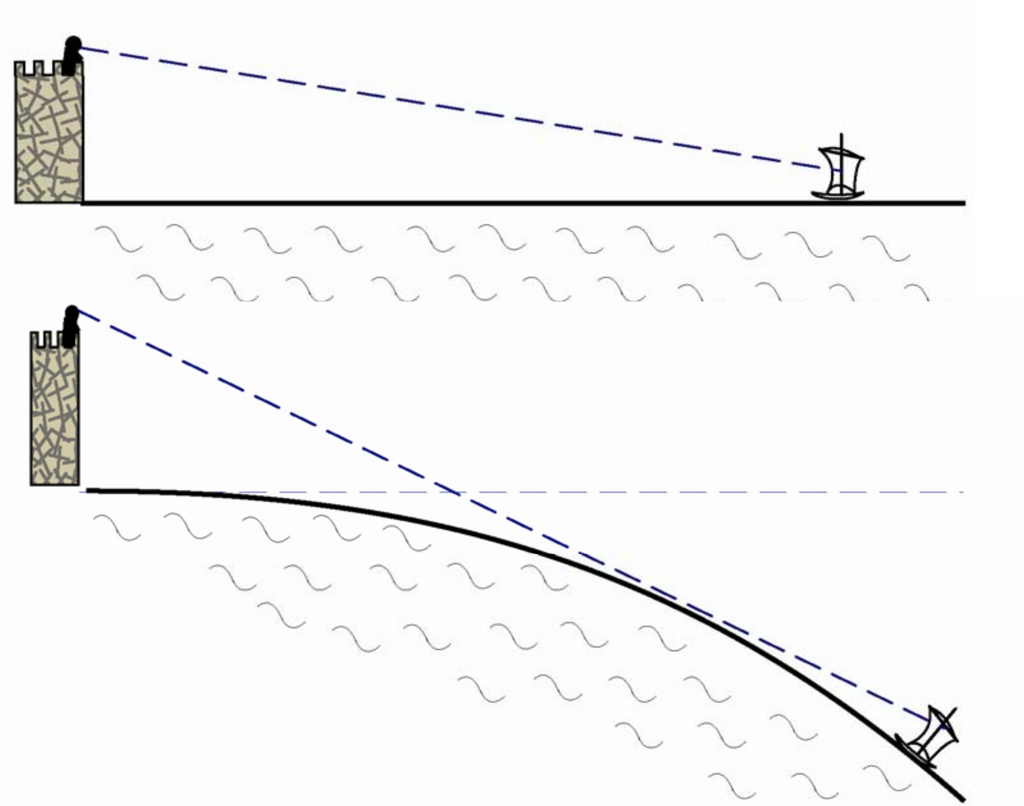
Ancient times Geodesy (6 century bc)

- Geodesy is a very old science. It comes from the first question mankind ask themselves : **what is the shape and the size of the earth ?**



If the Earth were flat, then one could see very far

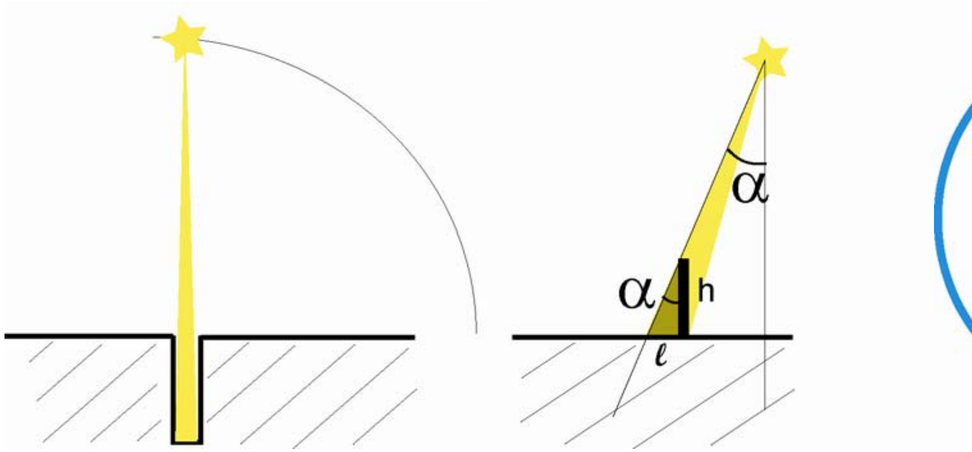
=> no horizon

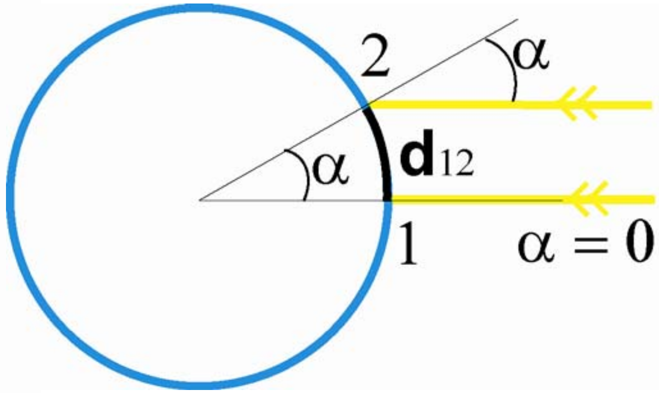
Because there is an horizon (i.e. objects disappear below the horizon)

⇒ **Earth is spherical**

Ancient times Geodesy (Eratosthene, 300 bc)

Size of the Earth : **circ = 360°/α * d_{12}** = **40000 km**





At one place on Earth, the Sun is vertical (lights the bottom of a well) only once a year

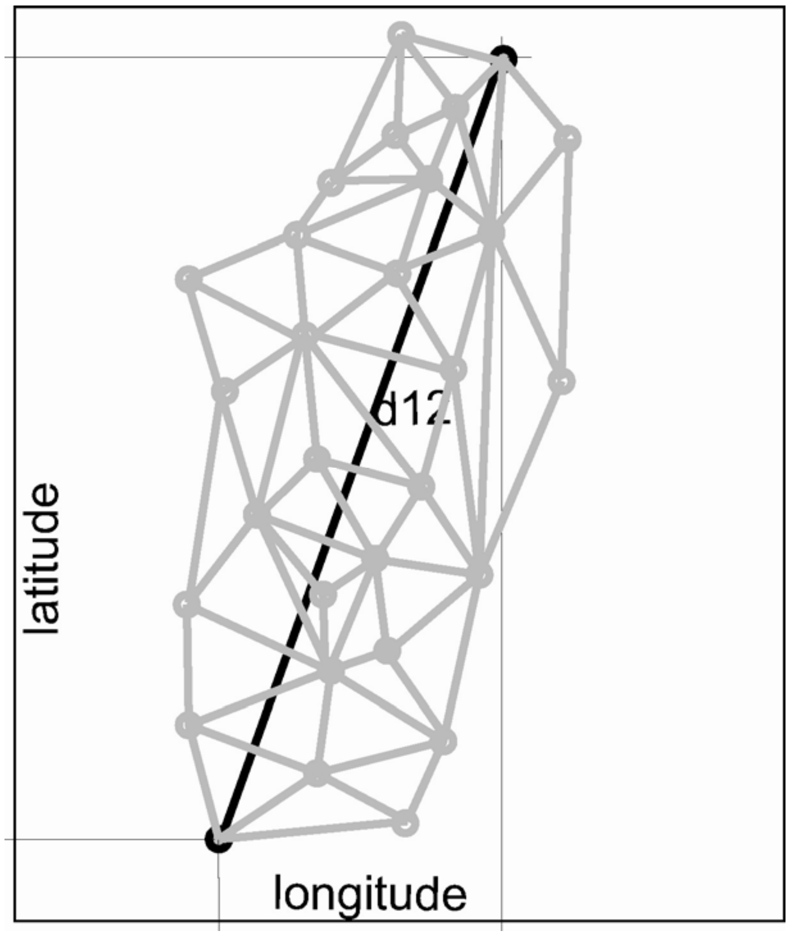
At the same time, at a different place, the Sun is not vertical

The angle can be measured from the length of the shadow of a vertical pole

The angle α of the sun light direction depends on the **local vertical** direction

=> Depends on the **latitude** of the site

«Modern» Geodesy (17th century)



A correction has to be made if distance is **not aligned** with longitude

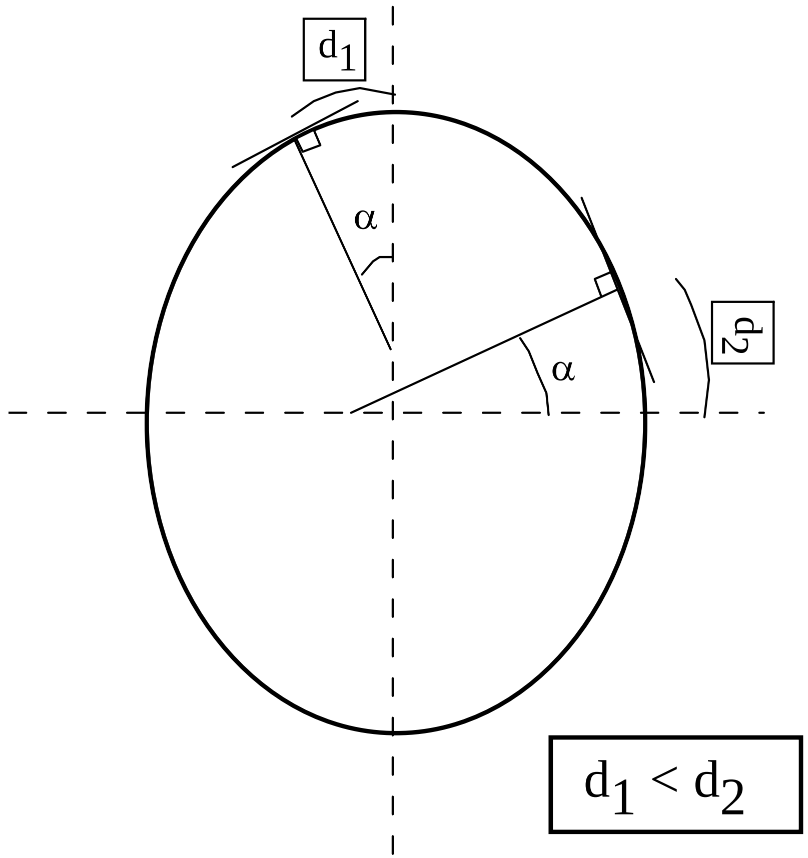
d_{12} can be computed from the **sum** (oriented) of many smaller distances

Measuring many (if not all) **distances** and **angles** within a network of points give the more accurate solution for d_{12}

The shape of the Earth (18th century)

Making those measurements, different people find different values for the length of an arc of **1°** at different places in Europe

- **Snellius (1617) : 104 km**
- **Norwood (1635) : 109 km**
- **Riccioli (1661) : 119 km**



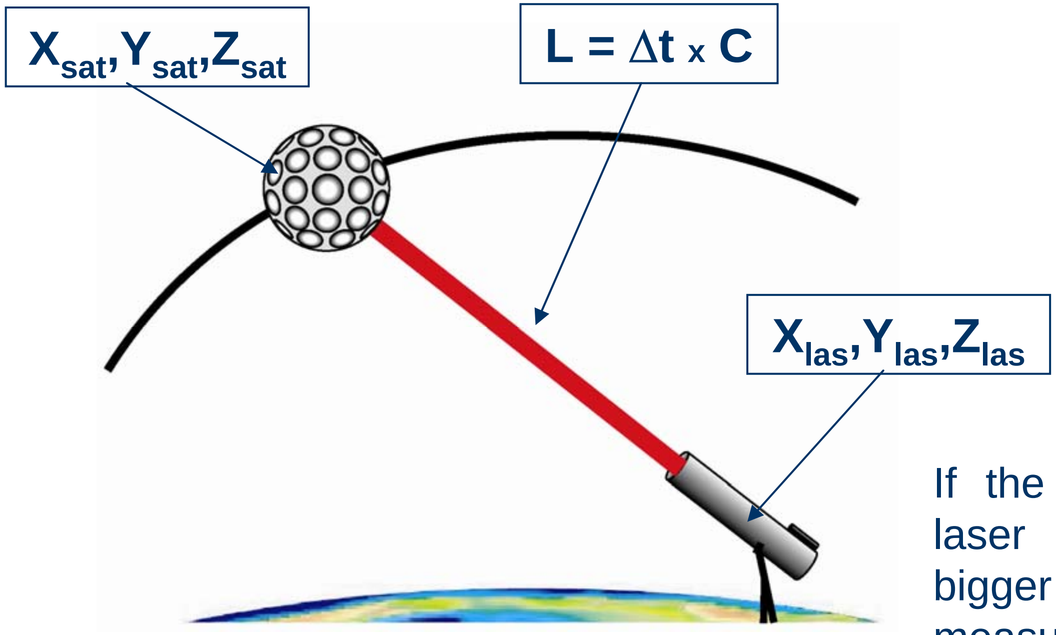
In France, **Picard** finds :

- **108 km** in the **north** of France
- **110 km** in the **south** of France

Earth surface deformation

Satellite Laser Ranging

High energy laser firing at satellites enable to determine the position of the satellite and then the Geoid, assuming the station position is know. On reverse, assuming one knows the satellite position (i.e. the earth gravity field), then by measuring the satellite-station distance one can determine the station position. The time is measured with a precision of about **0.1ns to 0.3 ns** (3.10^{-10} sec), which give a precision of about **3 to 10 cm** on the measured length, hence on the station position.



$$X_{las} = X_{sat} - L_x$$
$$Y_{las} = Y_{sat} - L_y$$
$$Z_{las} = Z_{sat} - L_z$$

$$pos_{las} = pos_{sat}(t_i) - L(t_i)$$

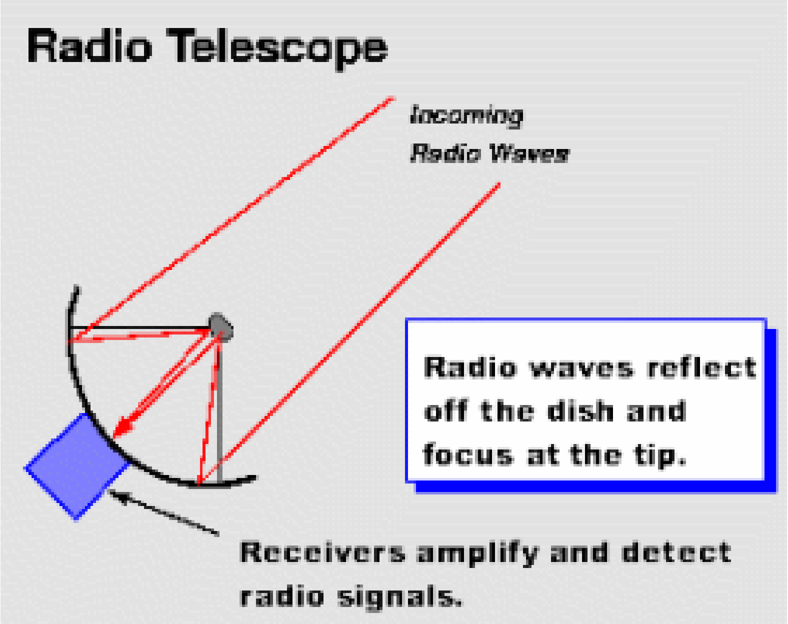
With : ti = time of ith measurement along the orbit

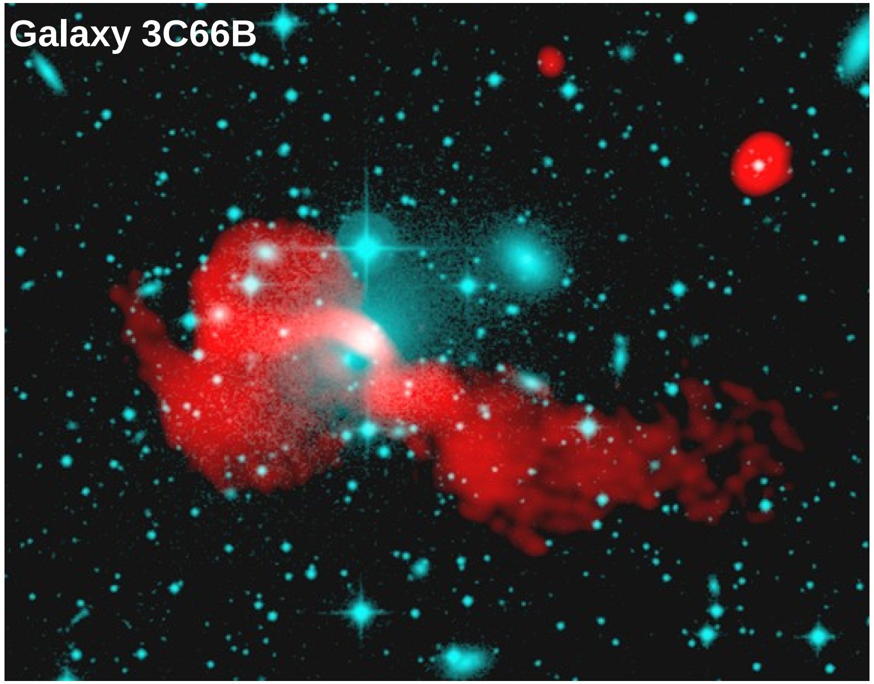
If the earth surface deforms, then the laser station moves. If this motion is bigger than a few cm, then the measurement detects it !

Earth surface deformation

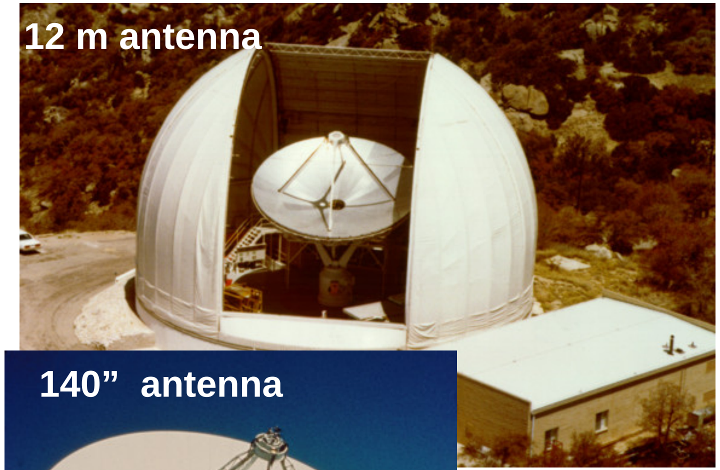
Radio Telescope principle

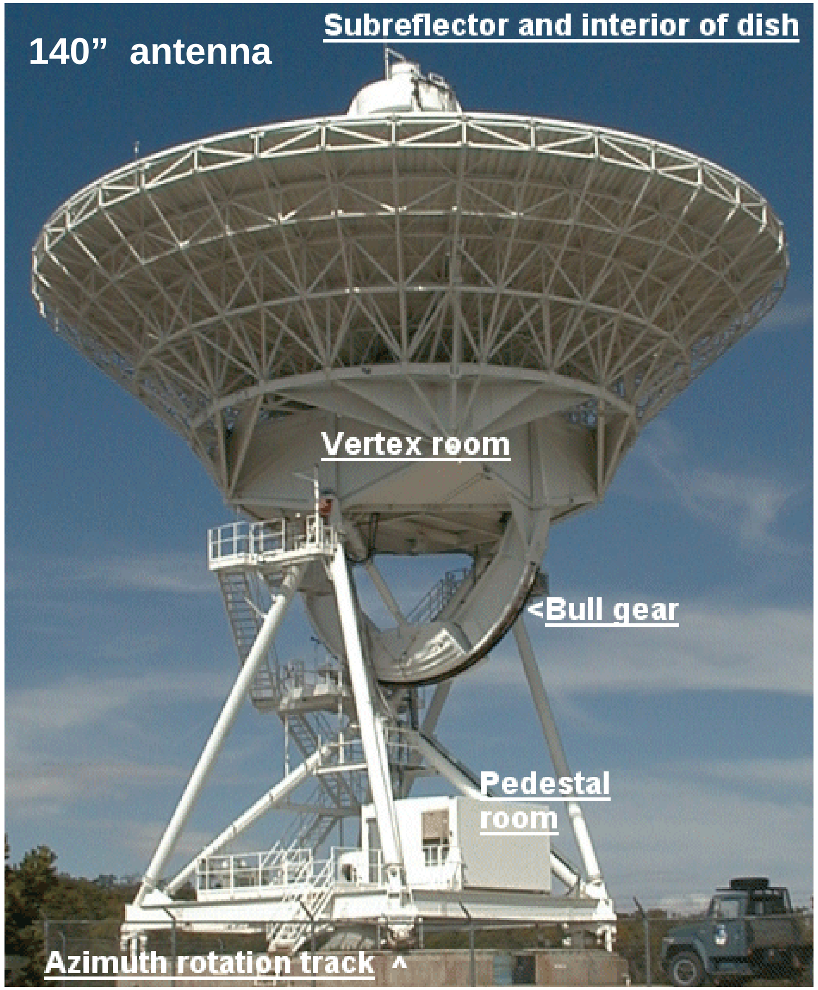
Radio telescopes are used to study naturally occurring radio emission from stars, galaxies, quasars, and other astronomical objects between wavelengths of about 10 meters (30 megahertz [MHz]) and 1 millimeter (300 gigahertz [GHz]). At wavelengths longer than about 20 centimeters (1.5 GHz), irregularities in the ionosphere distort the incoming signals. Below wavelengths of a few centimeters, absorption in the atmosphere becomes increasingly critical. the effective angular resolution and image quality **is limited only by the size of the instrument.**





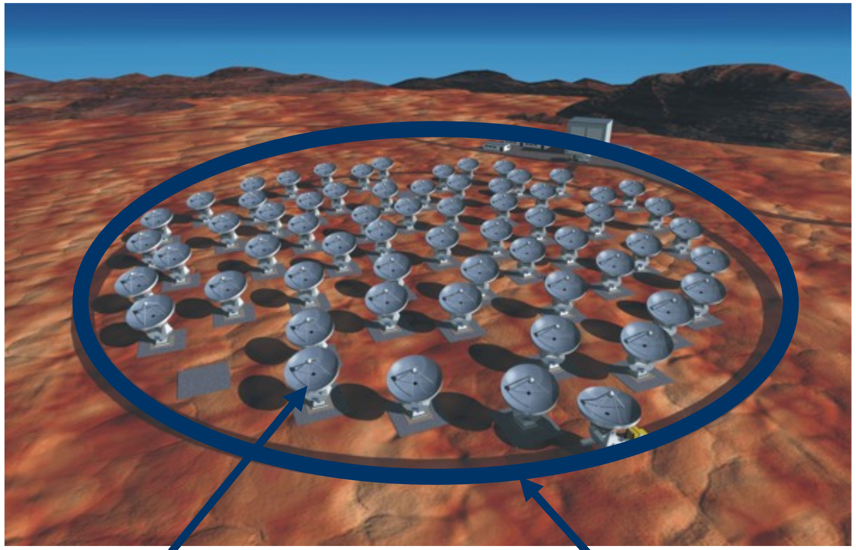
Bigger antennas





Very Large Base Interferometry (VLBI)

It is extremely difficult to built antennas bigger than 20-30 meters diameter…
But, one **single large** mirror (or antenna) can be replace by **many small** mirrors (or antenna). The size of the image wills be equivalent. Thus, an array of small antennas make a **virtual** big antenna of equivalent size the size of the array.

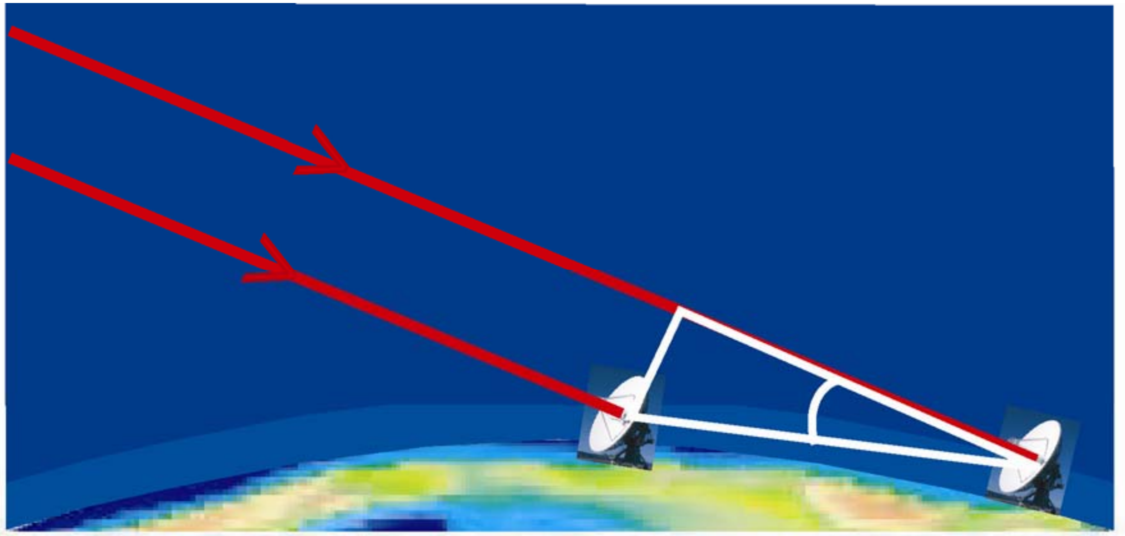


Single small antenna

virtual antenna

Very Large Base Interferometry (VLBI)

One can reconstruct a precise image of the observed object, knowing precisely the distances between the individual antennas. If these distances are not well known, then the image is fuzzy.
Again, reversing the problem, focusing a known image allow to determine the distances between stations.



The radio wavelength arrives at first antenna at time **t**, and at the second antenna at time **t + Δt**.

The additional distance is : **Δt .C**

Which we can easily convert into distance between stations (knowing the angle=difference in latitude)

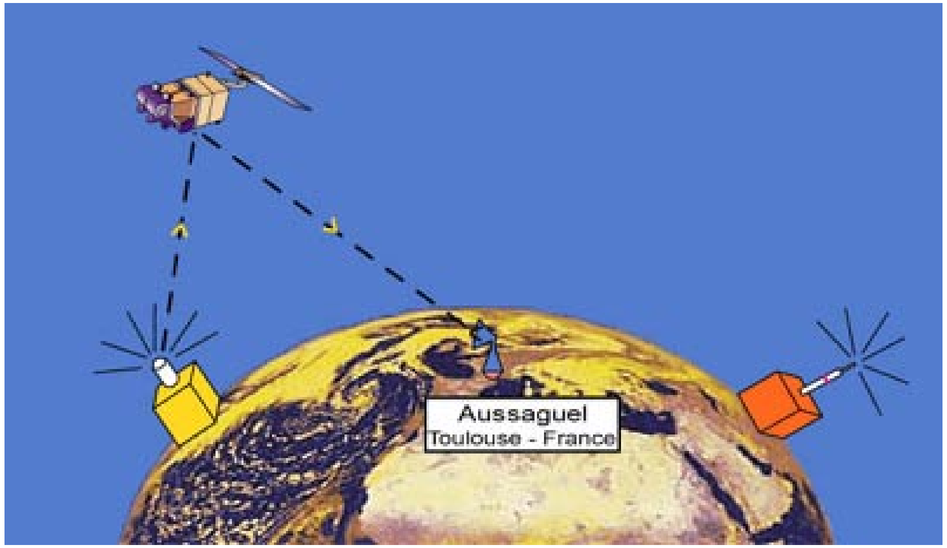
The obtained precision is around **1 millimeter** !

DORIS (Doppler system)

A wavelength is broadcasted by a ground station with a given frequency. A satellite is receiving this signal. Because the satellite is moving, the frequency it receives is shifted. This is the Doppler effect.

For a velocity **v**, the frequency ν will be shifted by a quantity equal to νx **v/c**

The complete formula for **V** not // to line of view is : $\nu' = \nu \frac{1 - \cos\phi \frac{v}{c}}{\sqrt{1 - \frac{v^2}{c^2}}}$



For a satellite velocity and position are linked by the Keplerian equation of its orbit.
Thus, measuring the Doppler shift allows to determine the Station to Satellite distance

DORIS (Doppler system)

The obtained precision on station position is around 1-3 cm

DORIS GLOBAL network
~60 stations covering the whole Globe

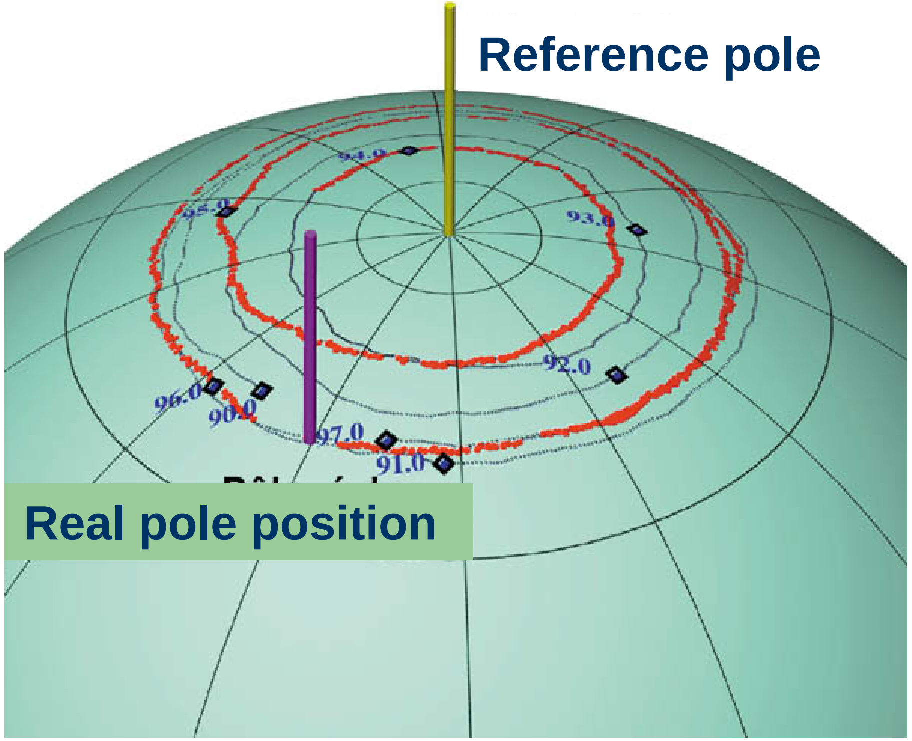
 

DORIS beacons

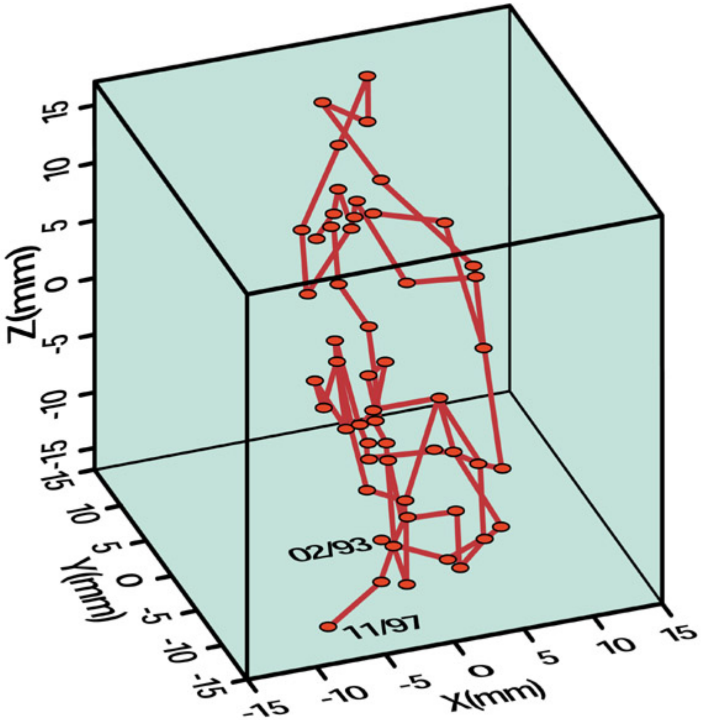
DORIS (Doppler system)

DORIS allow to detect motion of stations but also the motion of the whole network (as a polyhedron) in space. Thus we can determine the **oscillations of planet Earth.** These oscillations have a complex frequency contains from Milankovitch period (26 000 years) to Chandler Wobble (400 days) and daily adjustments due to atmospheric loads

Motion of the Earth axis of rotation

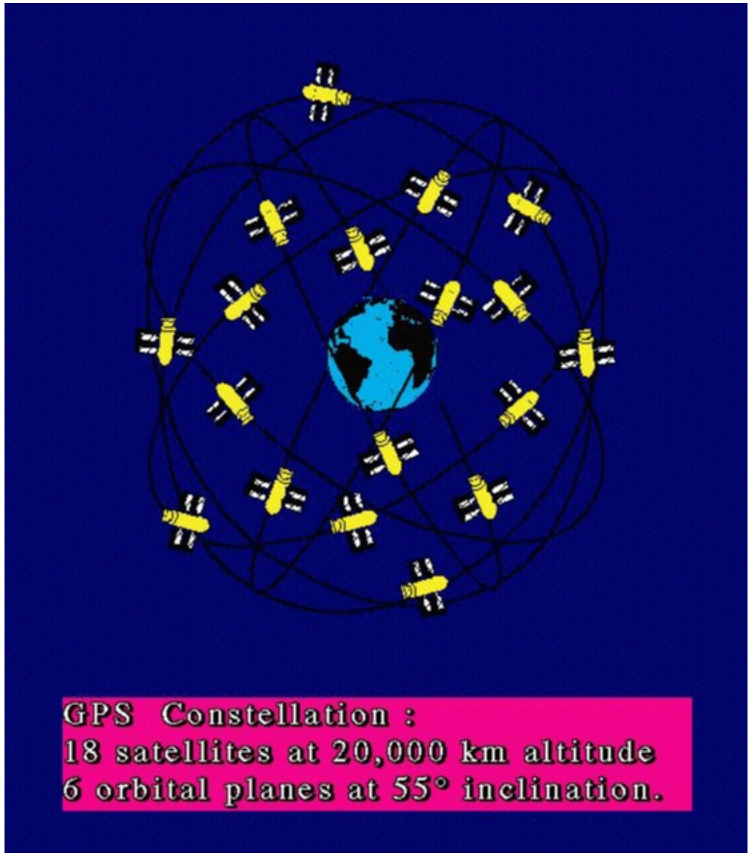


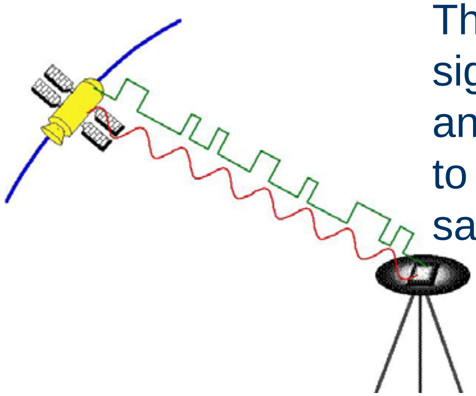
Motion of the Earth gravity center



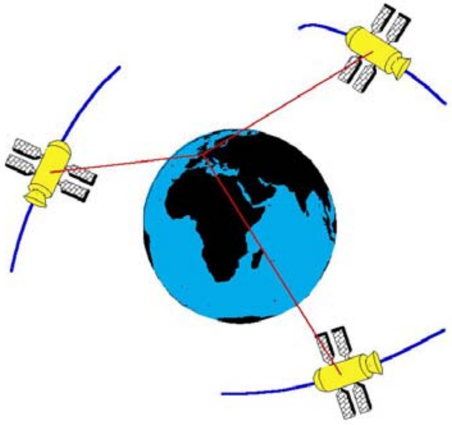
GPS (Global Positioning System)

GPS was created in the 80s' by the US Department of Defense for military purposes. The objective was to be able to get a precise position anywhere, anytime on Earth.



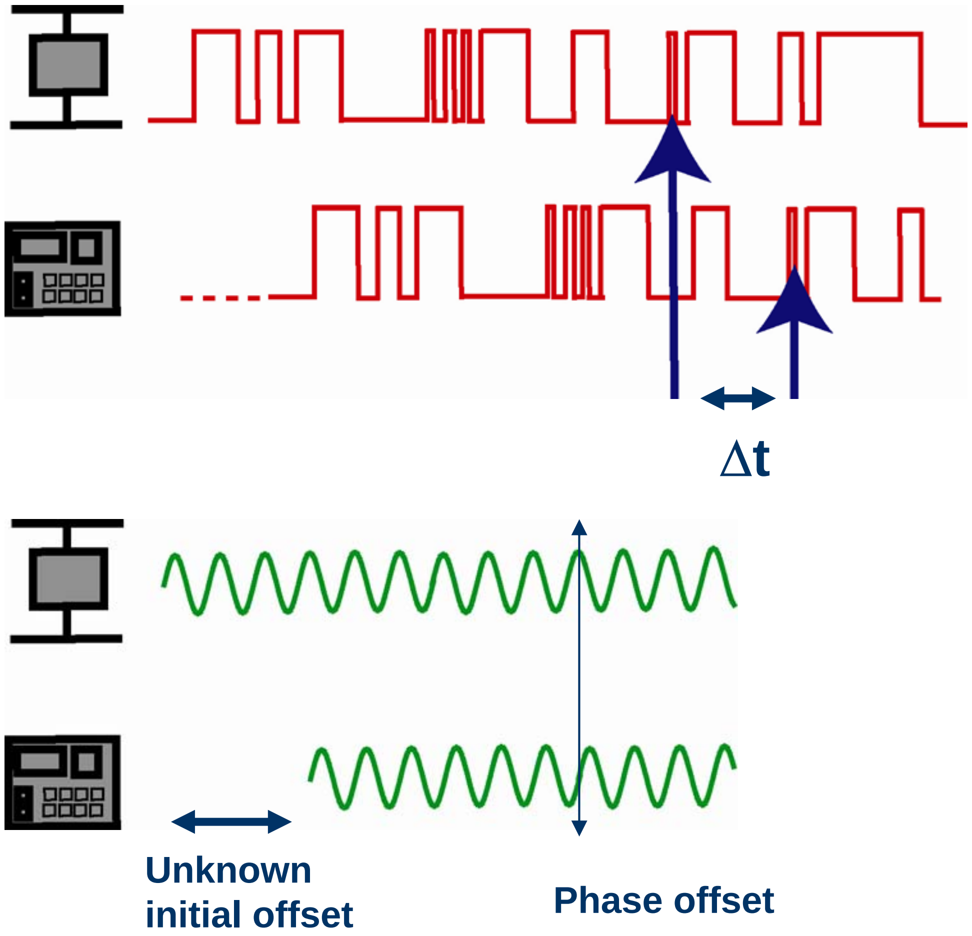


The satellites send a signal, received by a GPS antenna. Again, this allow to measure the distance satellite to antenna



With at least 3 satellites visible at the same time, we can compute instantaneously the station position. The precision can be as good as 1 millimeter

GPS (Global Positioning System)



pseudo-distance Measurement:

Accurate to 30 m if C/A code (pseudo frequency of 1 MHz)

Accurate to 10 m if P code (pseudo frequency of 10 MHz)

Easy because code never repeats itself over a long time, i.e. no ambiguity

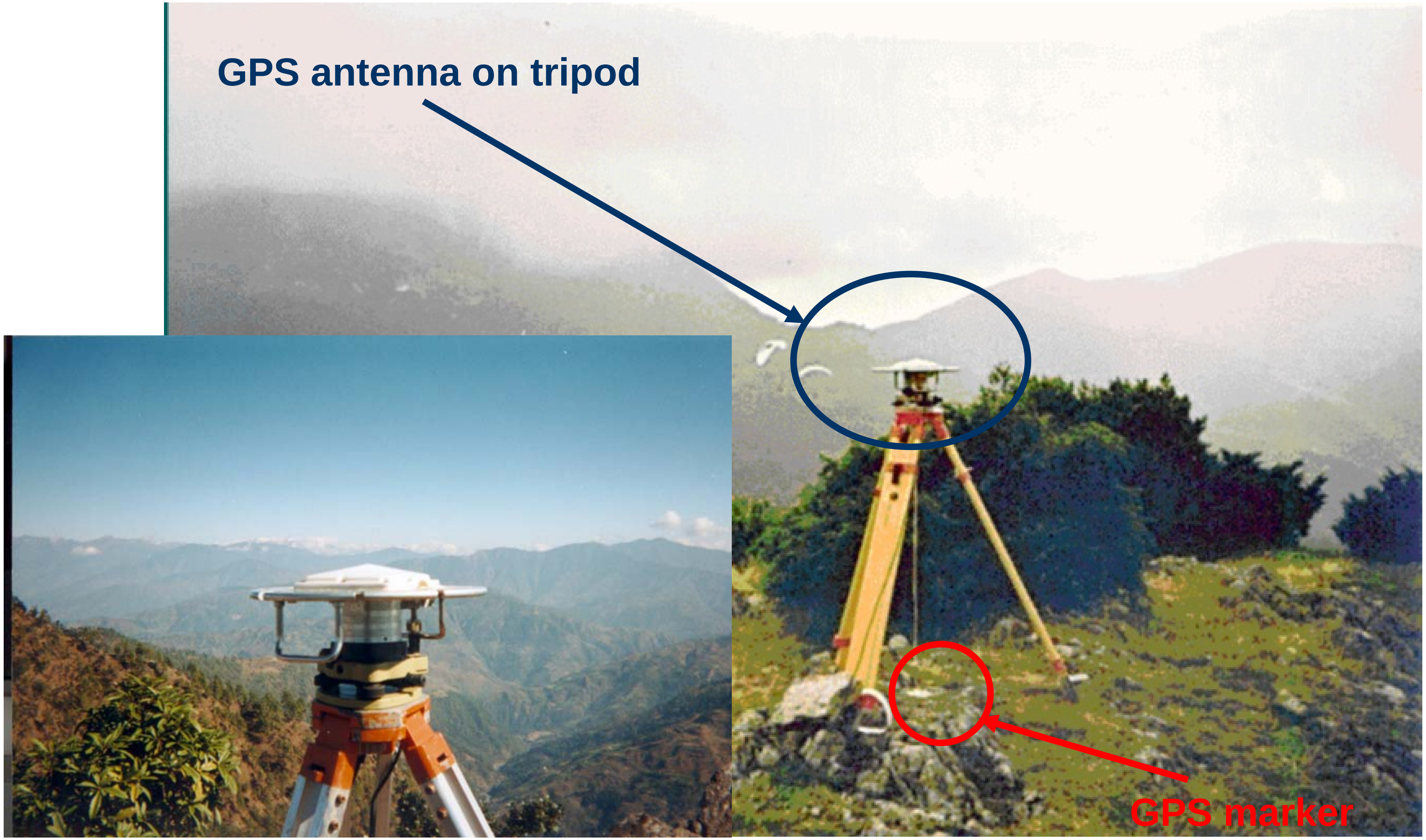
Phase Measurement:

Accurate to 20 mm on L1 or L2 (1.5 GHz)

But difficult because the initial offset is unknown.

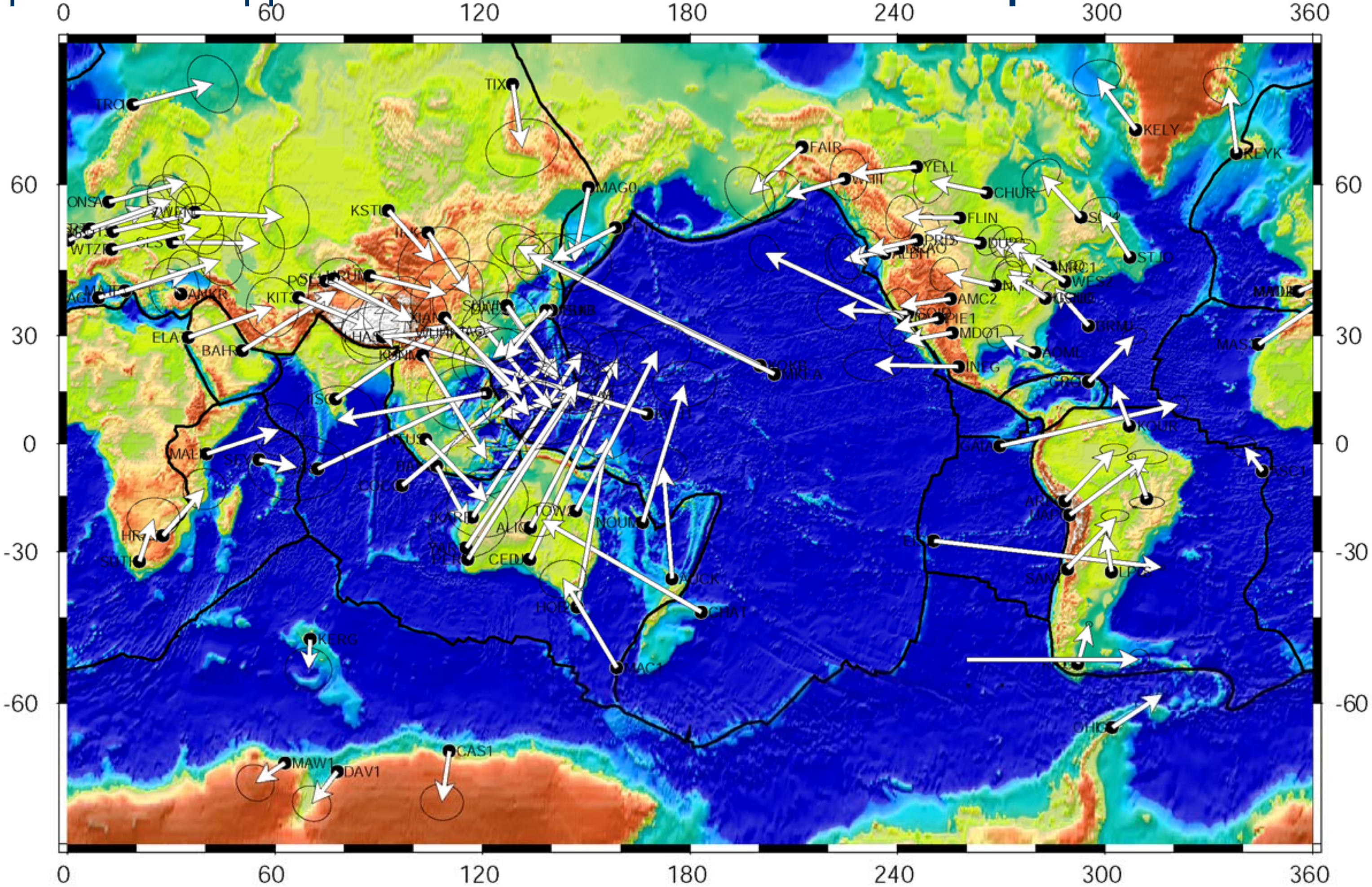
=> Post processing of a sequence of measurements on 1 satellite give final station position

GPS (Global Positioning System)

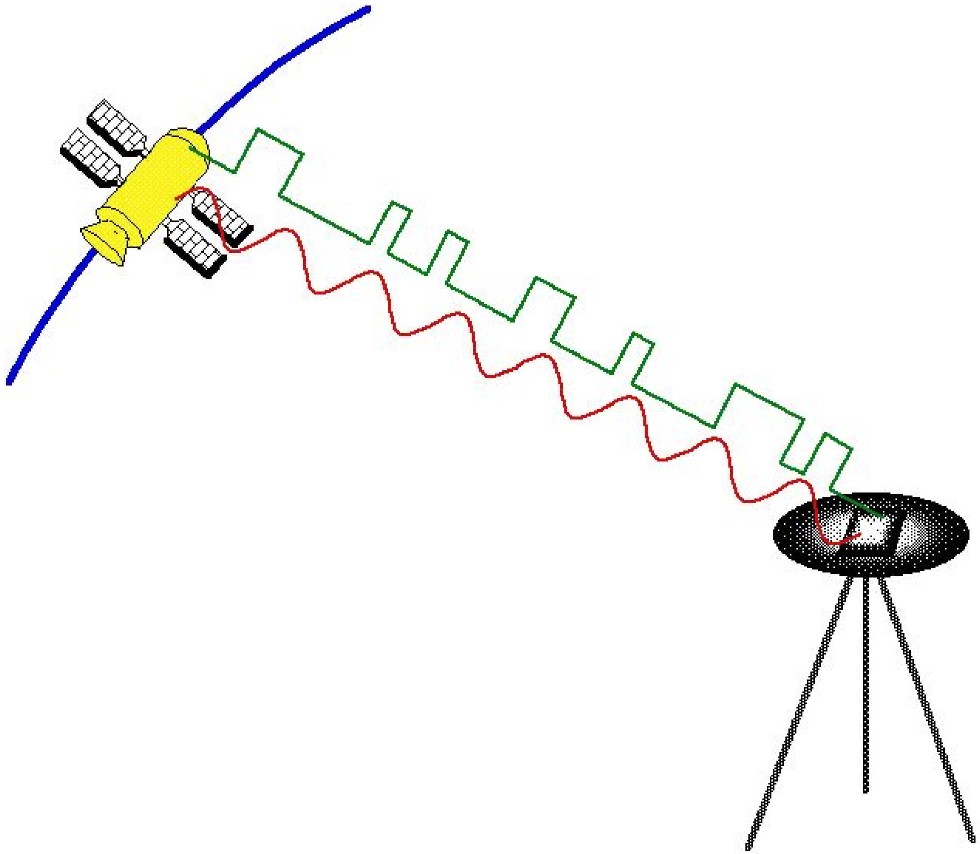


GPS (Global Positioning System)

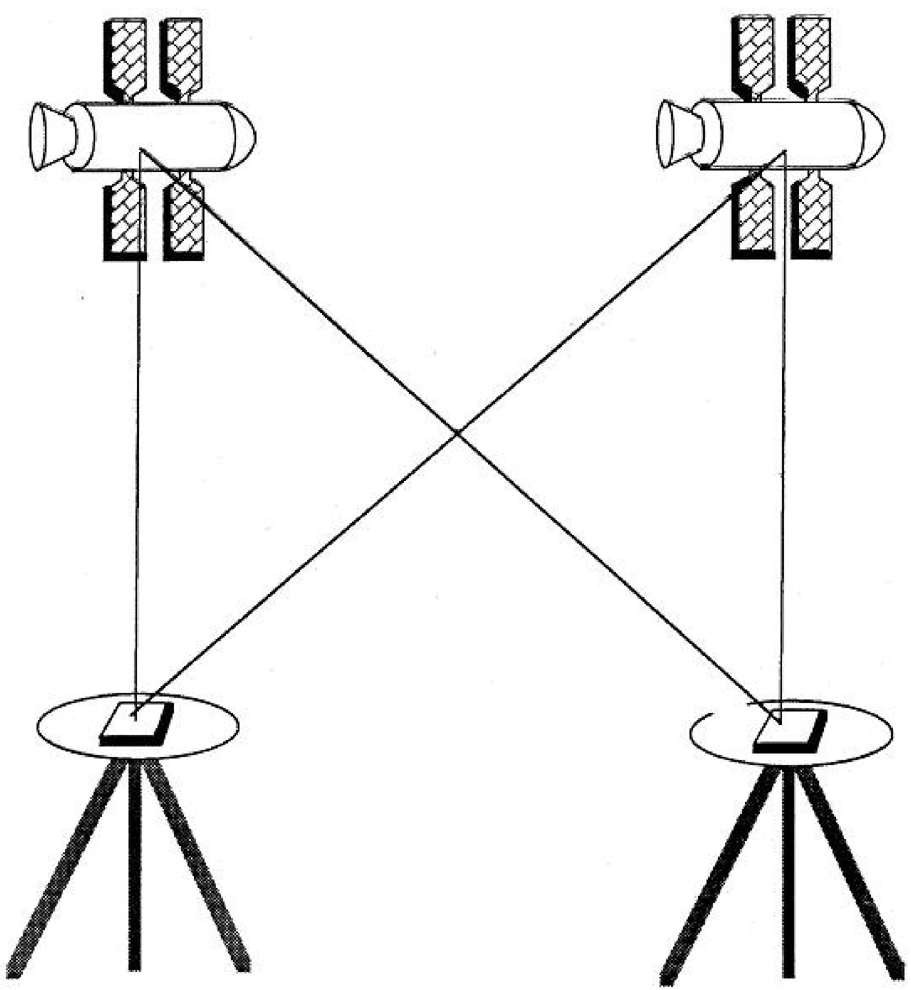
A spectacular application of GPS : the measurement of **plate tectonics**



Fundamentals of GPS



Double differences



One way phases are affected by **stations and satellites** clock uncertainties

Single differences are affected by

stations clock uncertainties

Or

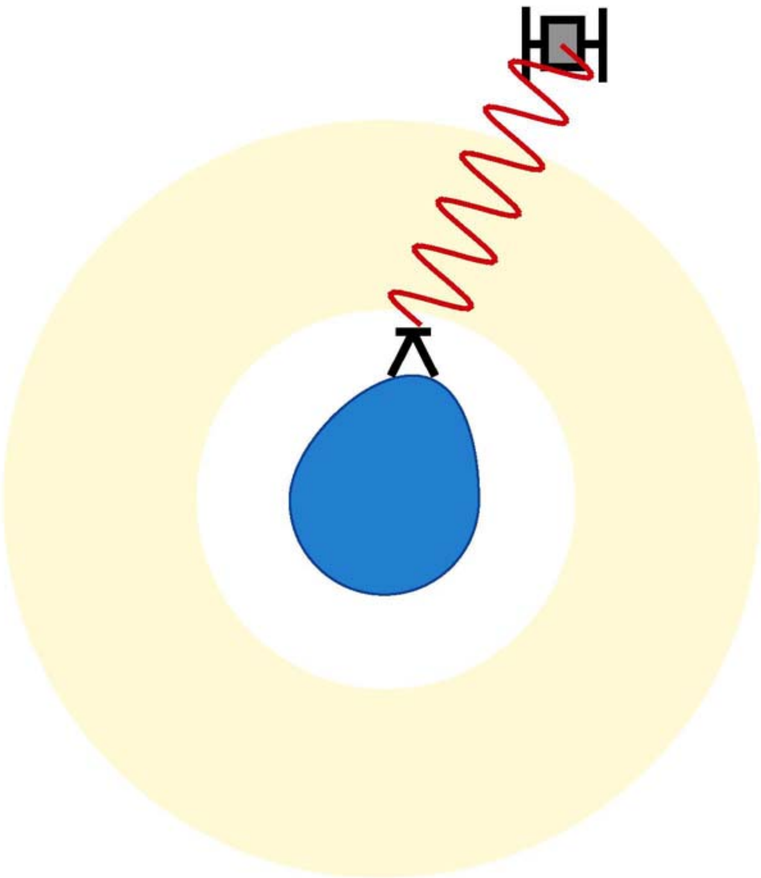
satellites clock uncertainties

Double differences Are free from all clock uncertainties **but**

=> Measurement of distances between points **(= baselines**)

=> **Relative positioning**

Other perturbation : The Ionosphere



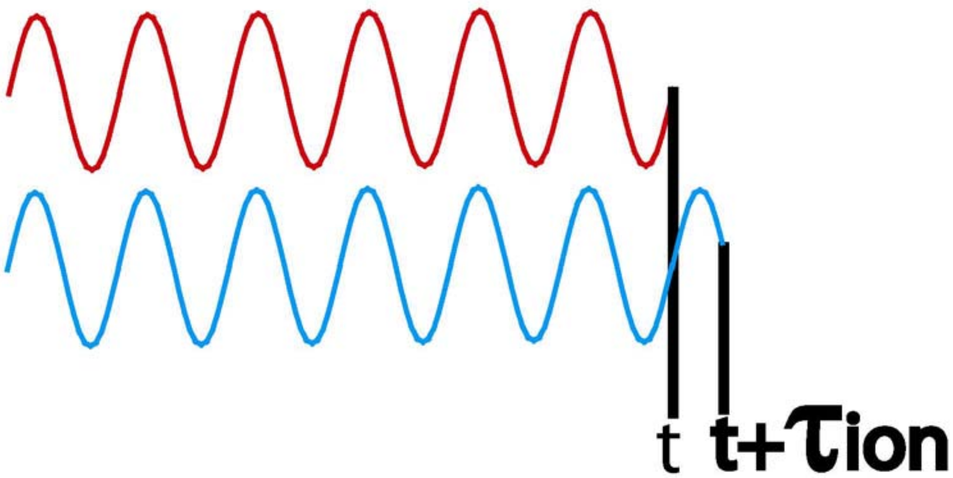
Correct measurement in an empty space

But the ionosphere perturbates propagation of electric wavelength …..

… and corrupts the measured distance

… and the inferred station position

Ionosphere theory



Ionospheric delay τion depends on :

• ionosphere contains in charged particules (ions and electrons) : **Ne**

• Frequency of the wave going through the ionosphere : f

$$\tau_{ion} = 1.35\ 10^{-7}\ Ne / f^2$$

Ionosphere : solution = dual frequency

Problem : **Ne** changes with time and is never known

solution : sample the ionosphere with 2 frequencies

$\tau ion_1 = 1.35\ 10^{-7}\ Ne / f_1^2$ $\tau ion_2 = 1.35\ 10^{-7}\ Ne / f_2^2$

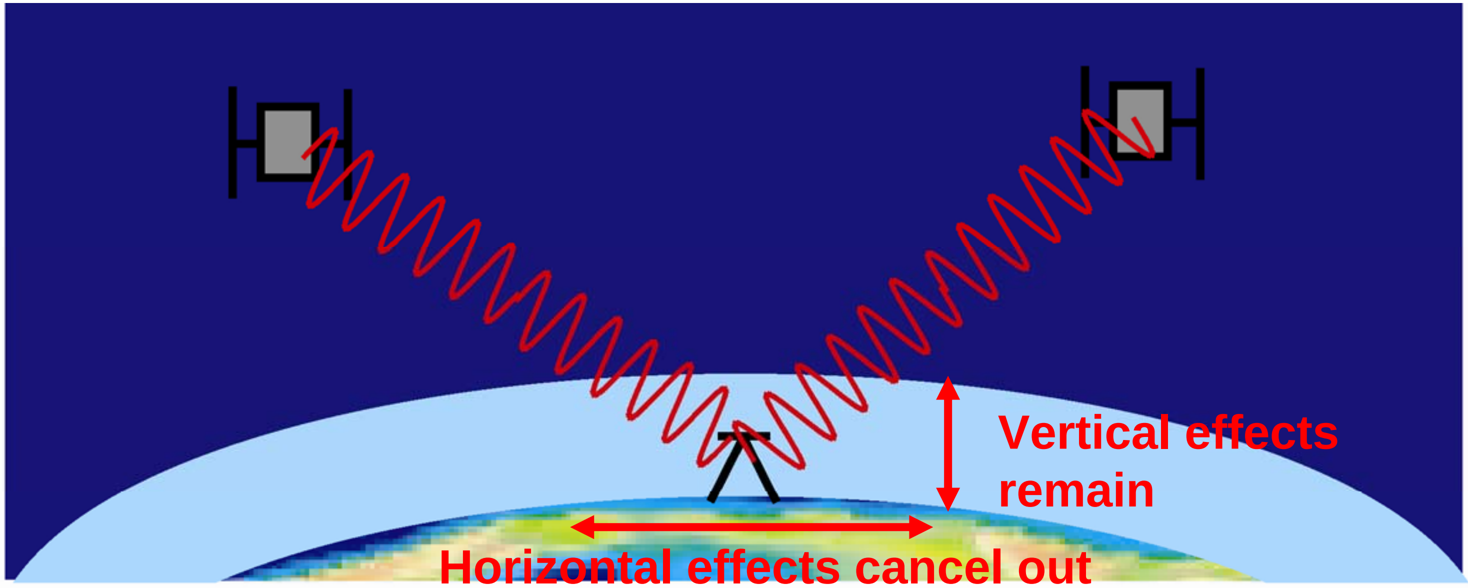
$\Rightarrow \tau ion_2 - \tau ion_1 = 1.35\ 10^{-7}\ Ne\ (1/ f_2^2 - 1/ f_1^2)$

$\Rightarrow Ne = \left[\tau ion_2 - \tau ion_1\right] / 1.35\ 10^{-7}\ (1/ f_2^2 - 1/ f_1^2)$

Using dual frequency GPS, allow to determine the number Ne and then to quantify the ionospheric delay on either L1 or L2.

(*in fact, GPS can and is used to make ionosphere Total Electron Containt (TEC) maps of the ionosphere*)

Second perturbation : The Troposphere

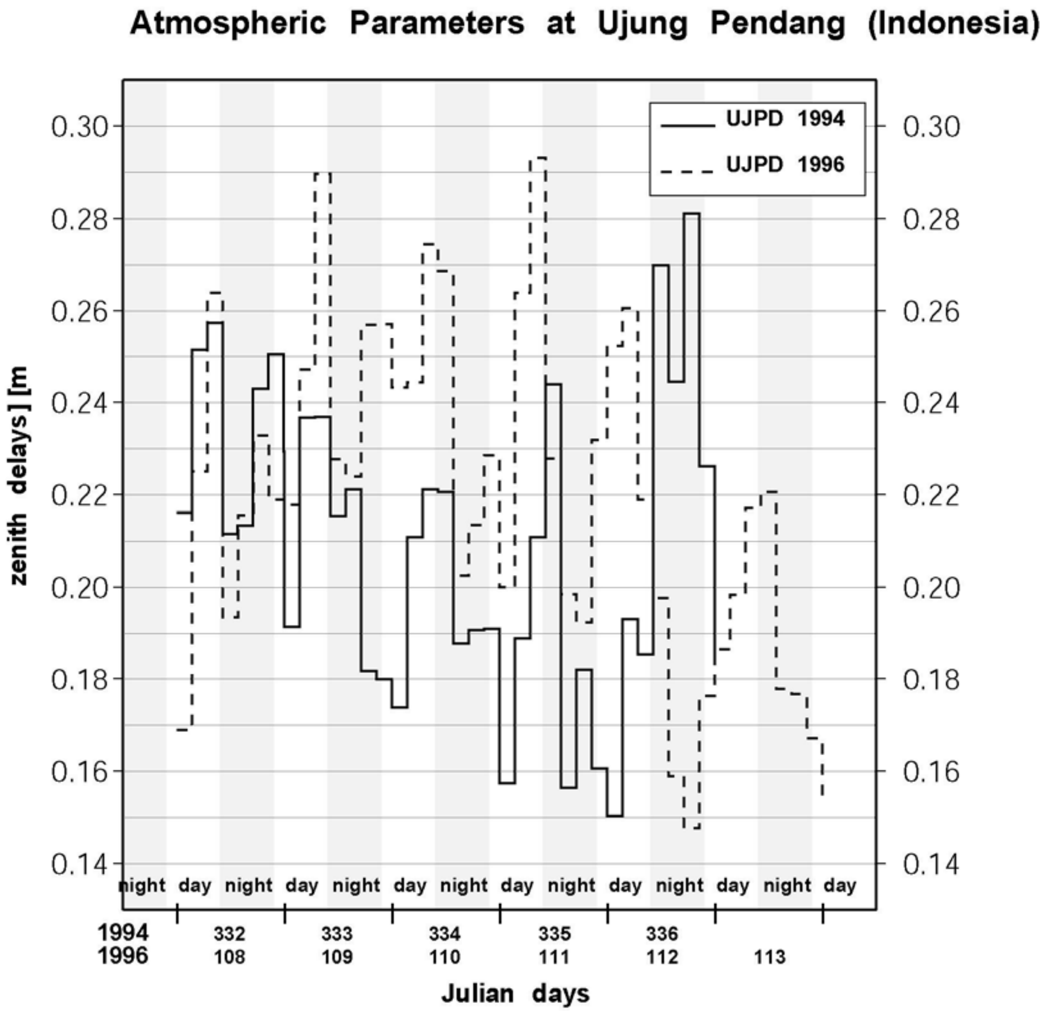


The troposphere (lower layer of the atmosphere) contains water. This also affects the travel time of radio waves.

But the troposphere is not dispersive (effect not inversely proportional to frequency), so the effect cannot be quantified by dual frequency system. Therefore there a position error of 1-50 cm.

Thanks to the presence of many satellites, the effect cancel out (more or less) in average, on the horizontal position. Only remains a vertical error called **Zenith tropospheric delay**

Troposphere zenith delay



The tropospheric zenith delay can be estimated from the data themselves…
if we measure every **30s** on **5** satellites, we have **1800** measurements in **3 hours.**
We only have **3** unknowns : station **lat**, **lon**, and **altitude** !

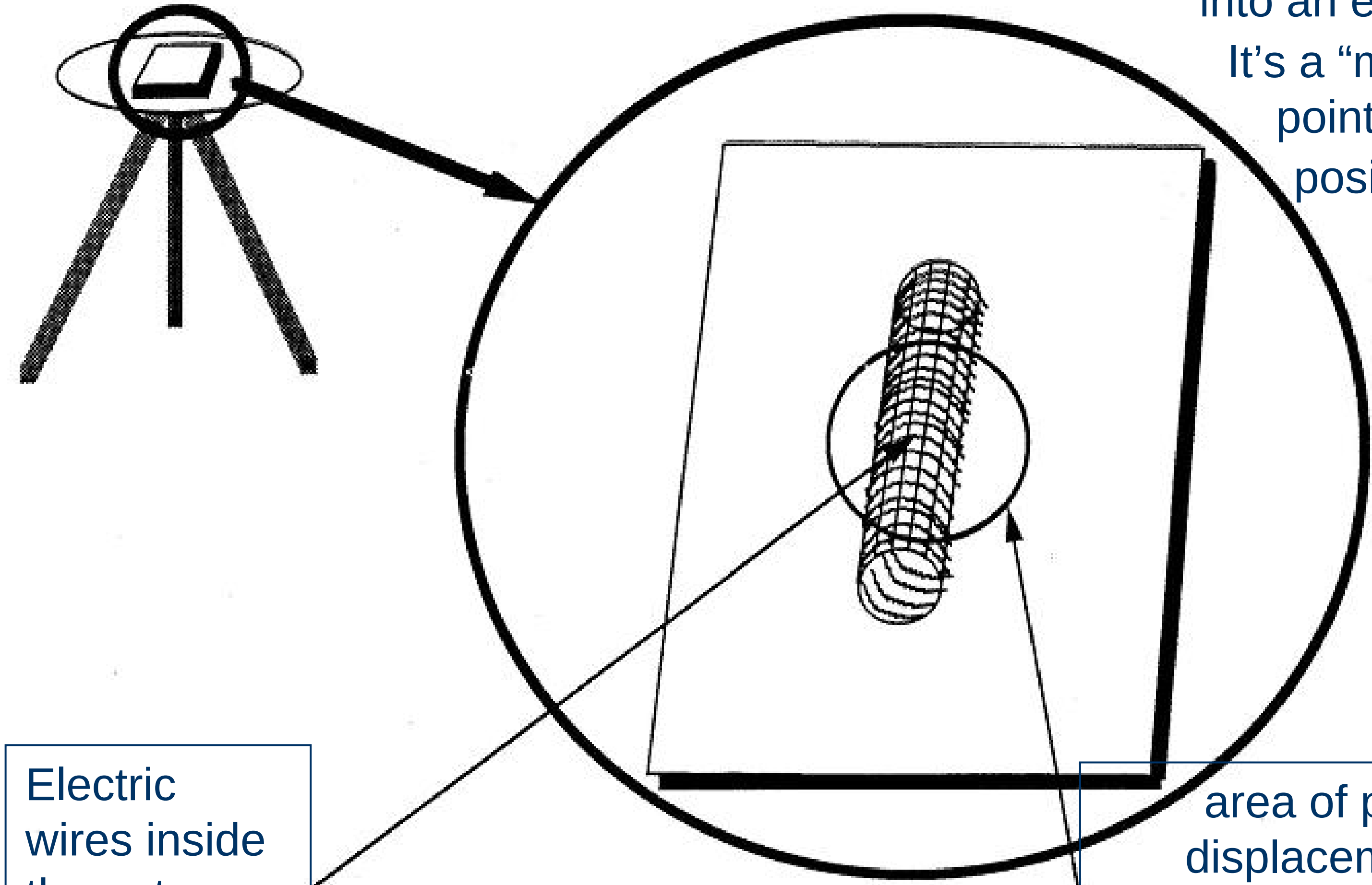
So we can add a new one :
1 Zenith delay every 3 hours

The curves show that the estimated Zenith delay vary from 15 cm to 30 cm with a very clear day/night cycle

Antenna phase center offset and variations

The Antenna phase center is the wire in which the radio wave converts into an electric signal.

It's a "mathematical" point, which exact position depends on the signal alignment with the wire (azimuth and elevation)

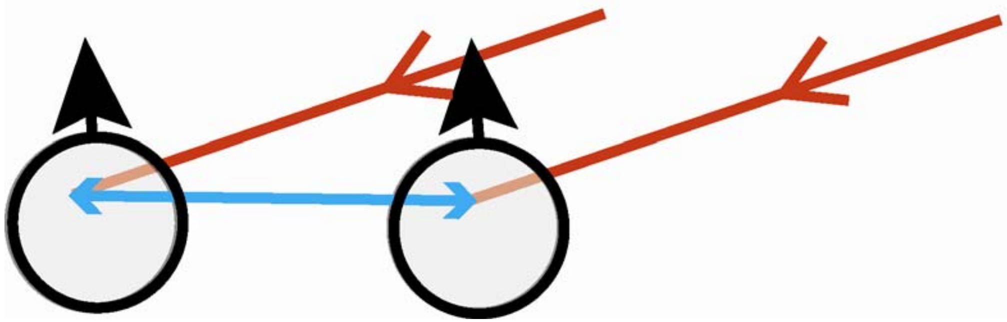


Electric wires inside the antenna

area of phase center displacement (~1 cm)

Antenna phase center offset and variations

Solution : use **identical** antennas, oriented in the **same direction**



As the signal rotates, the antenna phase centers move

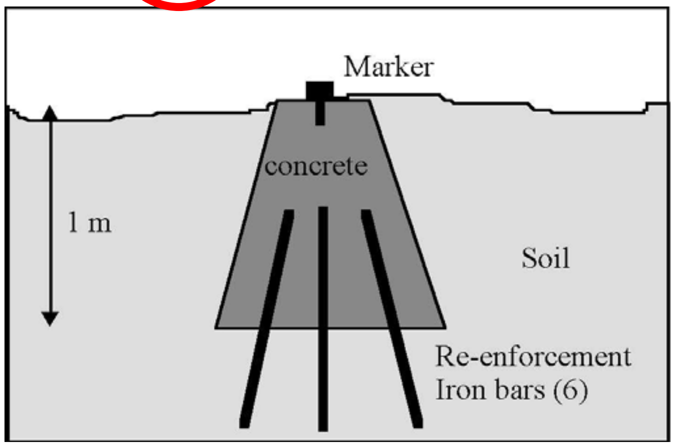
But they move the same quantity in the same direction if antennas are strictly identical because the incoming signal are the same (satellite is very far away)

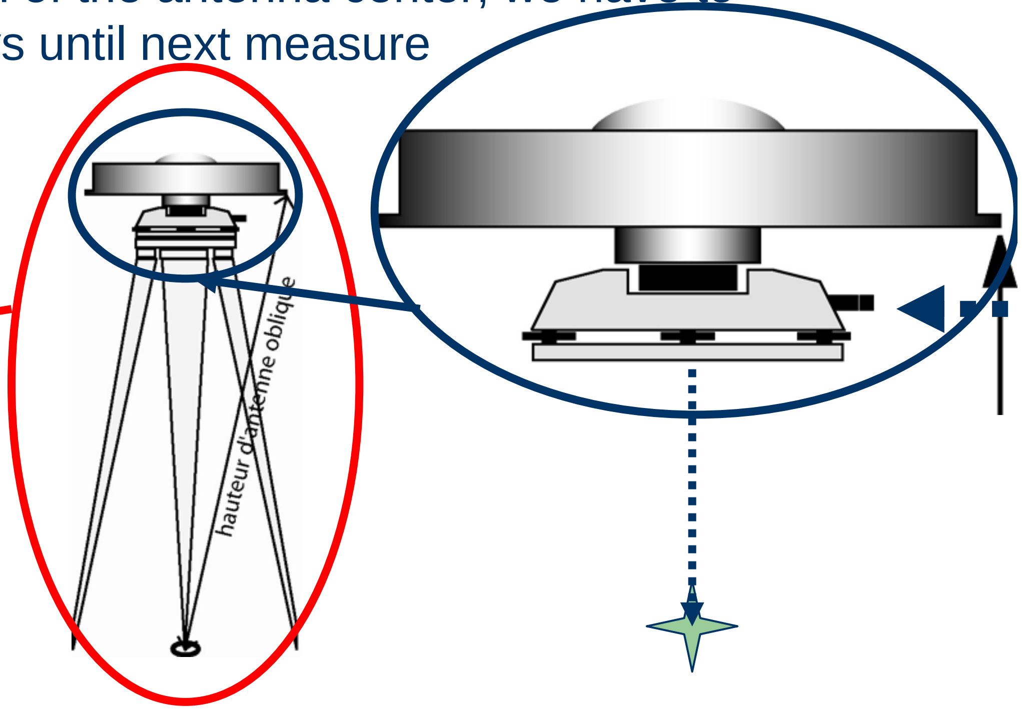
Therefore, the **baseline** between stations remains unchanged

But this works for small baselines only (less than a few 100 km)

Tripod and tribrachs source of errors

The measurement give the position of the antenna center, we have to **tie** it to the GPS marker which stays until next measure



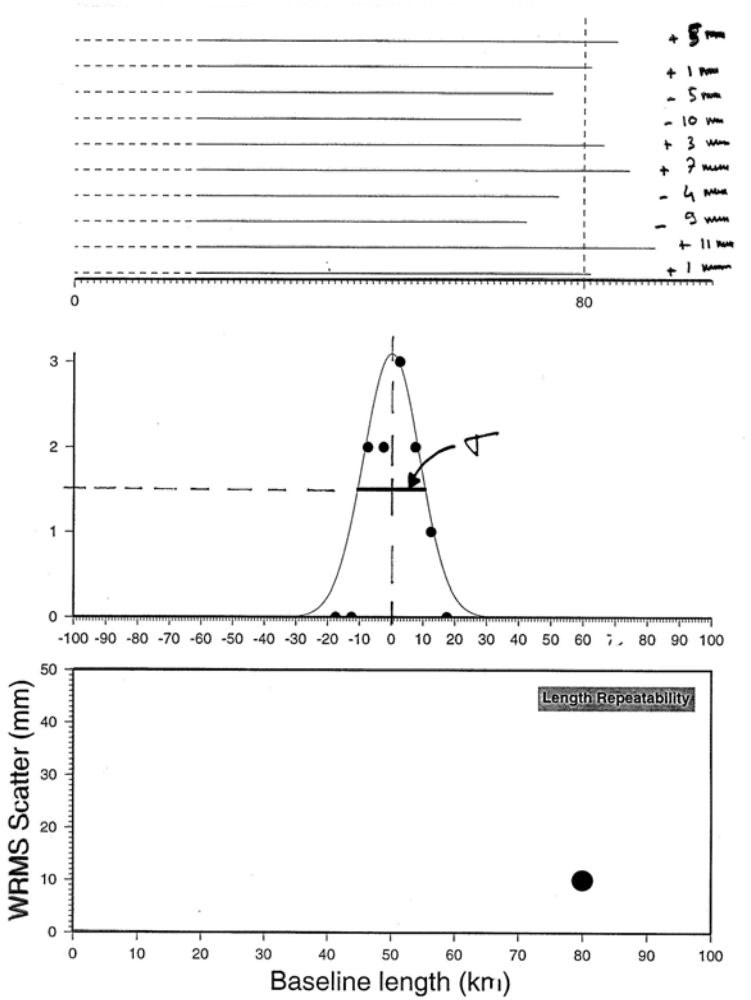


The antenna has to be leveled horizontally and centered perfectly on the mark. Then :

Horiz. position of marker = horiz. position of antenna

Altitude of marker = altitude of antenna – antenna height

Precision and repeatability



10 measurements of the same baseline give slightly different values :

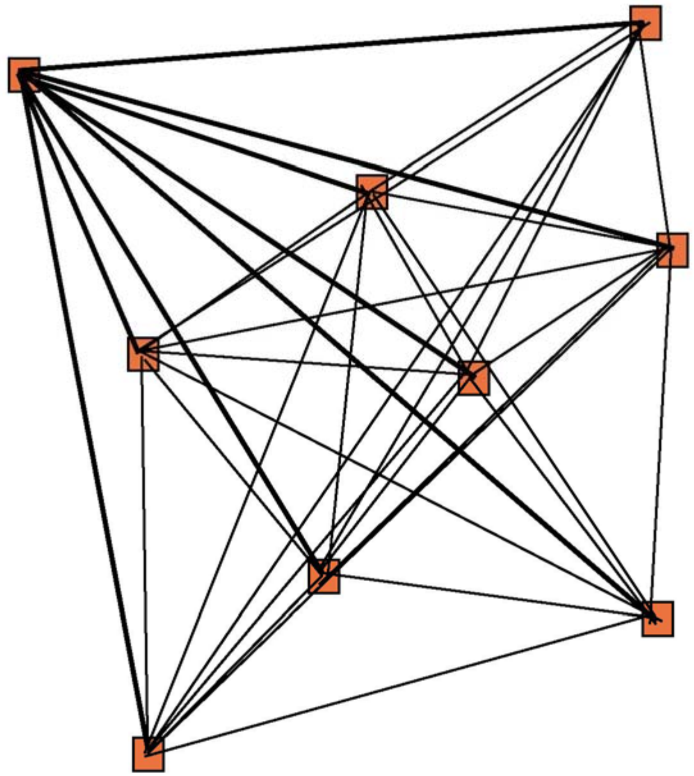
80 km +/- 10 mm

How many measurements are between 80 and 80+δ

The histogram curve is a Gaussian statistic

The baseline **repeatability** is the **sigma** of its Gaussian scatter

Network repeatabilities



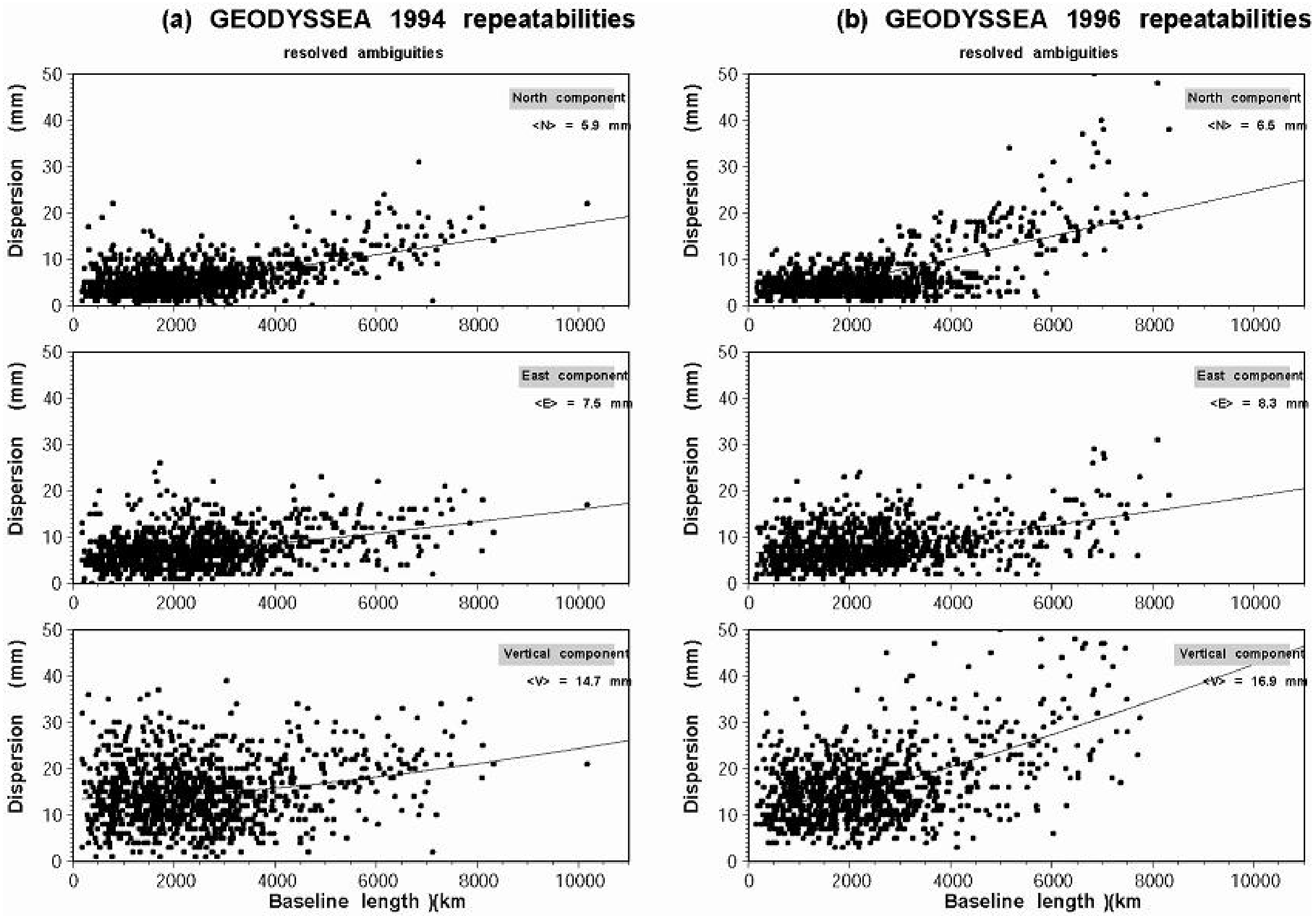
Network of N points (N=9)

(N-1) (=8) baselines from 1st station to all others

(N-2) (=7) baselines from 2nd station to all others
=> subtotal = (N-1)+(N-2)

total number of baselines
= (N-1)+(N-2)+…+1
= N(N-1)/2 (*36 in that case*)

Typical repeatabilities (60 points => ~1800 bsl)



Repeatabilities are much larger than formal uncertainties !

From position to velocity uncertainty

If one measures position P_1 at time t_1 and P_2 at time t_2 with precision ΔP_1 and ΔP_2, what is the velocity V and its precision ΔV ?

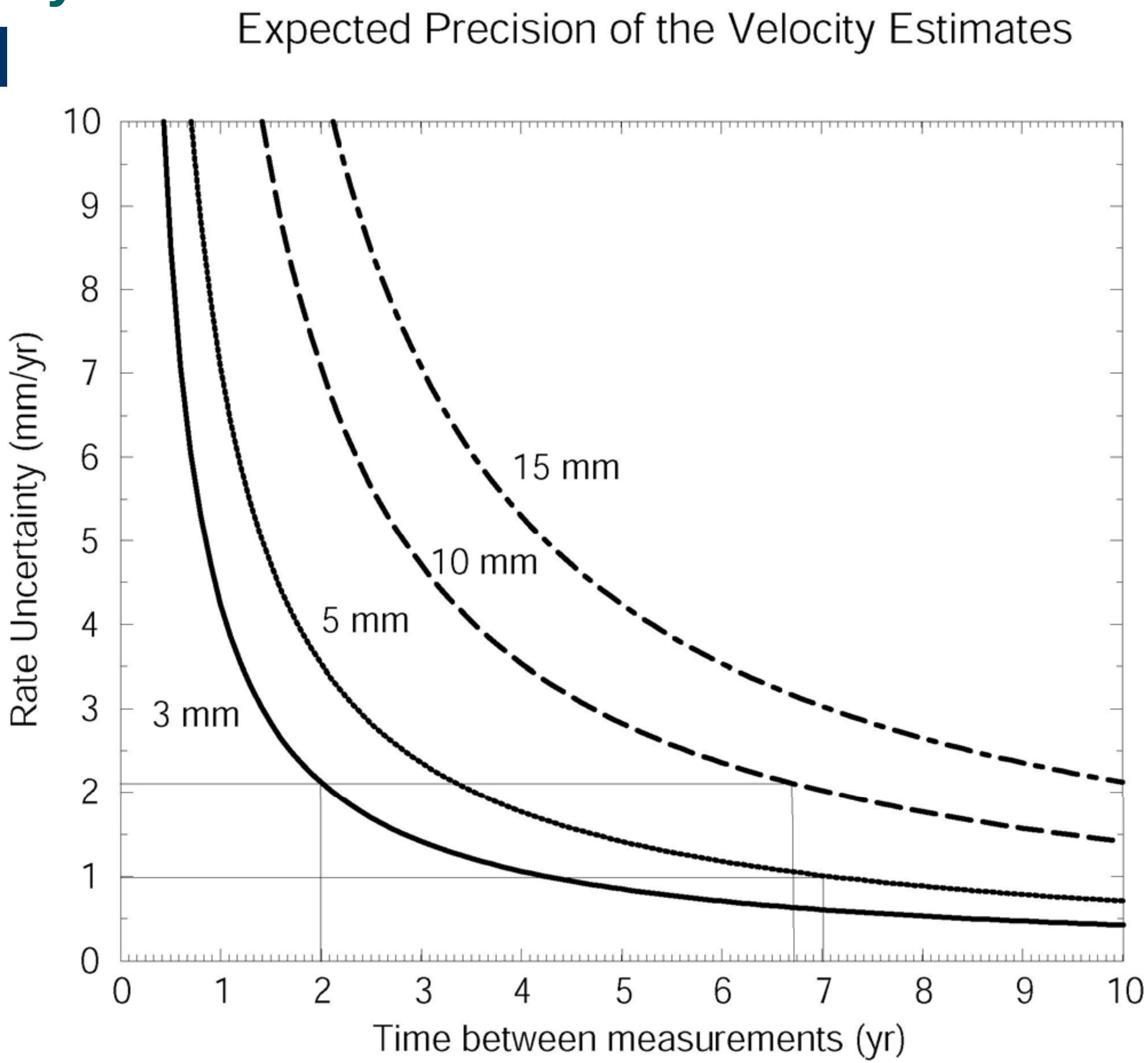
$$V = (P_2 - P_1) \; / \; (t_2 - t_1)$$

$$\Delta V = (\Delta P_2 + \Delta P_1) \; / \; (t_2 - t_1)$$ X

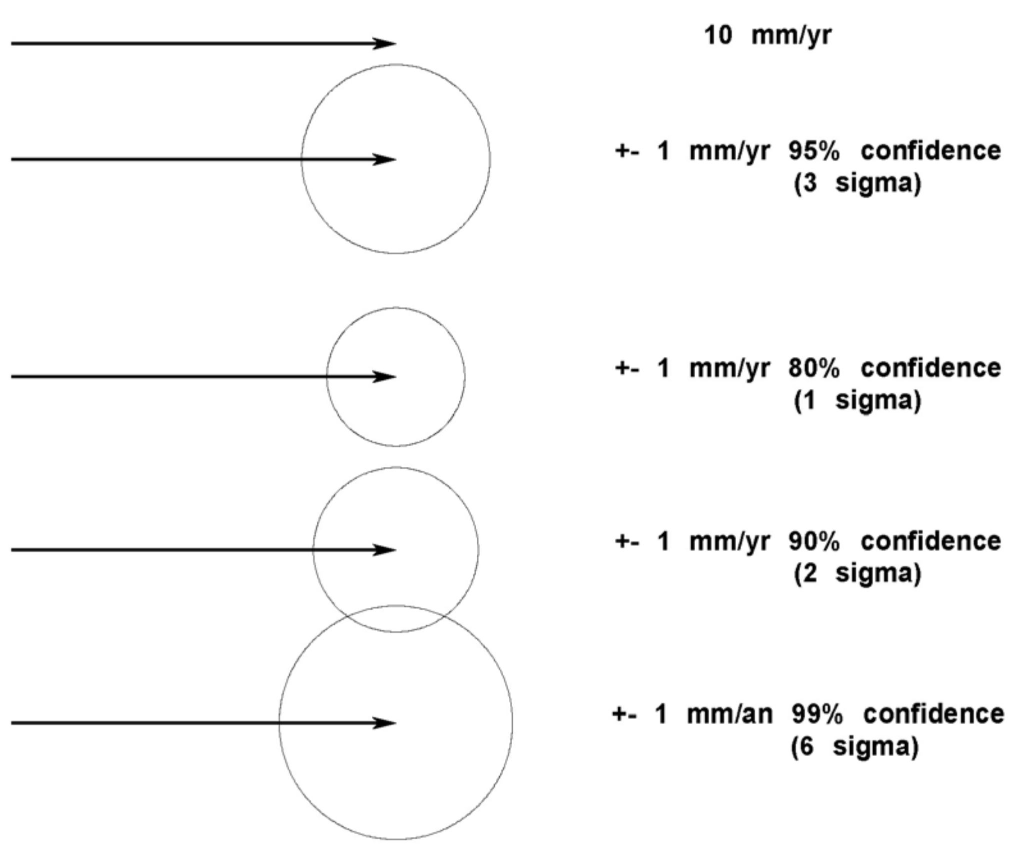
Uncertainties don’t add up simply, because sigmas involve probability.

$$\Delta V = \left[(\Delta P_2)^2 + (\Delta P_1)^2 \right]^{1/2} \Big/ \; (t_2 - t_1)$$

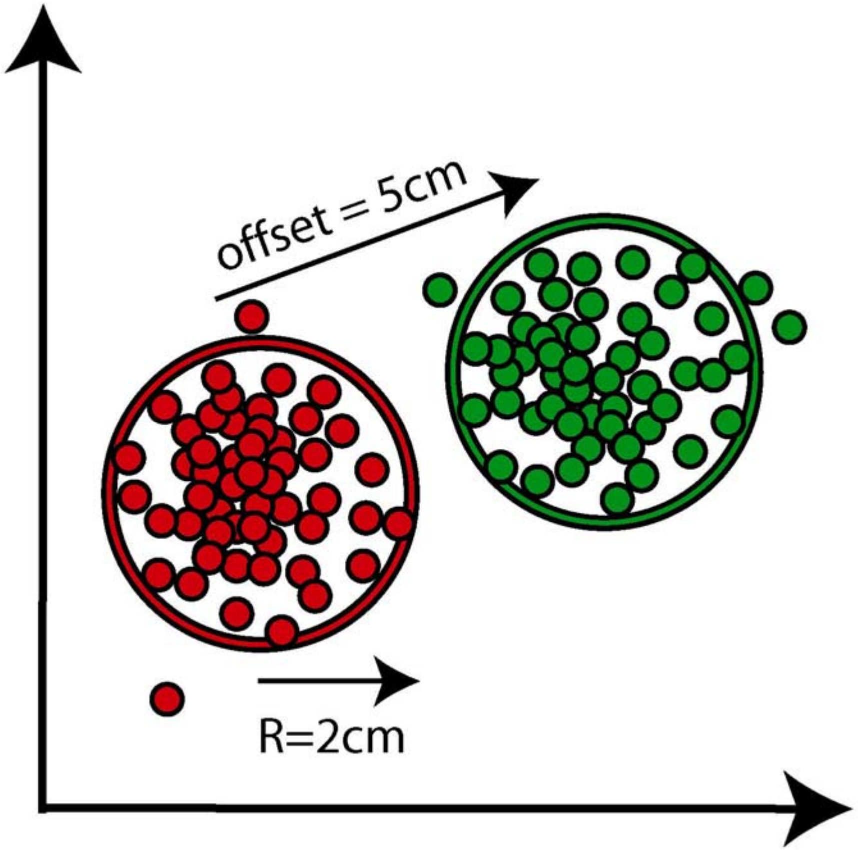
Velocity uncertainties



Velocity ellipses



Accuracy vs. precision (1)



Fix point :
measure 1 hour every 30 s

=> 120 positions

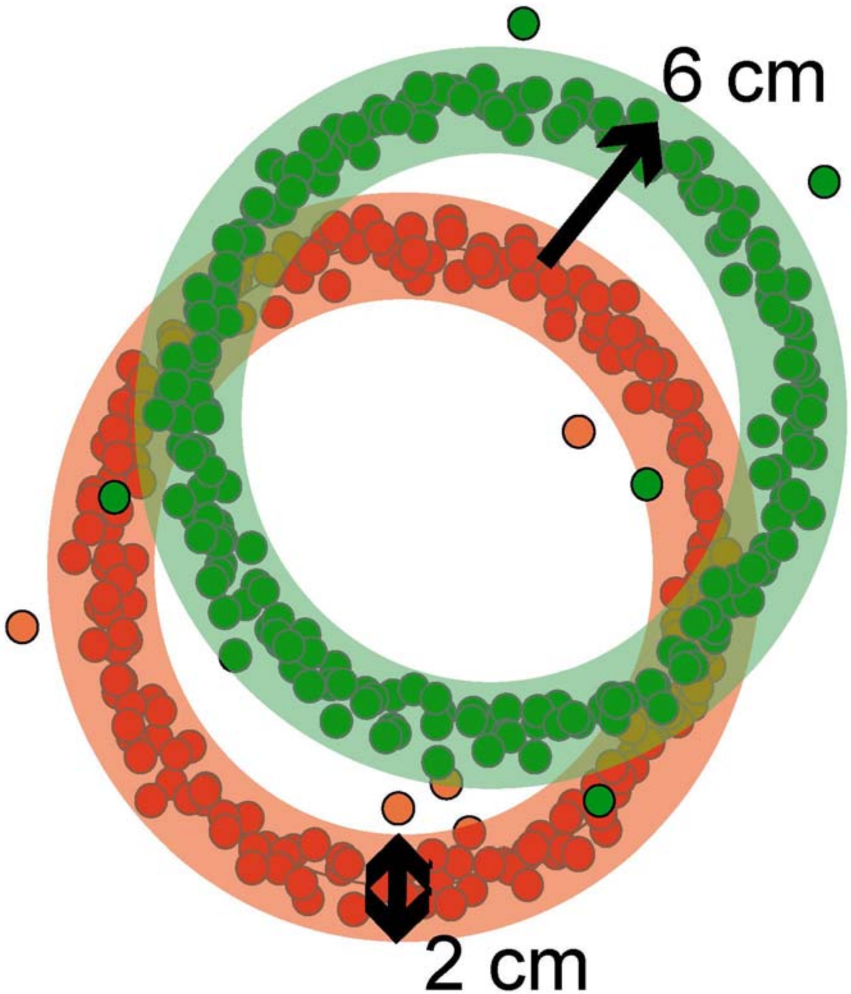
with dispersion ~+/- 2 cm

5 hours later, measure again
1 hour at the same location

=> Same dispersion but
constant offset of 5 cm

Precision = 2 cm
Accuracy = 5 cm

Accuracy vs. precision (2)



Measure path, 1 point every 10s

=> 1 circle with 50 points

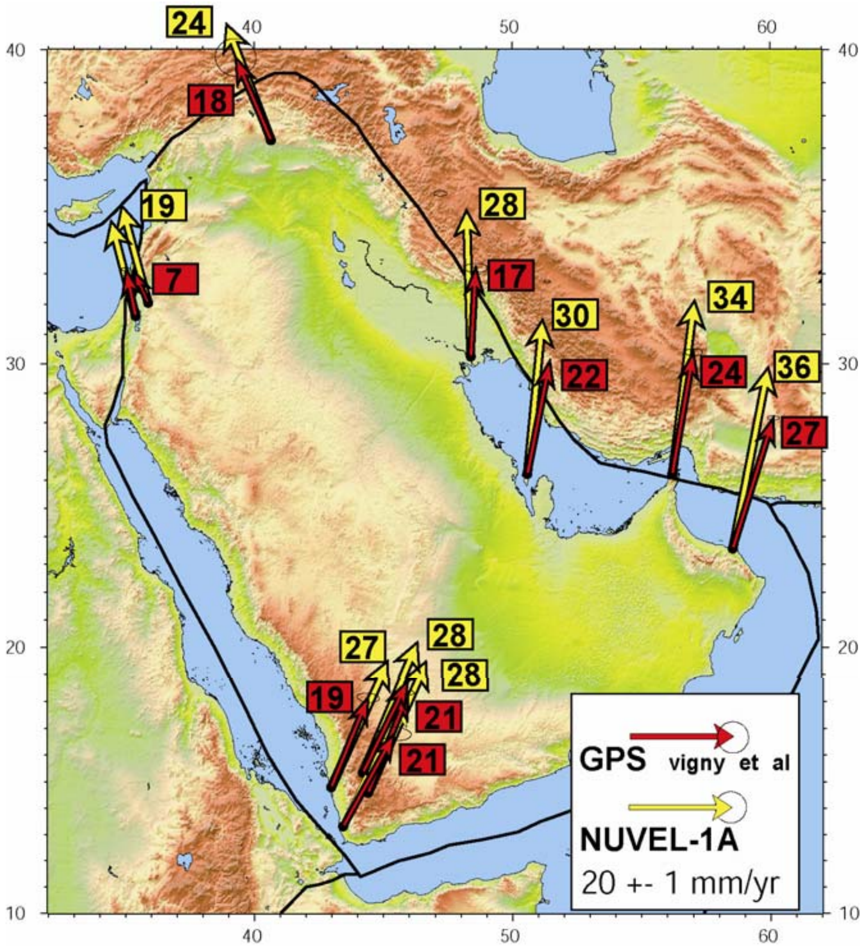
10 circles describe runabout with dispersion ~ 2 cm

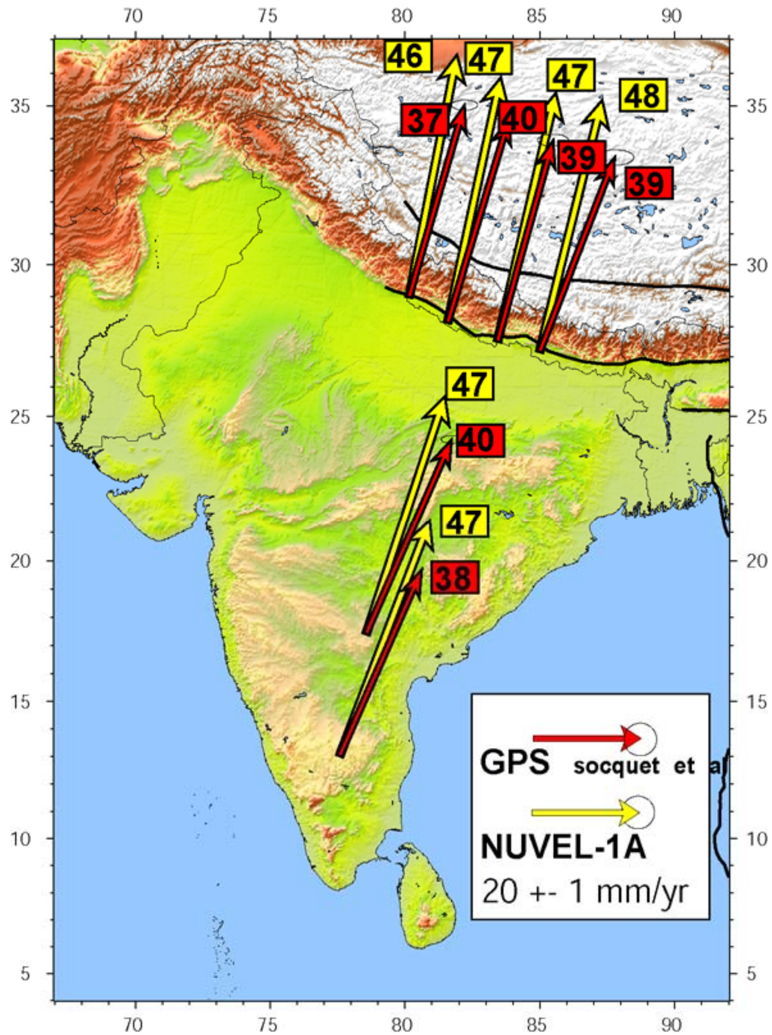
Next day, measure again

=> Same figure but constant offset of 6 cm

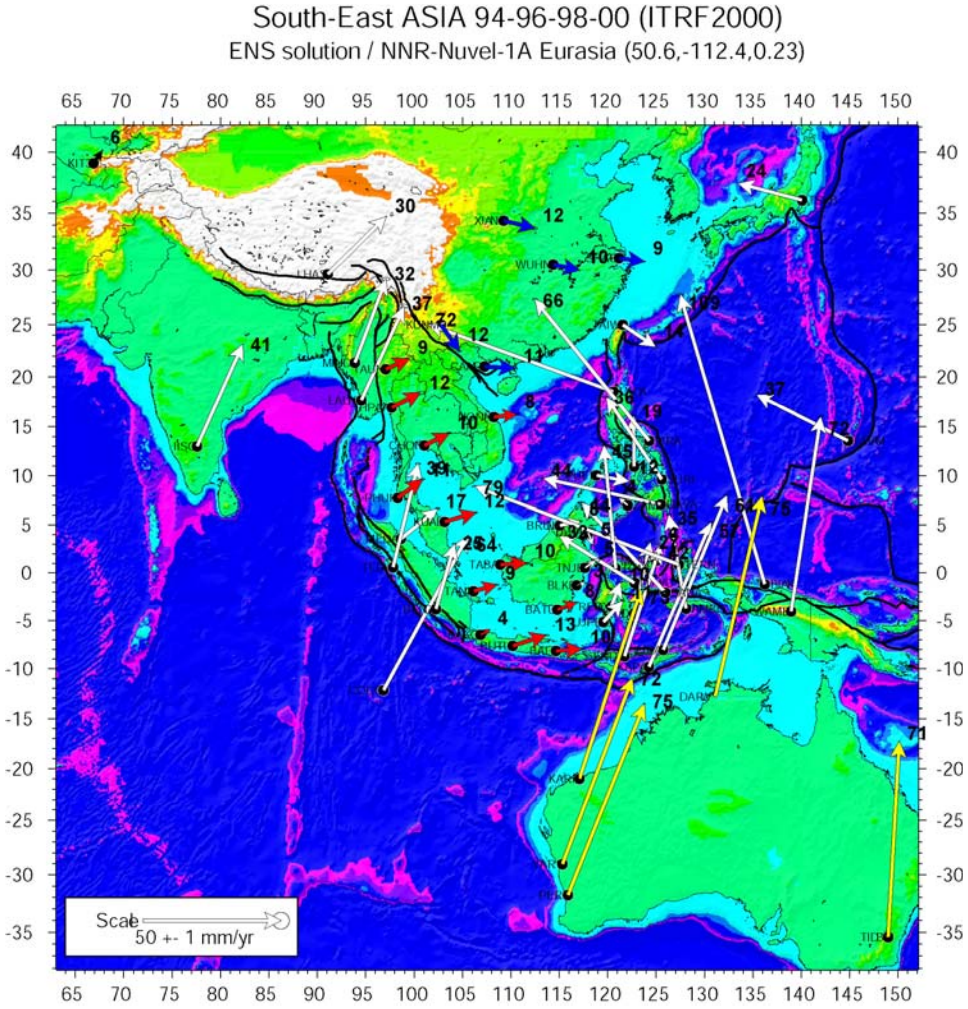
Precision = 2 cm
Accuracy = 6 cm

GPS finds Arabia, India and Nazca are slower





Rigid Sundaland



GPS campaigns with more than 60 sites allow to determine that :

- South-East Asia (red arrows) is an individual block which moves away from Eurasia (black arrows)
- South China (blue arrows) also moves away from Eurasia at around 10 mm/yr eastward

Strain rate and rotation rate tensors (1)

To asses plate deformation :

1. Look at station velocity residuals
2. Compute strain rate and rotation rate tensors

$$\text{Strain} = \frac{\text{Velocity}}{\text{Distance}} = \frac{\text{mm/yr}}{\text{km}} = \text{\% / yr}$$

Matrix tensor notation : $S_i^j = d(V_i) / d(x_j) = \begin{vmatrix} d(V_x) / d(x) & d(V_x) / d(y) \\ d(V_y) / d(x) & d(V_y) / d(y) \end{vmatrix}$

Theory says : [S] = [E] + [W]

	[E]	[W]
	Symetrical	Antisymetrical
	Strain rate	rotation rate

Strain rate and rotation rate tensors (2)

$$[E] = \tfrac{1}{2}\left([S] + [S]^T\right) = \begin{vmatrix} E_{11} & E_{12} \\ E_{12} & E_{22} \end{vmatrix} \qquad [W] = \tfrac{1}{2}\left([S] - [S]^T\right) = \begin{vmatrix} 0 & W \\ -W & 0 \end{vmatrix}$$

[E] has 2 Eigen values : ε_1, ε_2

ε_1 and ε_2 are extension/compression along principal direction defined by angle θ (defined as angle between ε_2 direction and north)

$$\varepsilon_1 = E_{11}\cos^2\theta + E_{22}\sin^2\theta - 2\,E_{12}\sin\theta\,\cos\theta$$

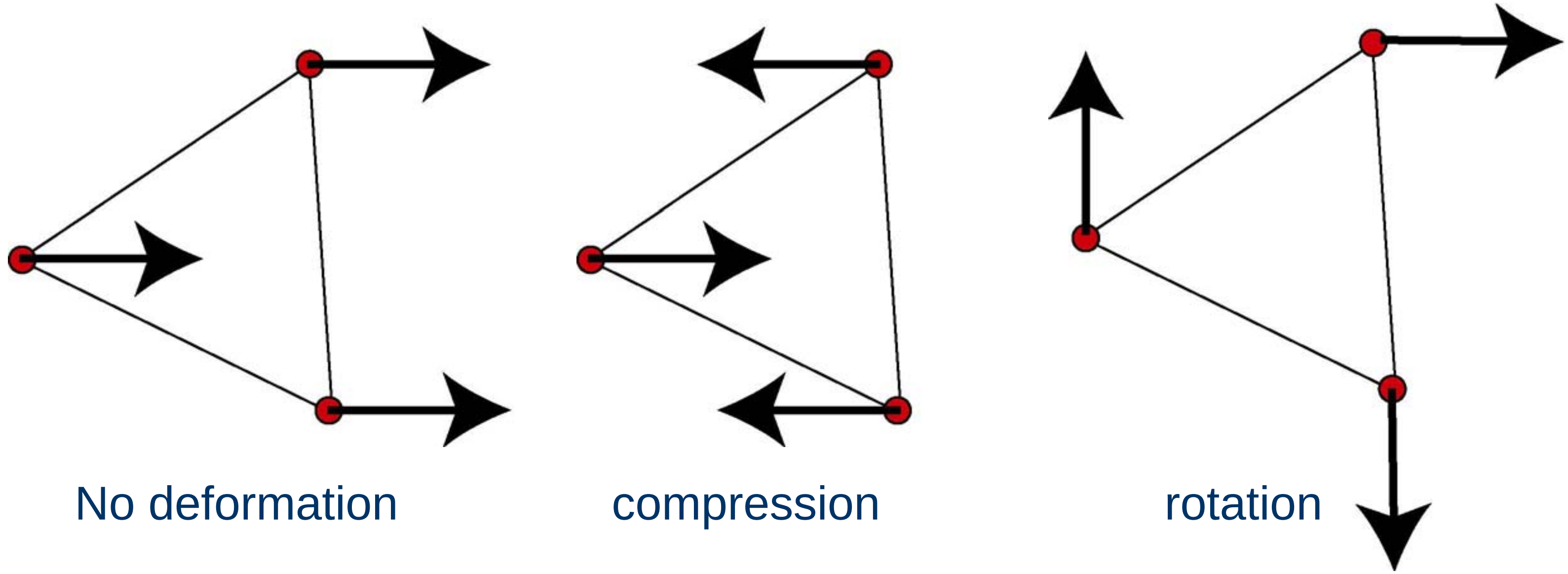
$$\varepsilon_2 = E_{11}\sin^2\theta + E_{22}\cos^2\theta - 2\,E_{12}\sin\theta\,\cos\theta$$

Strain rate and rotation rate tensors (3)

Minimum requirement to compute strain and rotation rates is :

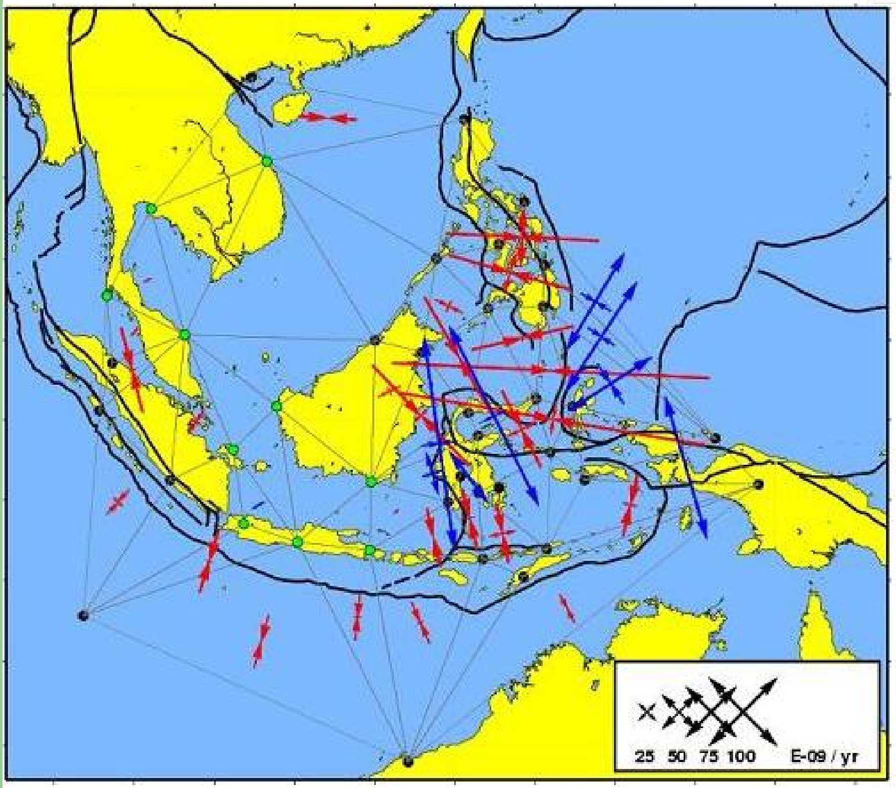
3 velocities (to allow to determine 3 values ε_1, ε_2, and W)

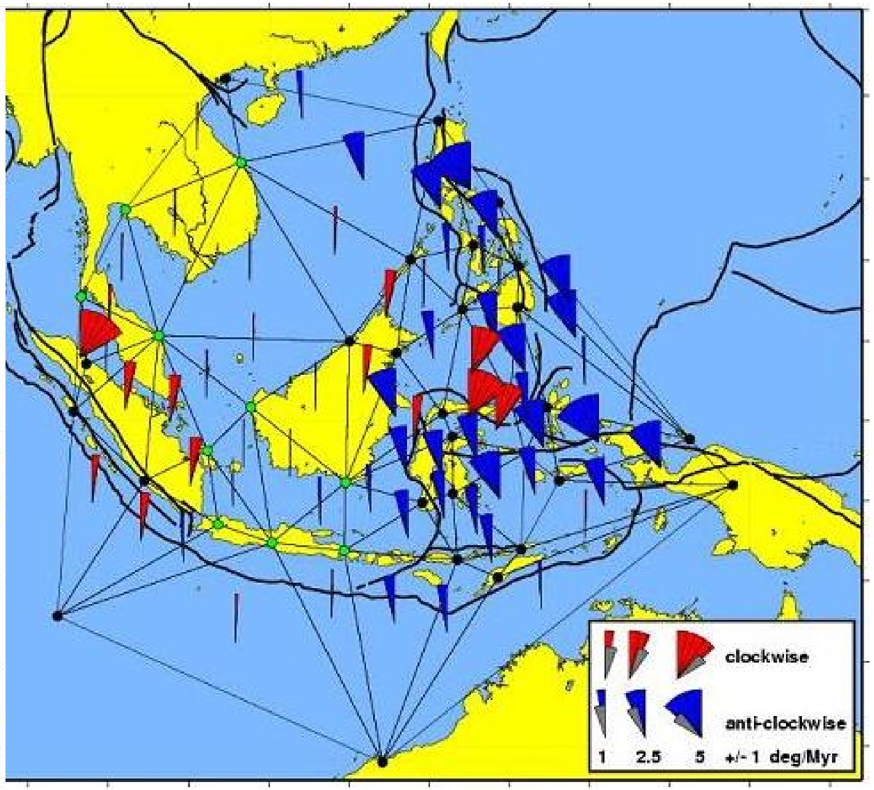
Therefore we can compute strain rate and rotation rate within any polygon, the minimum polygon being a triangle



Strain and rotations are unsensitive to reference frame

Strain and rotation in GEODYSSEA network





Strains :

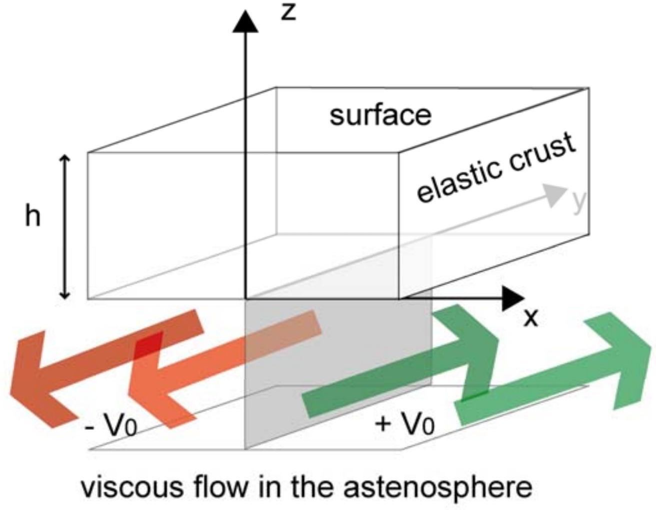
extension/compression/strike-slip

Rotations :

Anti-clockwise/clockwise

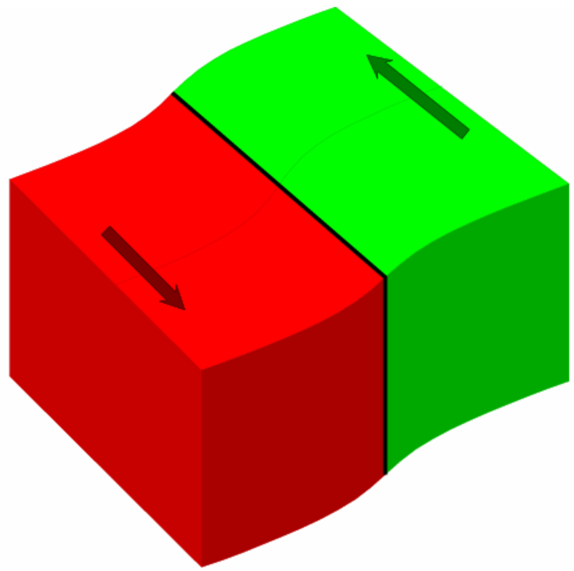
Mathematical formulation

$$U_y = K \arctan g\left(x/_z\right)$$



at the surface (z=h)

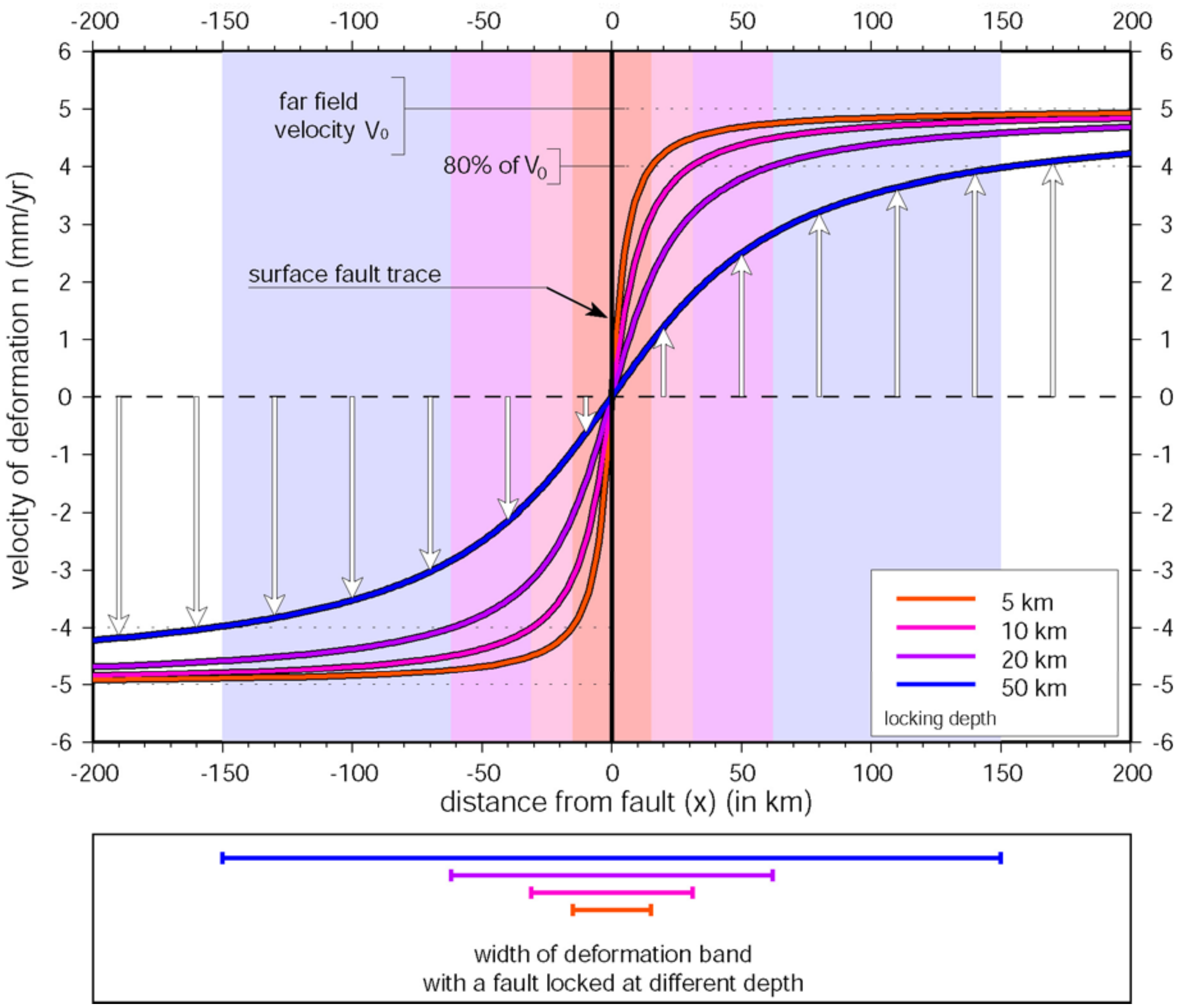
$$U_y = 2 \cdot {}^{V_0}/_{\Pi} \arctan g\left({}^{x}/_{h}\right)$$



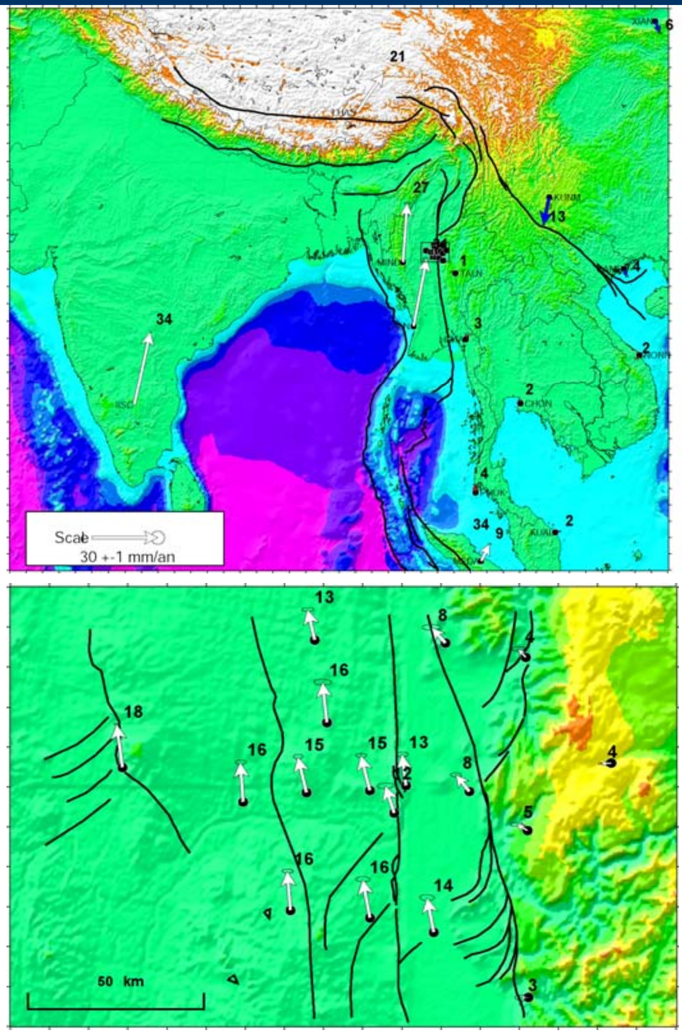
The expected profile of deformation across a strike slip fault we should see at the surface of the earth (if the crust is elastic) is shape like an **arctangant** function. The exact shape depends on the thickness of the elastic crust, also called the **locking depth**.

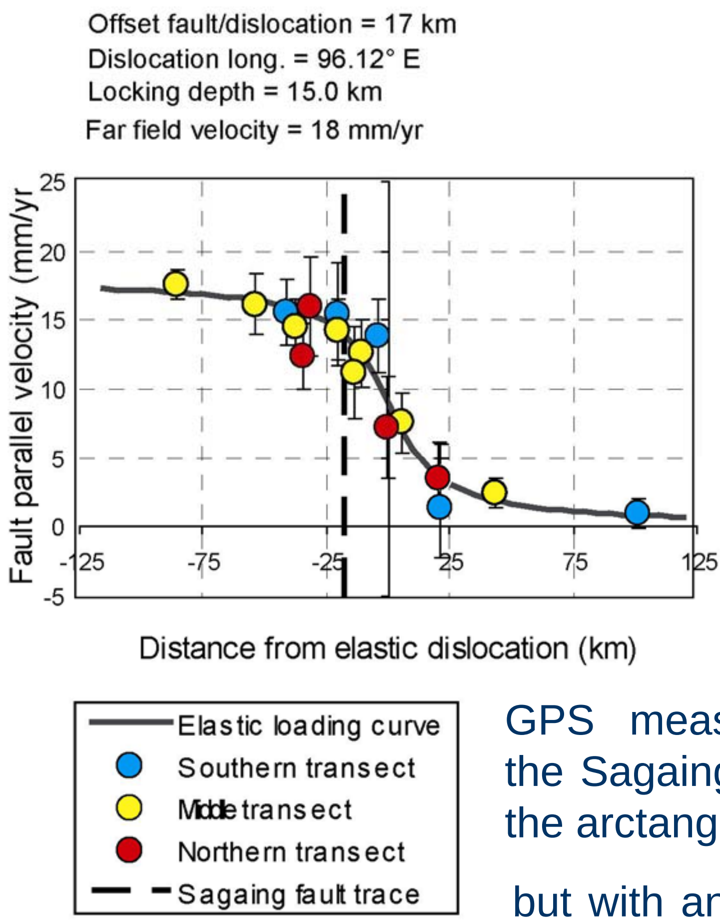
Arctang profiles

$$U_y = 2 \cdot {}^{V_0}/_{\Pi} \text{ arctang } ({}^{x}/_{h})$$



Sagaing Fault, Myanmar

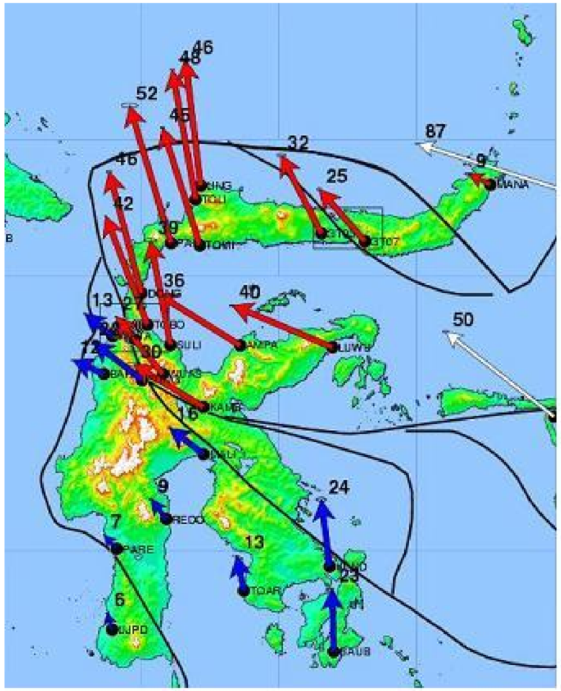


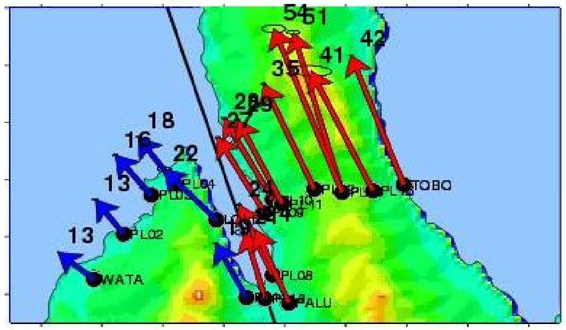


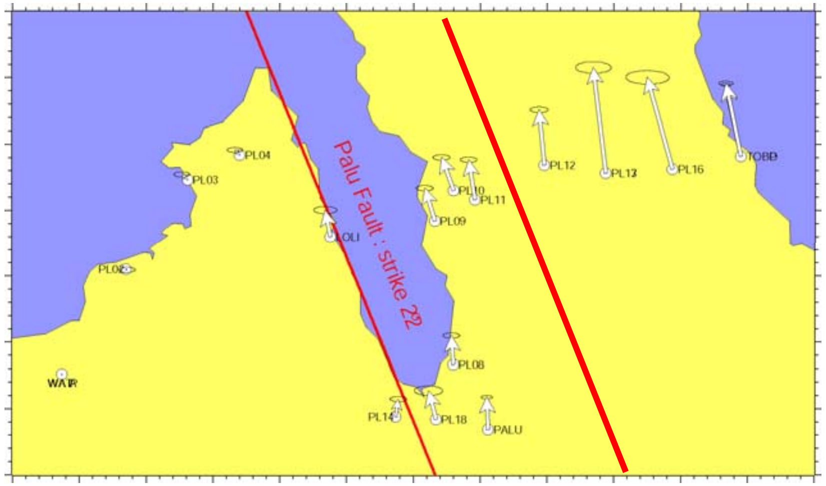
GPS measurement on the Sagaing fault fit well the arctang profile

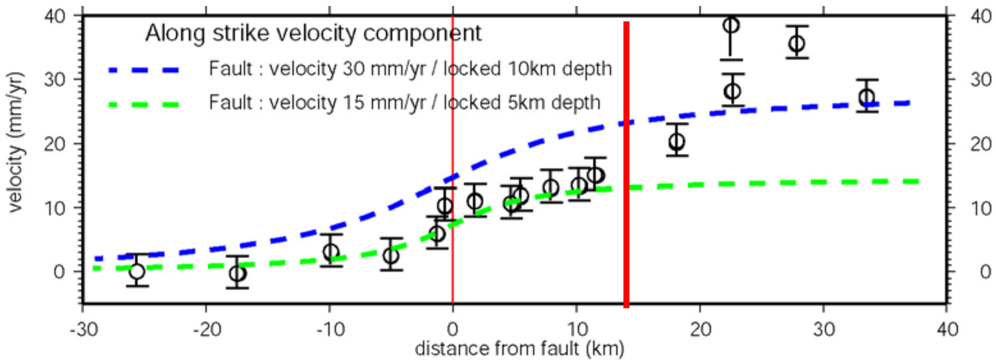
but with an offset of 10-15 km

Palu Fault, Sulawesi



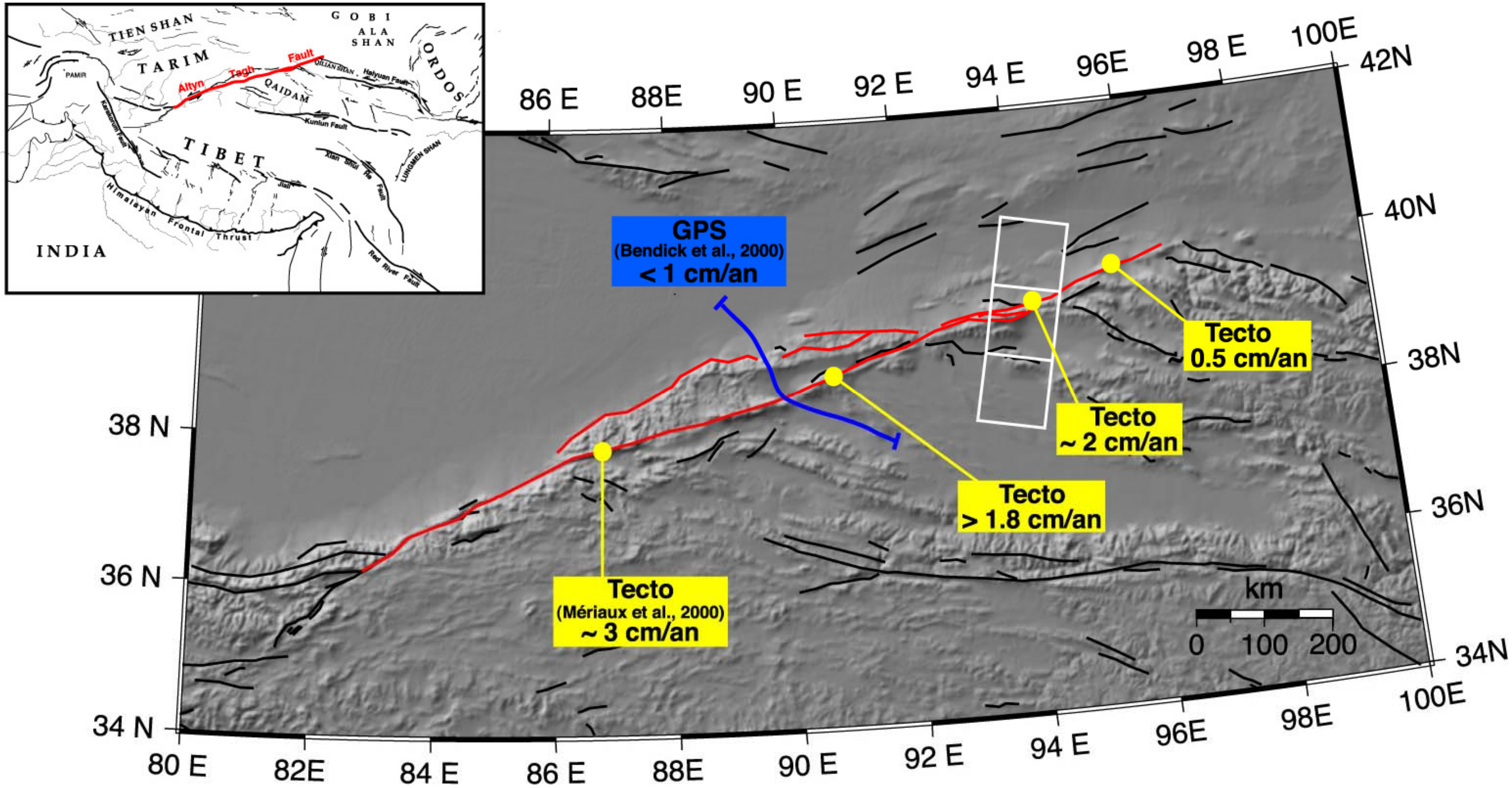






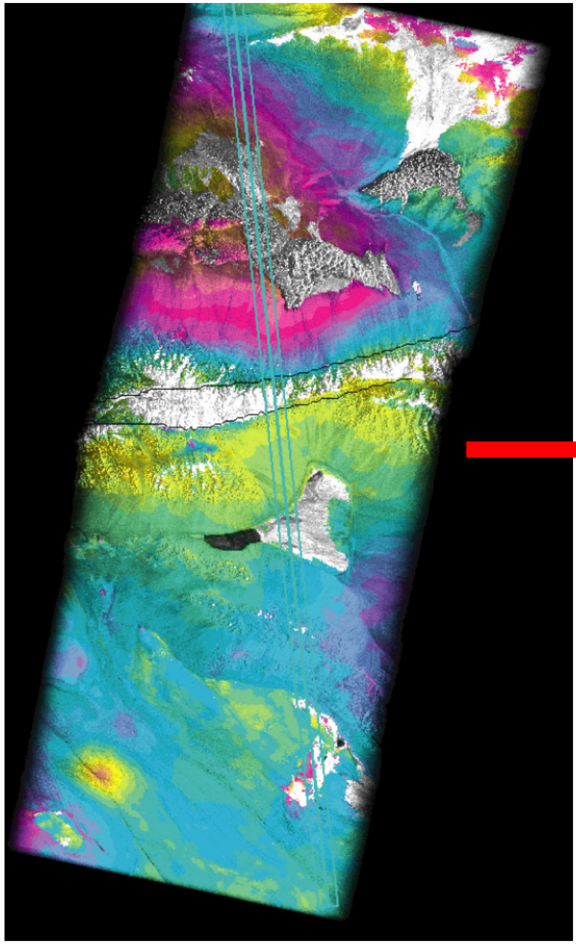
Part of the GPS data on Palu fault fits well an arctang profile. But wee need a second fault to explain all the data

Altyn Tagh Fault, China



Altyn Tagh Fault, China (INSAR)

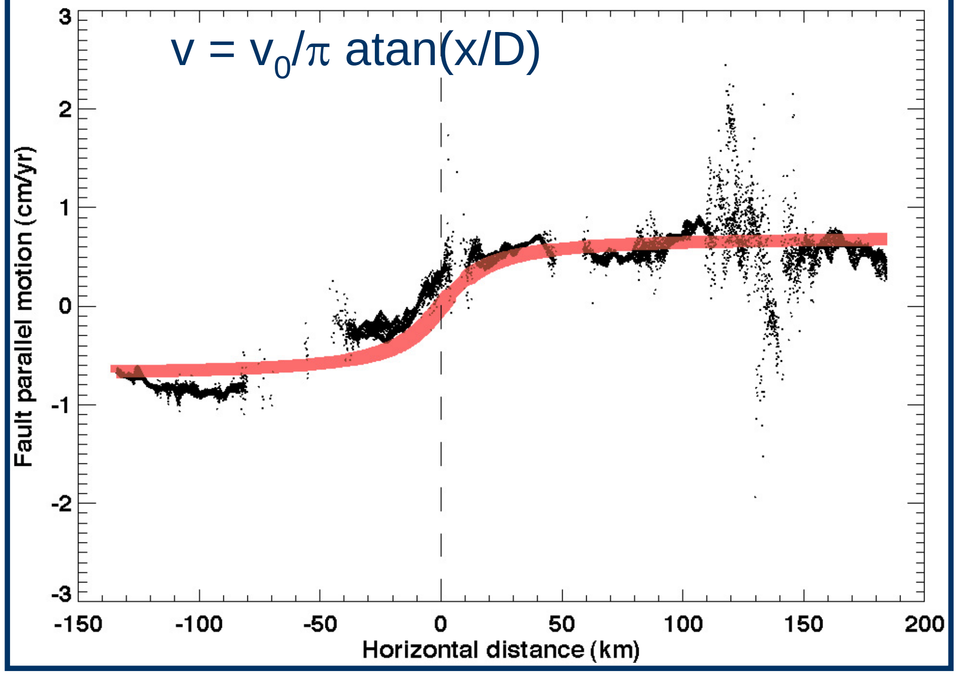
Interferogram Nov. 1995/ Nov. 1999



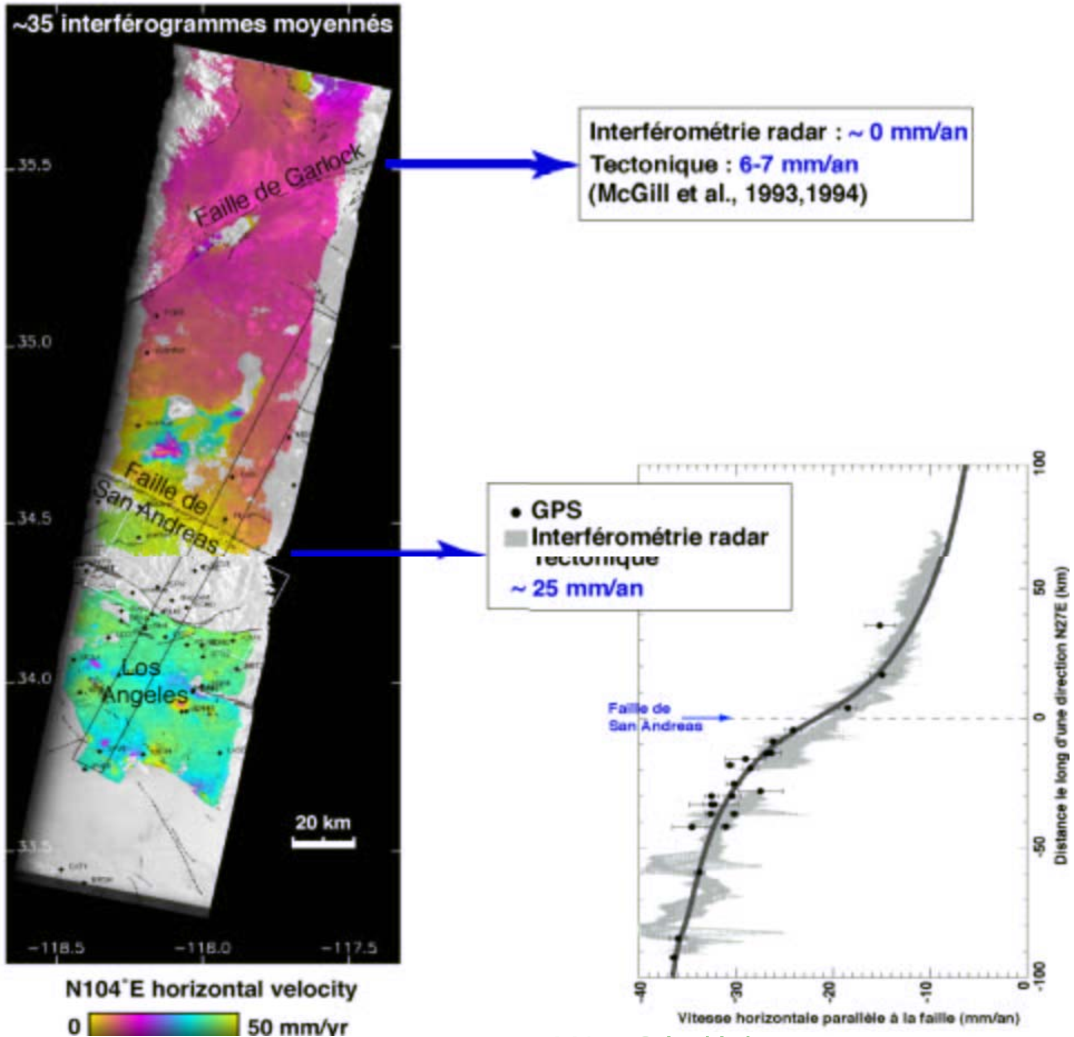
1 color cycle = 28 mm LOS displacement

Fault-parallel velocity :

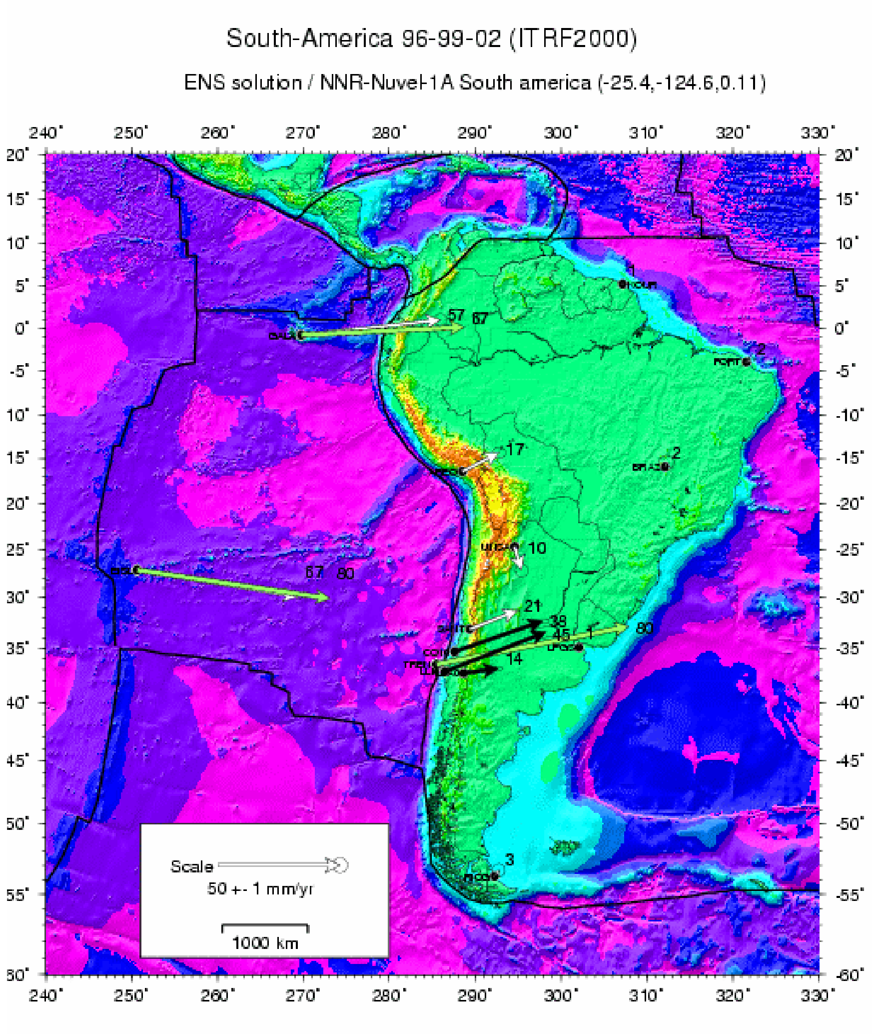
Slip rate V_0 = 1.4 cm/yr
Locking depth D = 15 km

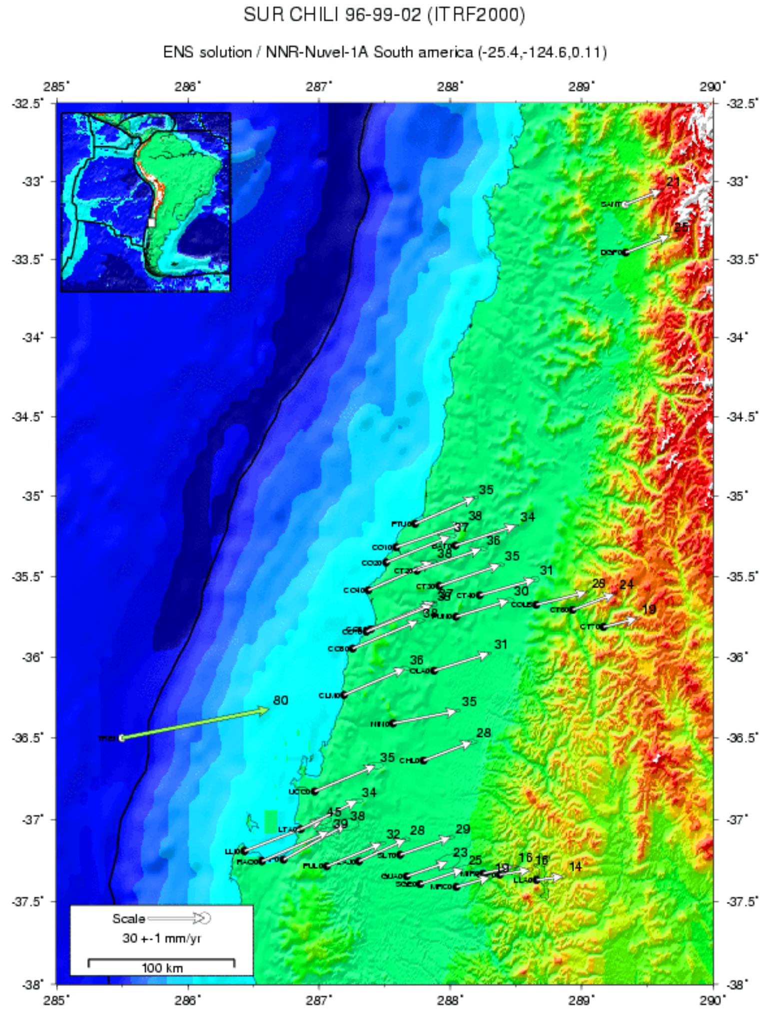


San Andreas Fault, USA (INSAR)

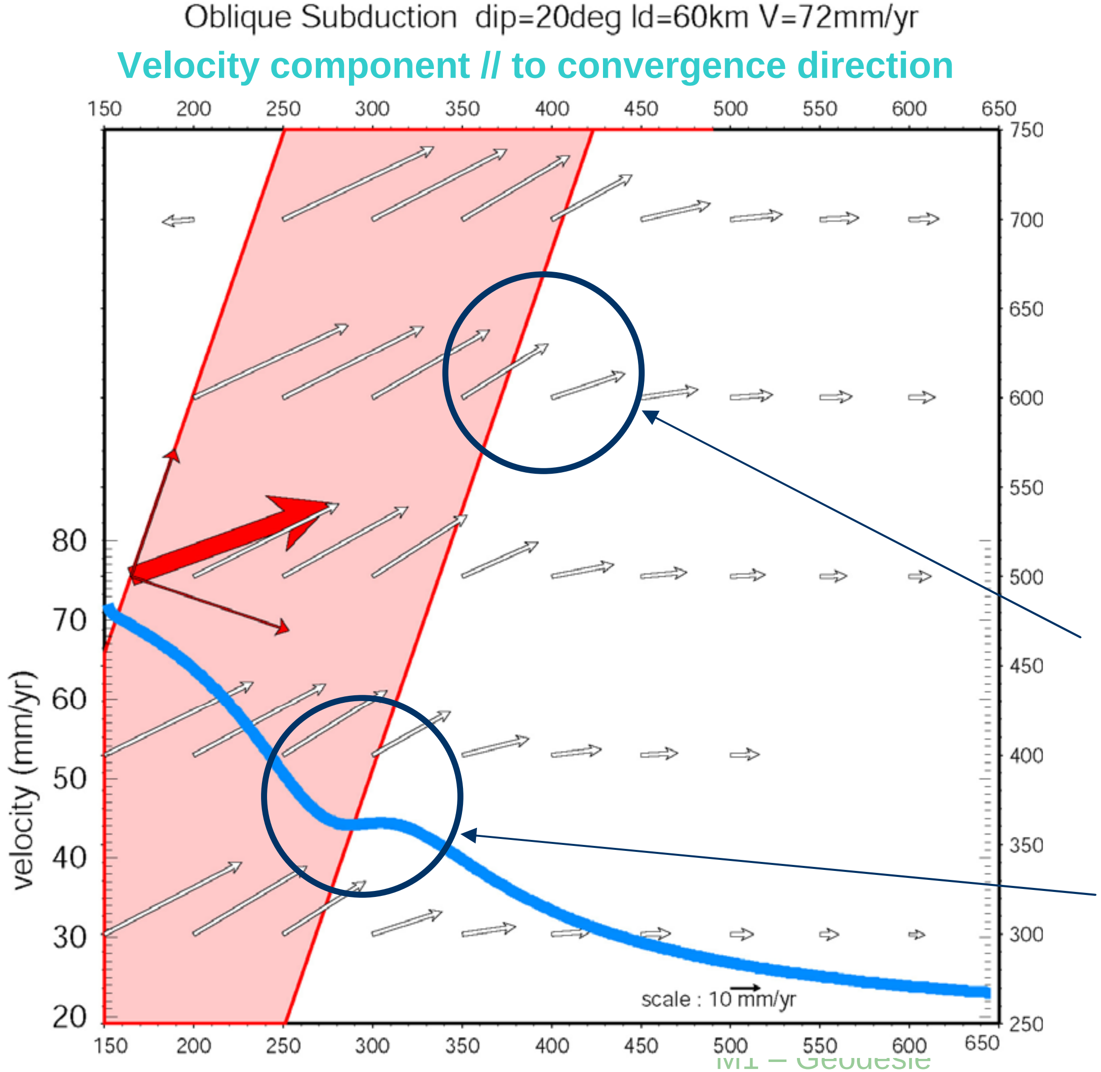


Subduction in south America





Subduction modeling



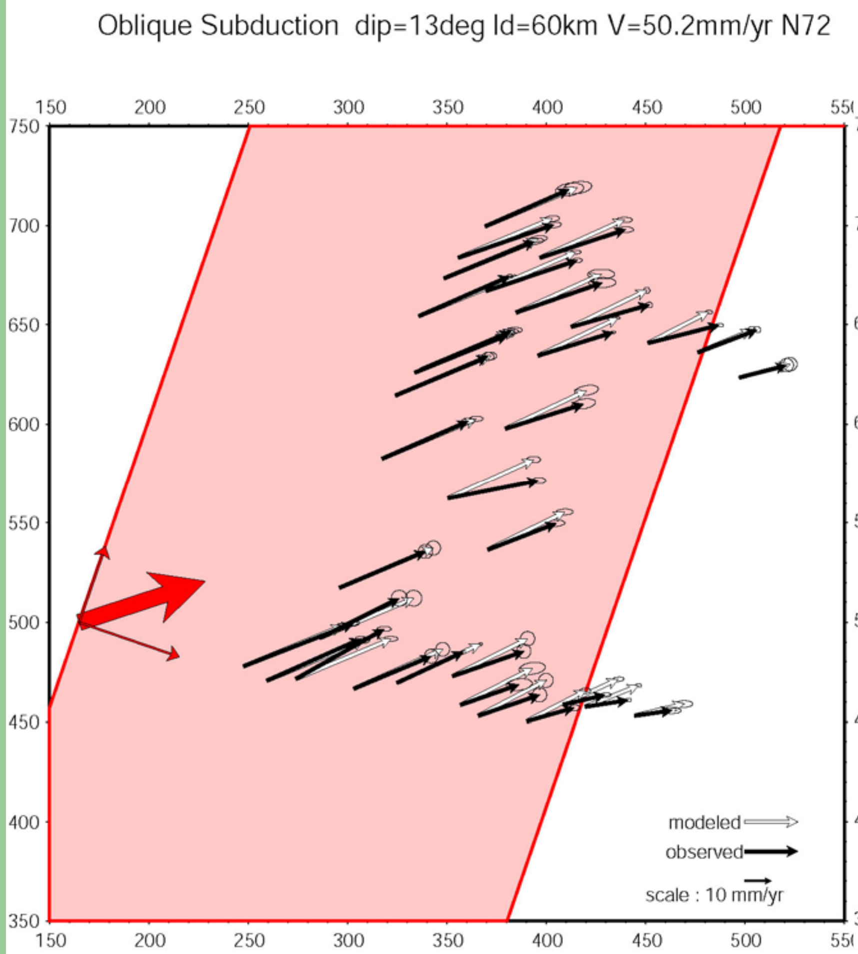
In the case of a subduction (dippping fault with downward slip) we use Okada's formulas.

We find a very large deformation area (> 500 km) because the dipping angle is only 22°

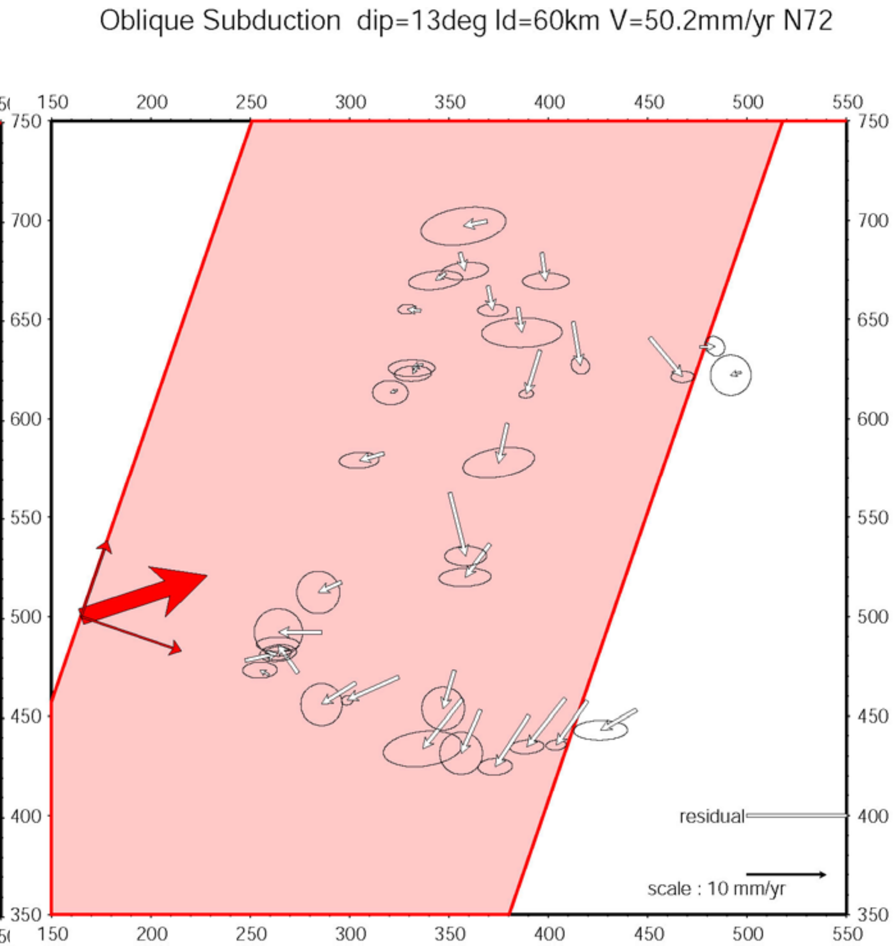
With oblique slip we predict the surface vector will start to rotate at the vertical of the end of the subduction plane

The profile of the velocity component // to the convergence shows this with a flat portion at this location

Subduction parameter adjustments



Model and data



Residuals