left[\qquad - xy \dfrac{2R+d}{R^3(R+d)^2}\right] \\[2ex]
I_5^{\,0} = \dfrac{\mu}{\lambda + \mu}\left[\dfrac{1}{R(R+d)} - x^2 \dfrac{2R+d}{R^3(R+d)^2}\right]
\end{cases}
$$

$$
\begin{cases}
p = y \cos\delta + d \sin\delta \\
q = y \sin\delta - d \cos\delta \\
R^2 = x^2 + y^2 + d^2 = x^2 + p^2 + q^2.
\end{cases}
$$

Sagaing Fault, Myanmar



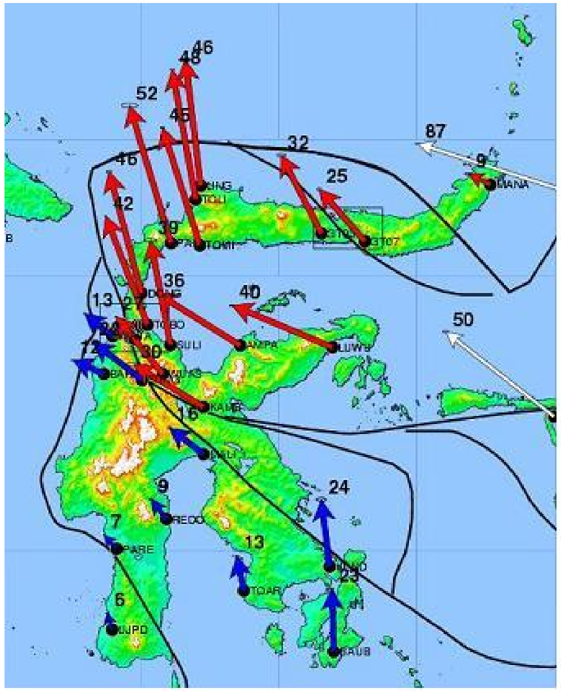


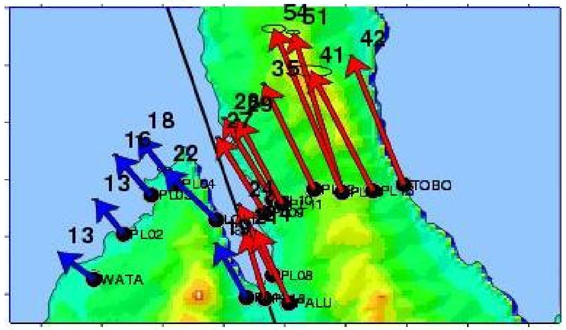


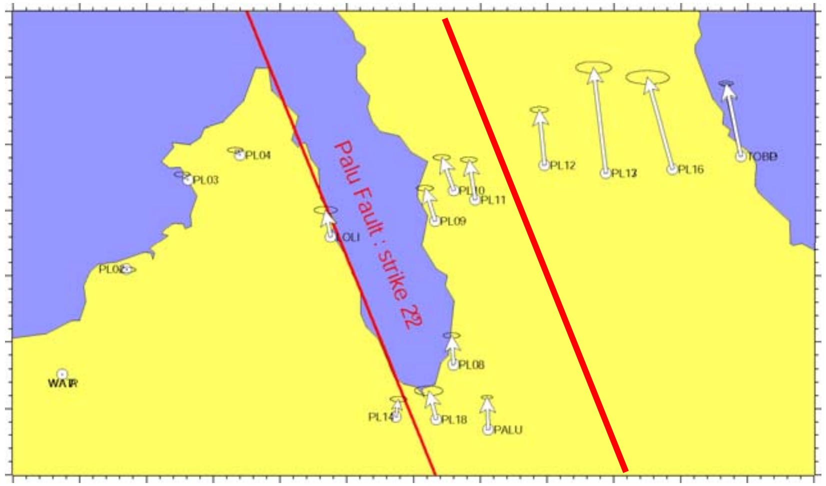
GPS measurement on the Sagaing fault fit well the arctang profile

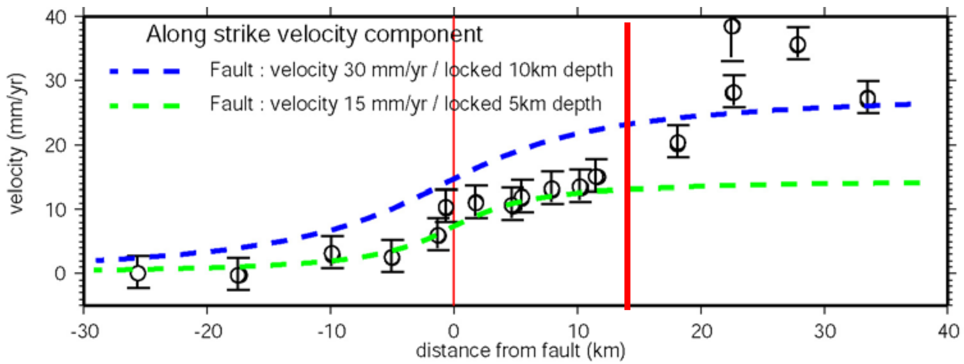
but with an offset of 10-15 km

Palu Fault, Sulawesi



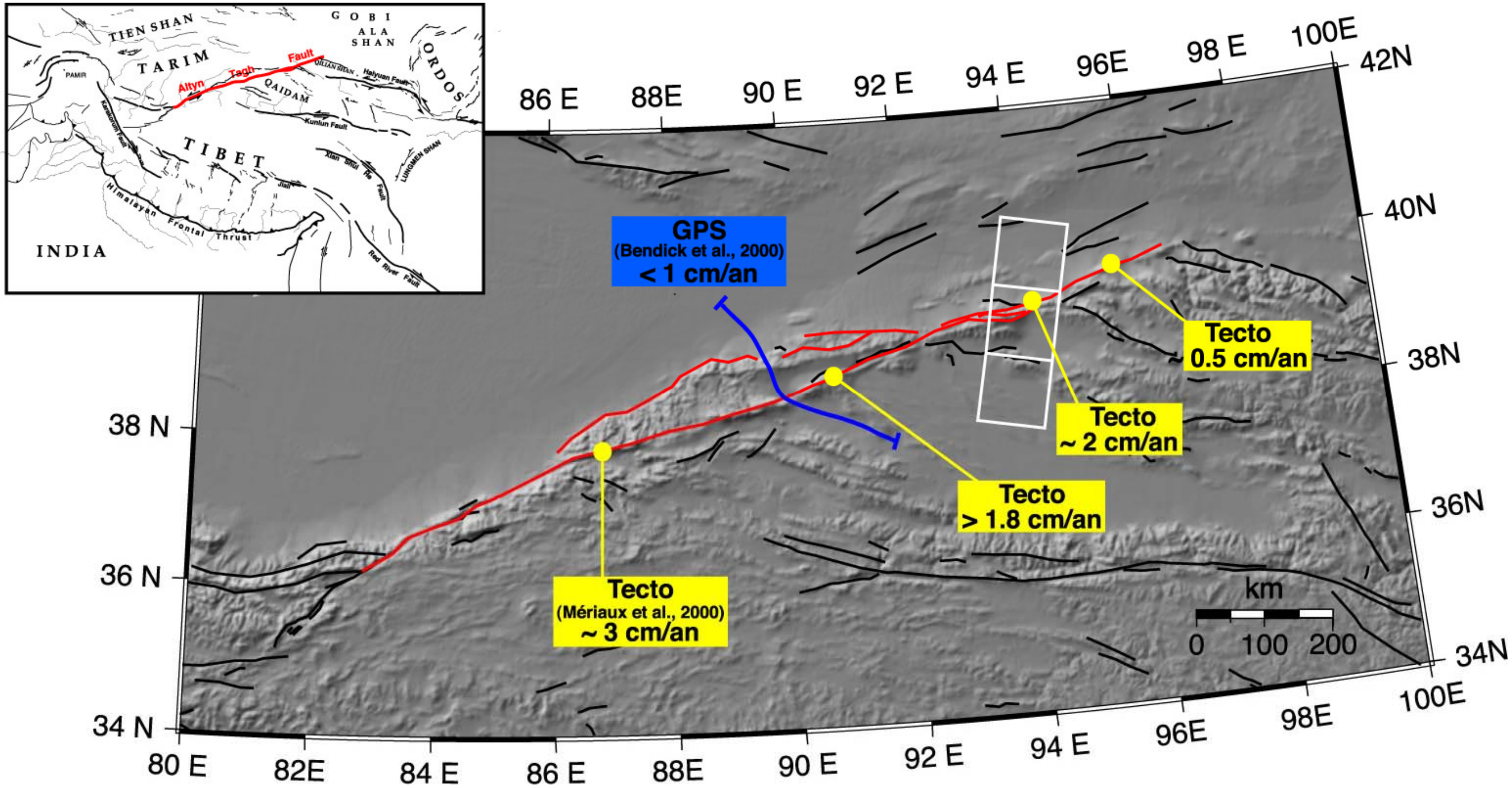






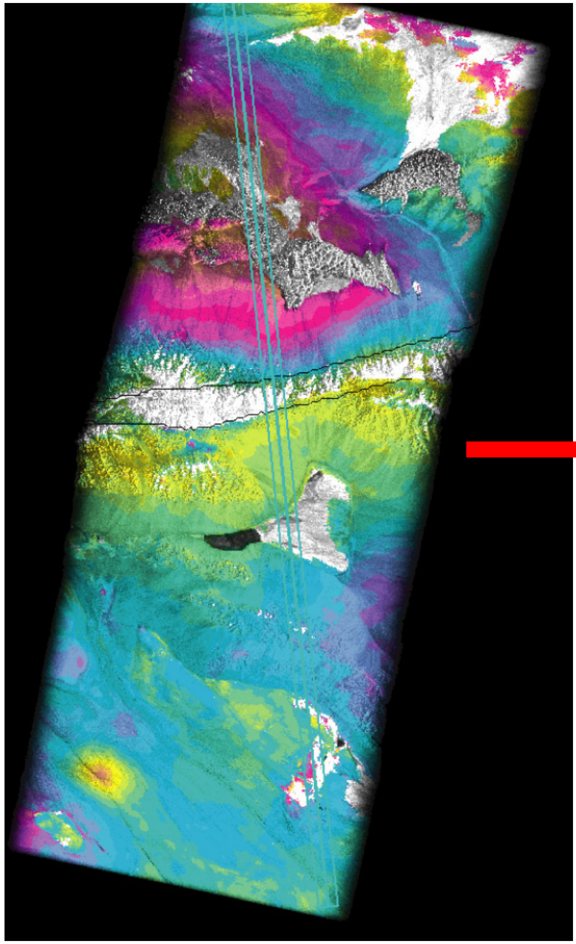
Part of the GPS data on Palu fault fits well an arctang profile. But wee need a second fault to explain all the data

Altyn Tagh Fault, China



Altyn Tagh Fault, China (INSAR)

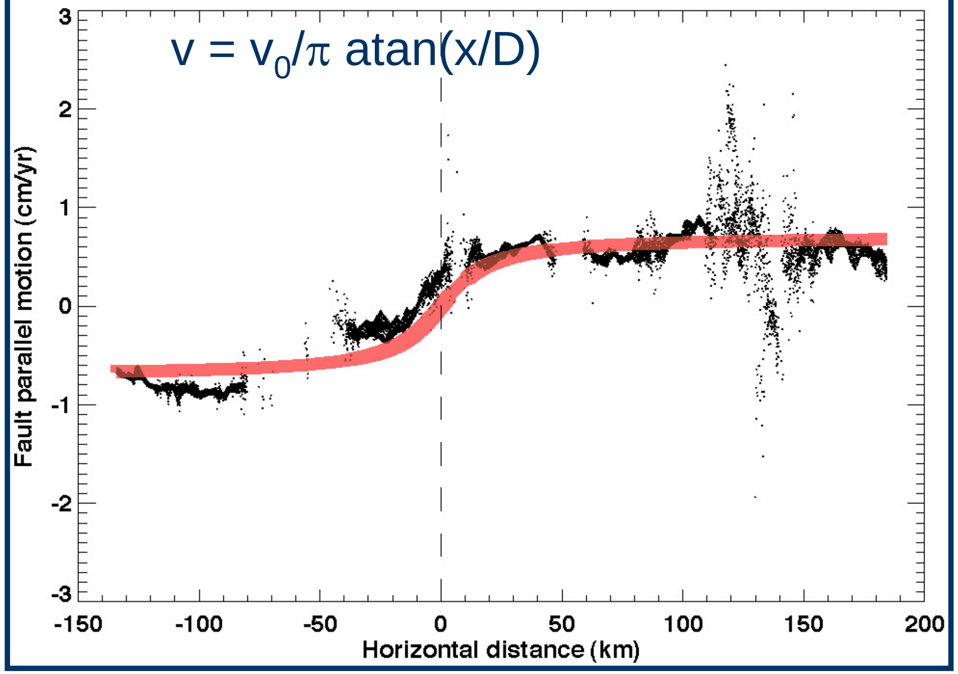
Interferogram Nov. 1995/ Nov. 1999



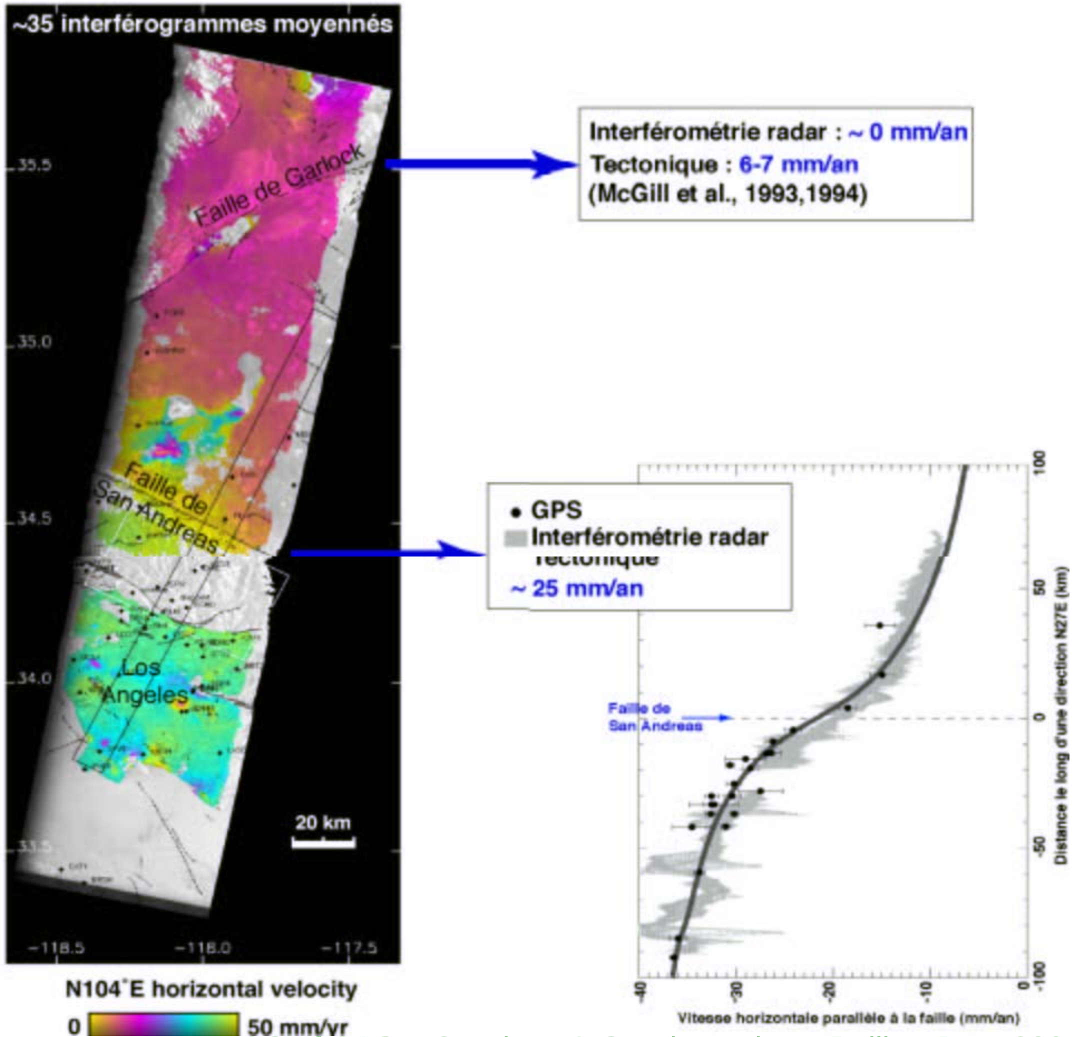
1 color cycle = 28 mm LOS displacement

Fault-parallel velocity :

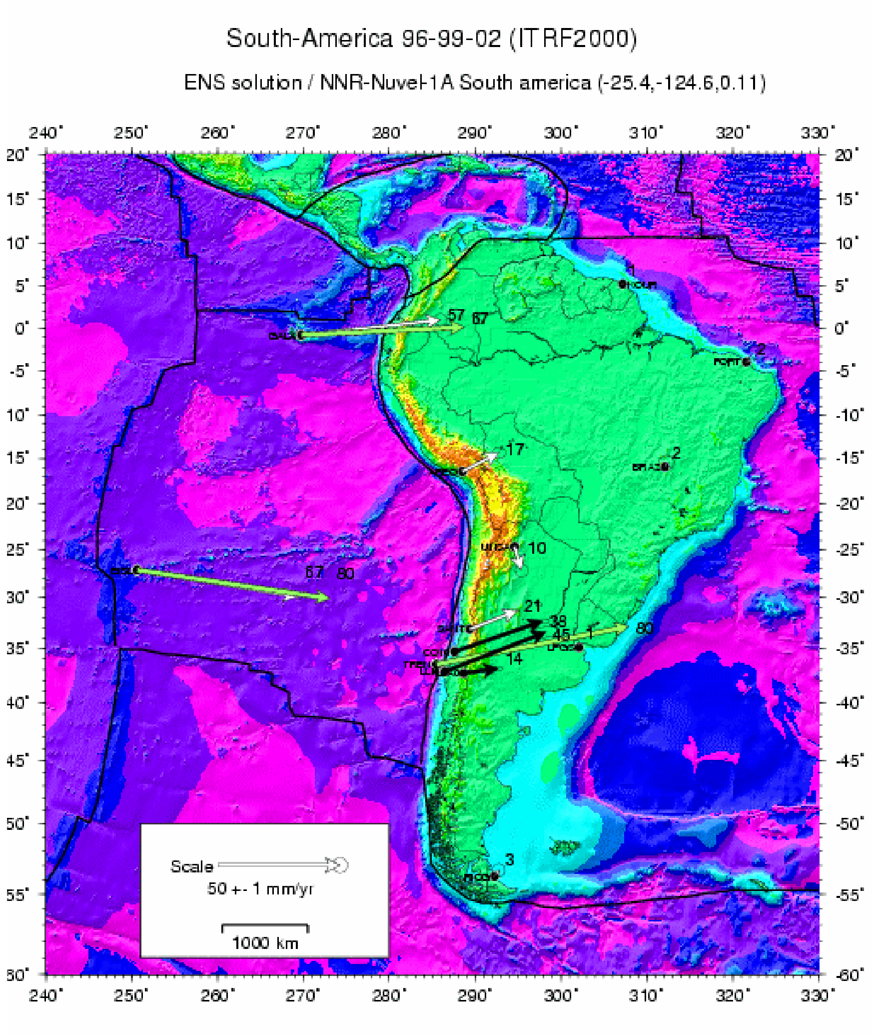
Slip rate V_0 = 1.4 cm/yr
Locking depth D = 15 km

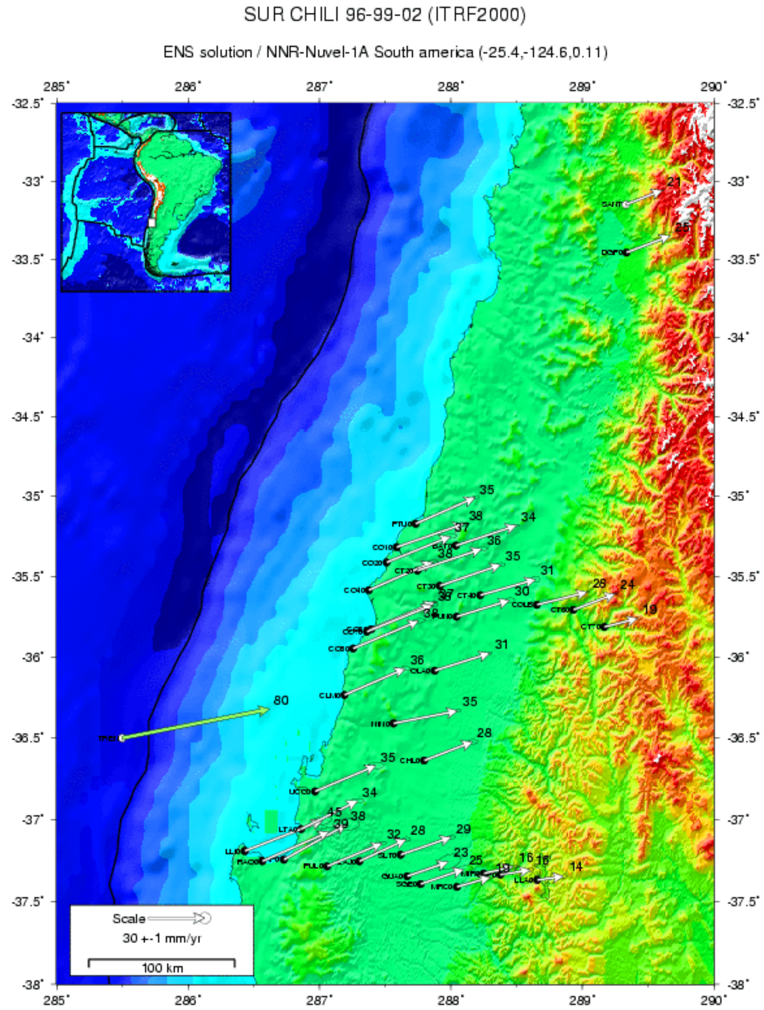


San Andreas Fault, USA (INSAR)

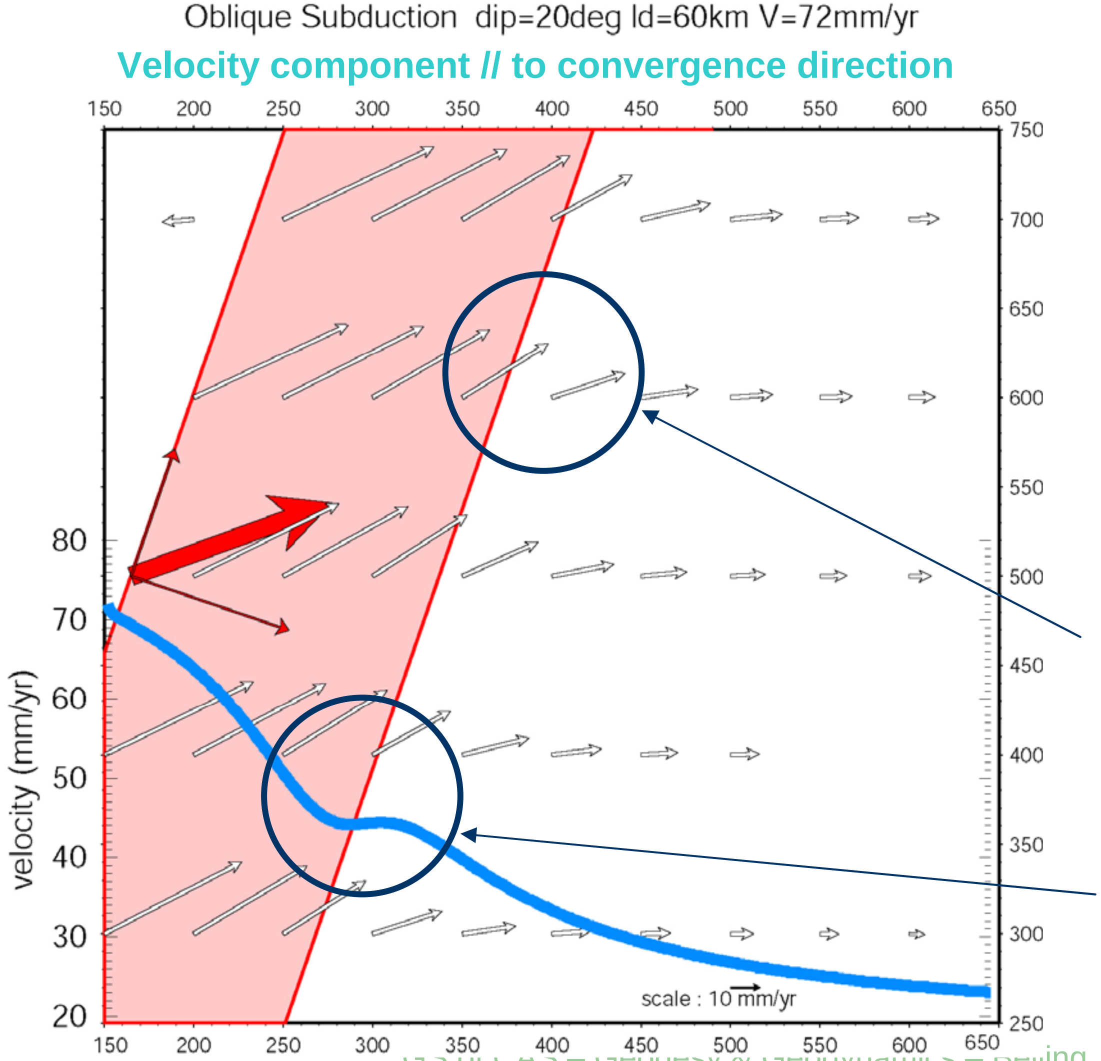


Subduction in south America





Subduction modeling



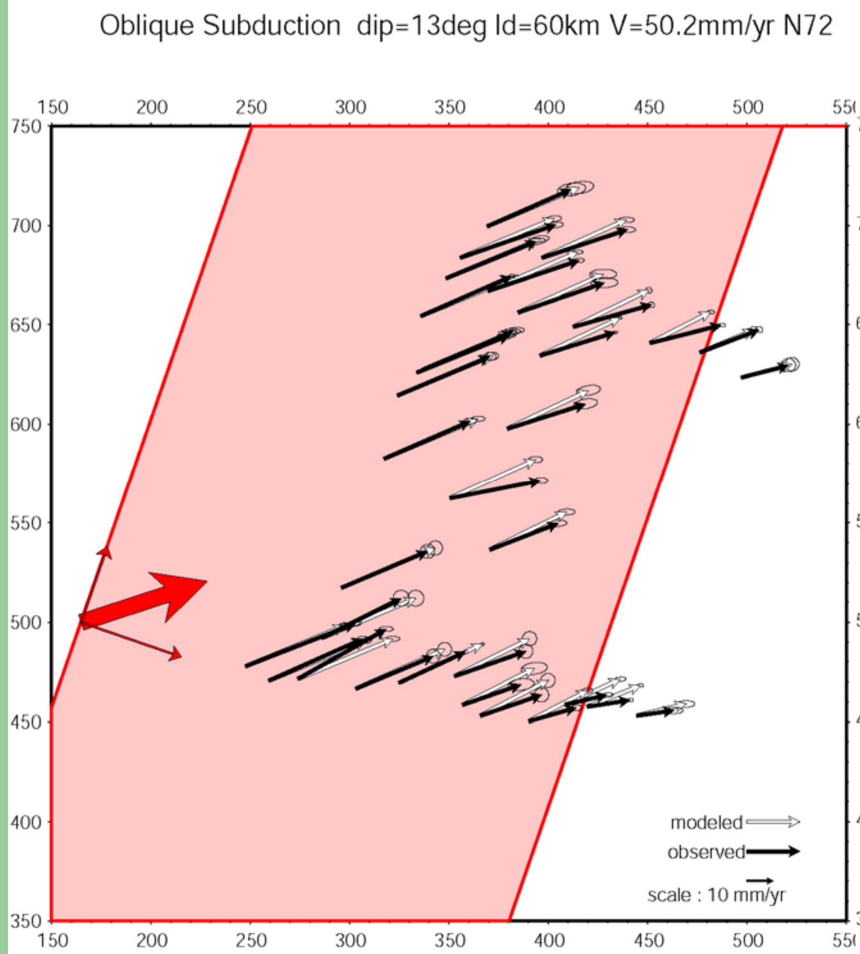
In the case of a subduction (dippping fault with downward slip) we use Okada's formulas.

We find a very large deformation area (> 500 km) because the dipping angle is only 22°

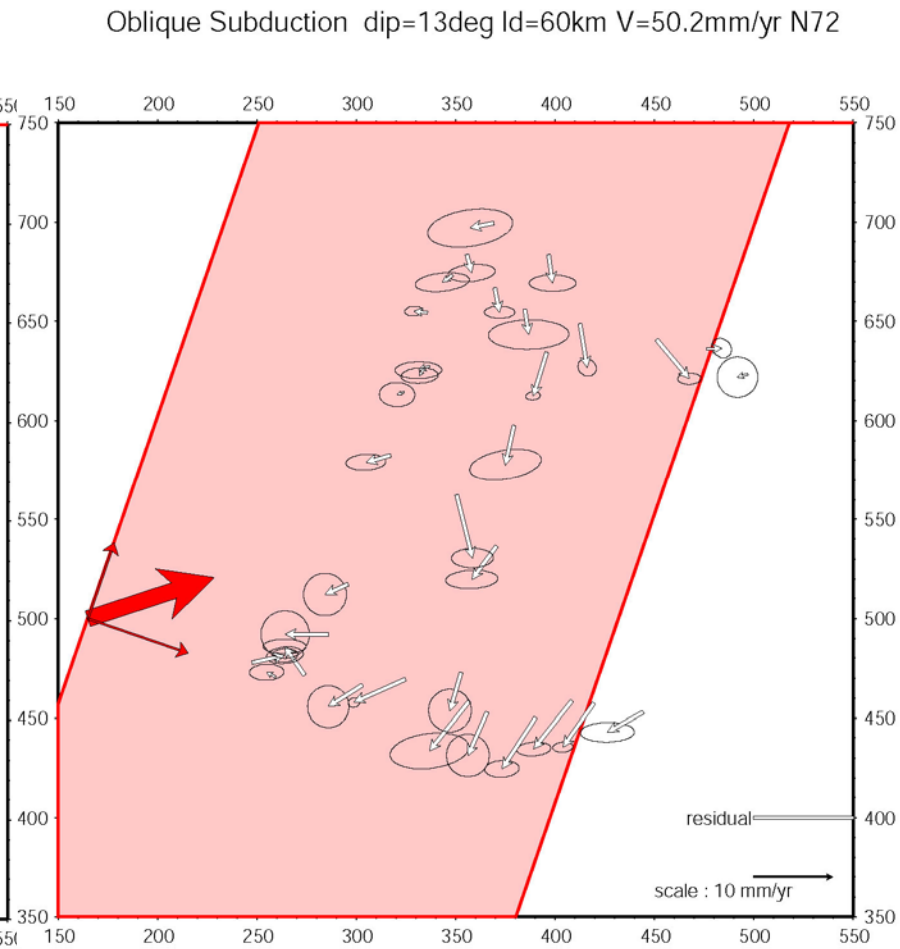
With oblique slip we predict the surface vector will start to rotate at the vertical of the end of the subduction plane

The profile of the velocity component // to the convergence shows this with a flat portion at this location

Subduction parameter adjustments



Model and data



Residuals