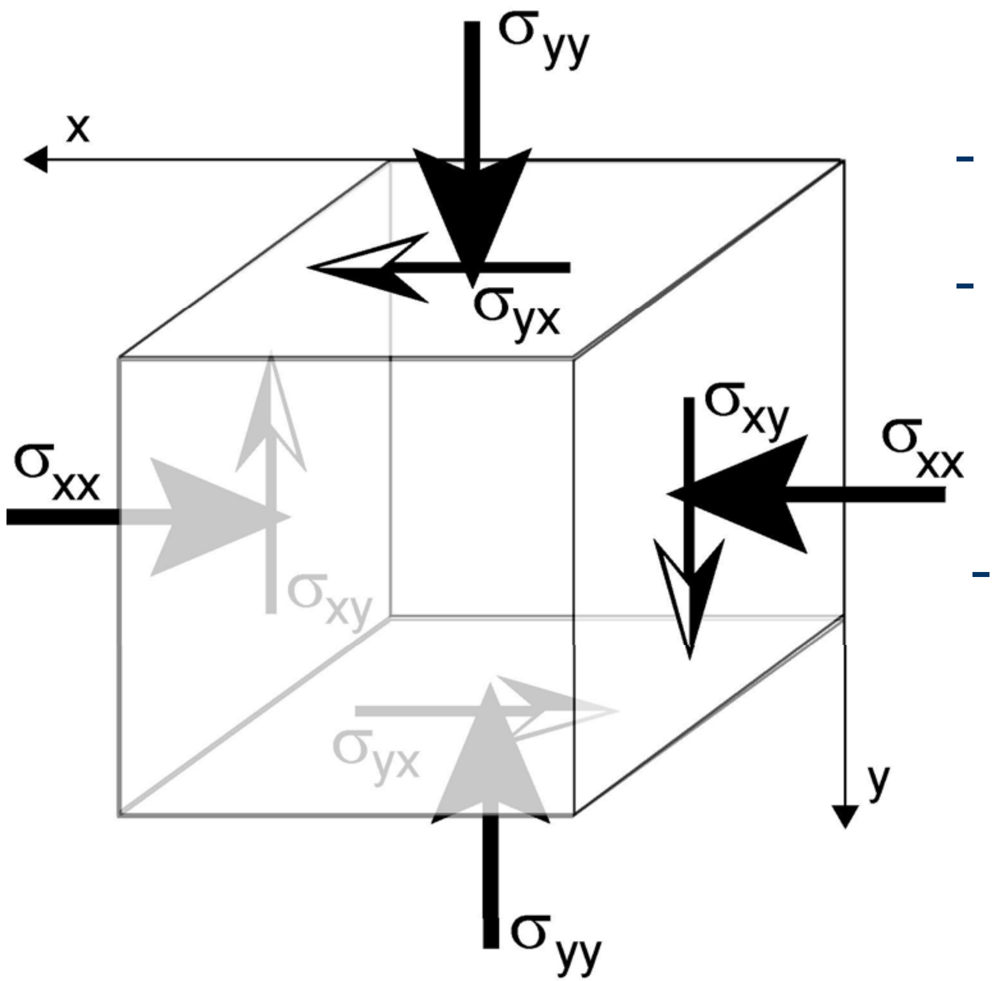
DEFORMATION PATTERN IN ELASTIC CRUST

- Stress and force in 2D
- Strain : normal and shear
- Elastic medium equations
- Vertical fault in elastic medium => arctangent
- General elastic dislocation (Okada's formulas)

Stress in 2D



- Force = σ x surface

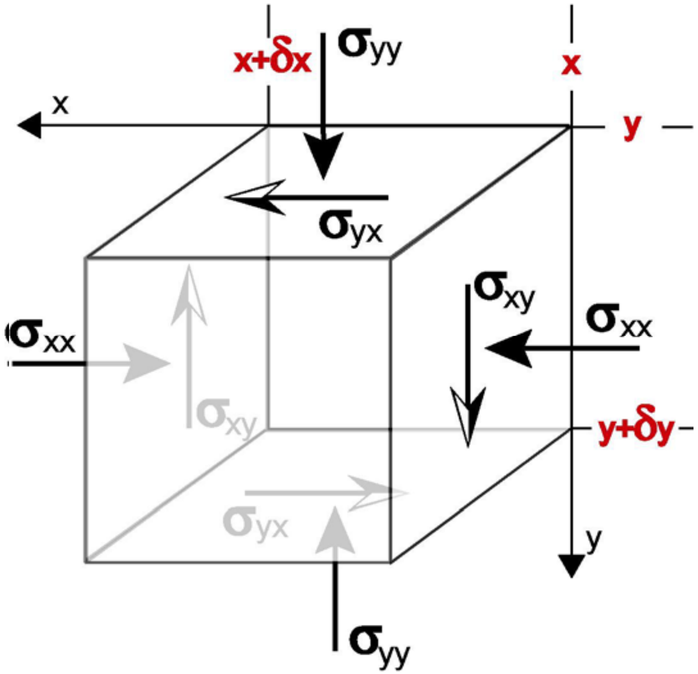
- no rotation =>

$$\sigma_{xy} = \sigma_{yx}$$

- only 3 independent components :

$$\sigma_{xx} , \sigma_{yy} , \sigma_{xy}$$

Applied forces



Normal forces on x axis :

$$= \sigma_{xx}(x).\,\delta y - \sigma_{xx}(x+\delta x).\,\delta y$$

$$= \delta y\left[\sigma_{xx}(x). - \sigma_{xx}(x+\delta x)\right]$$

$$= -\,\delta y\, {}^{d\sigma_{xx}}/_{dx}\,.\,\delta x \qquad (1)$$

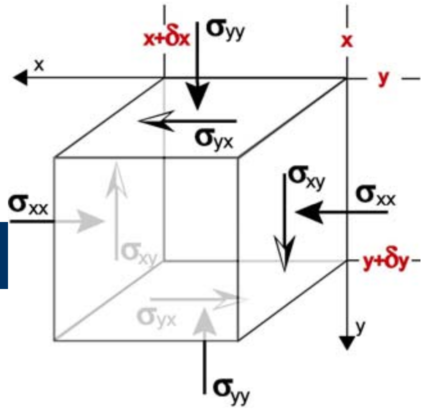
Shear forces on x axis :

$$= \sigma_{yx}(y).\,\delta x - \sigma_{yx}(y+\delta y).\,\delta x$$

$$= -\,\delta x\, {}^{d\sigma_{yx}}/_{dy}\,.\,\delta y \qquad (2)$$

Total on x axis = (1)+(2) $= \left[{}^{d\sigma_{xx}}/_{dx} + {}^{d\sigma_{yx}}/_{dy}\right]\delta x\,\delta y$

Forces Equilibrium



Total on x axis = $\left[\frac{d\sigma_{xx}}{dx} + \frac{d\sigma_{yx}}{dy}\right] \delta x \, \delta y$

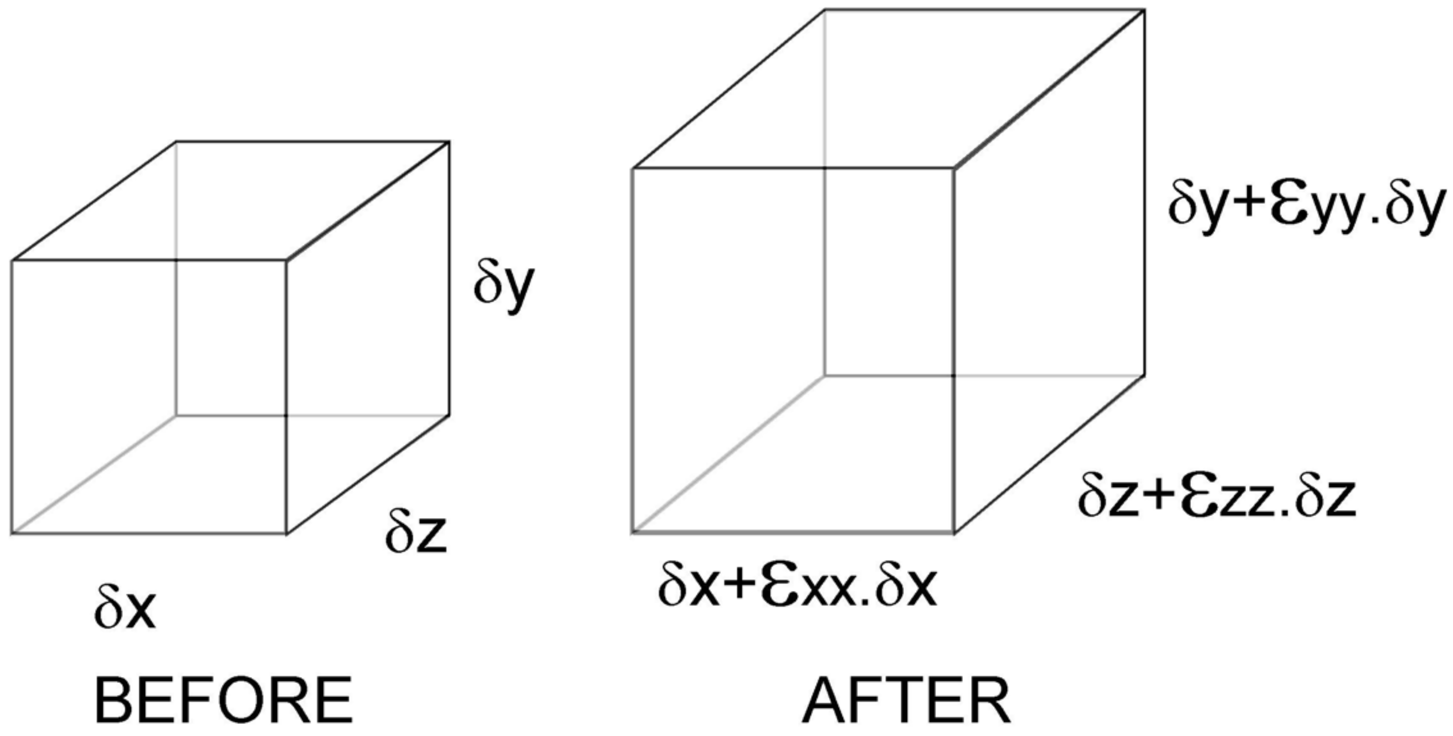
Total on y axis = $\left[\frac{d\sigma_{yy}}{dy} + \frac{d\sigma_{yx}}{dx}\right] \delta y \, \delta x$

Equilibrium =>

$$\left[\frac{d\sigma_{yy}}{dy} + \frac{d\sigma_{yx}}{dx}\right] = 0$$

$$\left[\frac{d\sigma_{xx}}{dx} + \frac{d\sigma_{yx}}{dy}\right] = 0$$

Normal strain : change length (not angles)

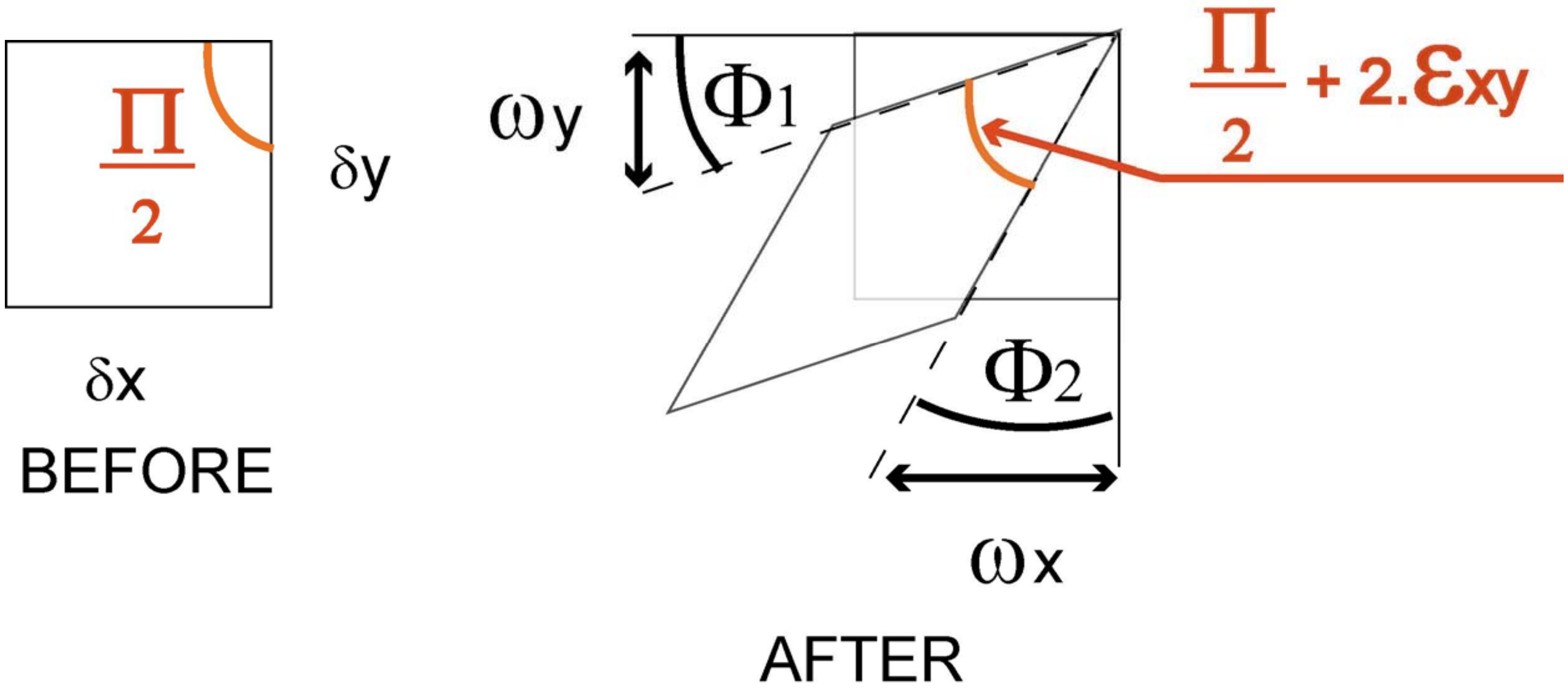


- Change of length proportional to length

- ε_{xx}, ε_{yy}, ε_{zz} are normal component of **strain**

nb : If deformation is small, change of volume is $\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$ (neglecting quadratic terms)

Shear strain : change angles



$$\varepsilon_{xy} = -\tfrac{1}{2}\left(\Phi_1 + \Phi_2\right) = \tfrac{1}{2}\left(\frac{d\omega_y}{dx} + \frac{d\omega_x}{dy}\right)$$

$\varepsilon_{xy} = \varepsilon_{yx}$ (obvious)

Solid elastic deformation (1)

- Stresses are proportional to strains
- No preferred orientations

$$\sigma_{xx} = (\lambda+2G)\,\varepsilon_{xx} + \lambda\,\varepsilon_{yy} + \lambda\,\varepsilon_{zz}$$

$$\sigma_{yy} = \lambda\,\varepsilon_{xx} + (\lambda+2G)\,\varepsilon_{yy} + \lambda\,\varepsilon_{zz}$$

$$\sigma_{zz} = \lambda\,\varepsilon_{xx} + \lambda\,\varepsilon_{yy} + (\lambda+2G)\,\varepsilon_{zz}$$

- λ and G are *Lamé* parameters

The material properties are such that a principal strain component ε *produces a stress* $(\lambda+2G)\varepsilon$ *in the same direction and stresses* $\lambda\varepsilon$ *in mutually perpendicular directions*

Solid elastic deformation (2)

Inversing stresses and strains give :

$$\varepsilon_{xx} = {}^1/_E\,\sigma_{xx} - {}^\nu/_E\,\sigma_{yy} - {}^\nu/_E\,\sigma_{zz}$$

$$\varepsilon_{yy} = -{}^\nu/_E\,\sigma_{xx} + {}^1/_E\,\sigma_{yy} - {}^\nu/_E\,\sigma_{zz}$$

$$\varepsilon_{zz} = -{}^\nu/_E\,\sigma_{xx} - {}^\nu/_E\,\sigma_{yy} + {}^1/_E\,\sigma_{zz}$$

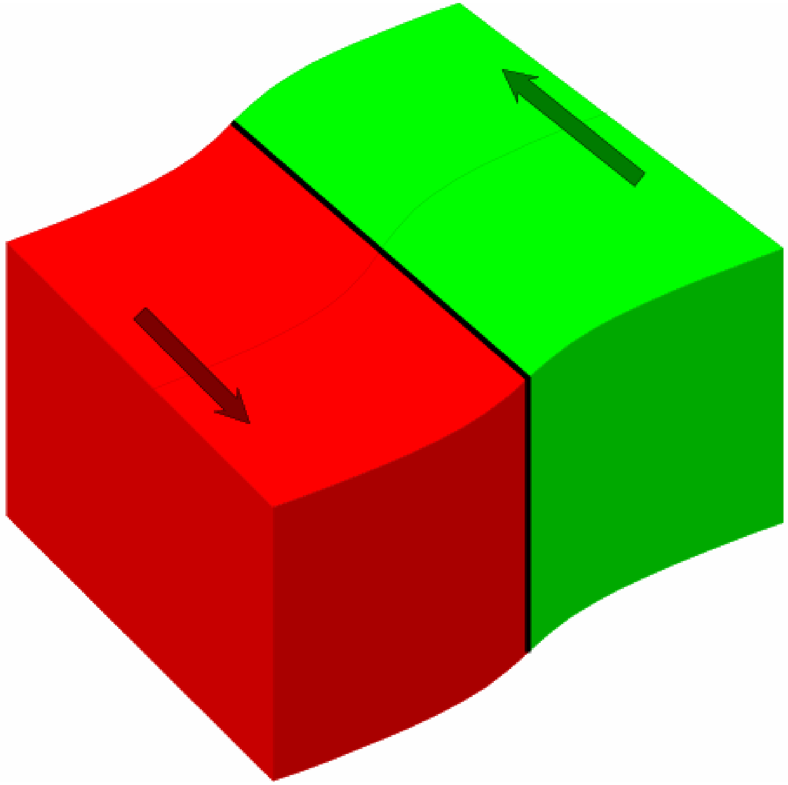
- E and ν are *Young*'s modulus and *Poisson*'s ratio

a principal stress component σ produces

a strain ${}^1/_E\,\sigma$ in the same direction and

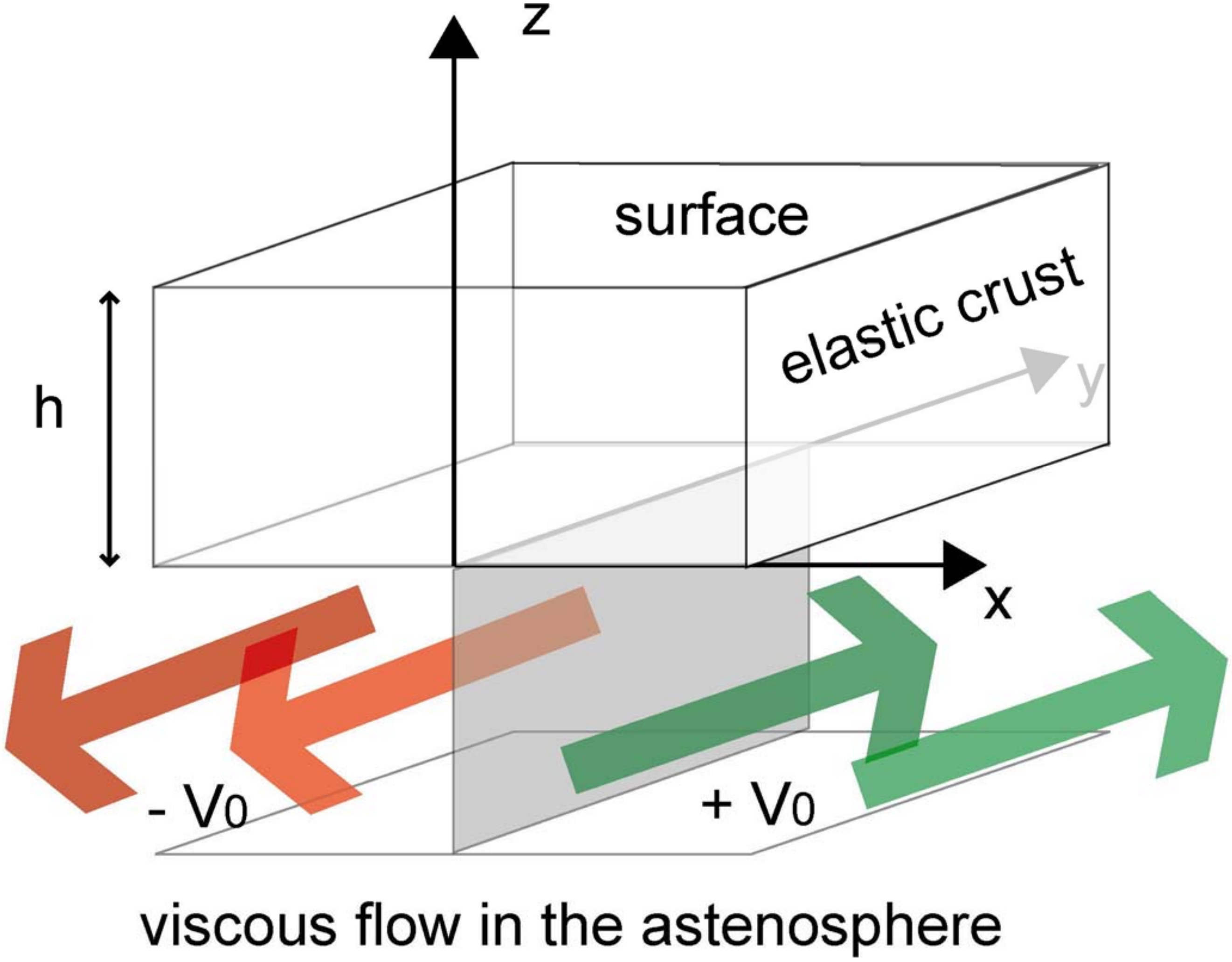
strains ${}^\nu/_E\,\sigma$ in mutually perpendicular directions

Elastic deformation across a locked fault

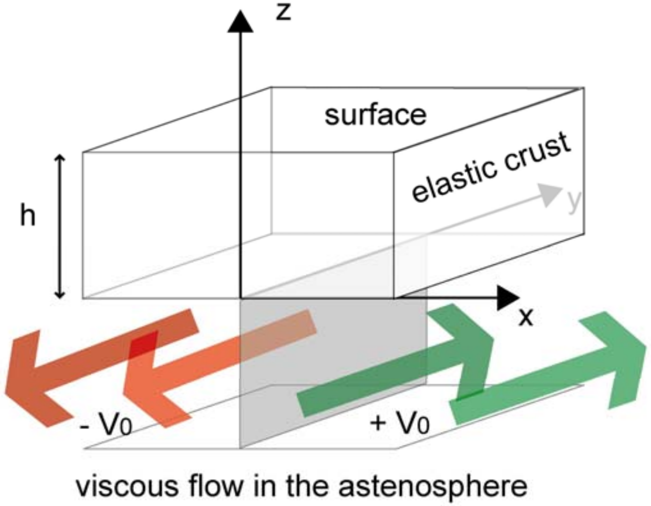


What is the shape of the accumulated deformation ?

Mathematical formulation



Mathematical formulation



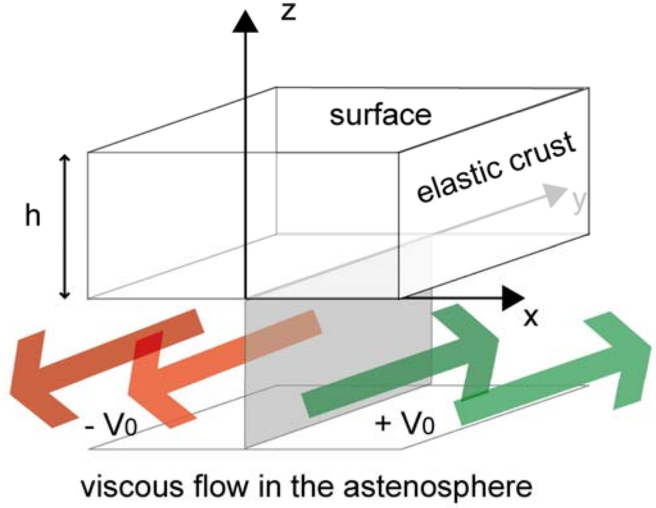
•Symetry => all derivative with y = 0

$\varepsilon_{yy} = 0$

•No gravity => $\sigma_{zz} = 0$

•What is the displacement field U in the elastic layer ?

Mathematical formulation



• Elastic equations :

(1) $\sigma_{xx} = (\lambda+2G)\,\varepsilon_{xx} + \lambda\,\varepsilon_{zz}$

(2) $\sigma_{yy} = \lambda\,\varepsilon_{xx} + \lambda\,\varepsilon_{zz}$ $\sigma_{xy} = 2G\,\varepsilon_{xy}$ $\sigma_{xz} = 2G\,\varepsilon_{xz}$

(3) $\sigma_{zz} = \lambda\,\varepsilon_{xx} + (\lambda+2G)\,\varepsilon_{zz}$ $\sigma_{yz} = 2G\,\varepsilon_{yz}$

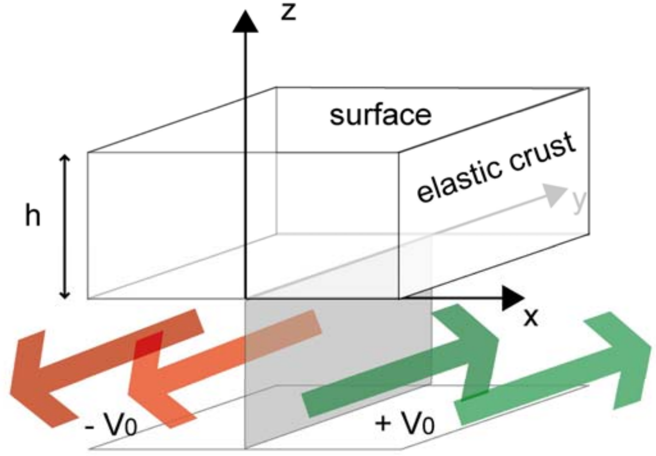
(3) + $\sigma_{zz} = 0$ => $\lambda\,\varepsilon_{xx} + \lambda\,\varepsilon_{zz} = -2G\,\varepsilon_{zz}$

↳ and (2) => $\sigma_{yy} = \lambda\,\varepsilon_{xx} + \lambda\,\varepsilon_{zz} = -2\,G\,\varepsilon_{zz}$

=> $\varepsilon_{xx} = -\,(2G + \lambda)/\,\lambda\,\varepsilon_{zz}$

↳ and (1) => $\sigma_{xx} = \left[-\,{(\lambda+2G)^2}/{\lambda} + \lambda\right]\varepsilon_{zz}$

Mathematical formulation



viscous flow in the astenosphere

• Force equilibrium along the 3 axis

$$\text{(x)} \quad d\sigma_{xx}/dx + \cancel{d\sigma_{yx}/dy} + d\sigma_{xz}/dz = 0$$

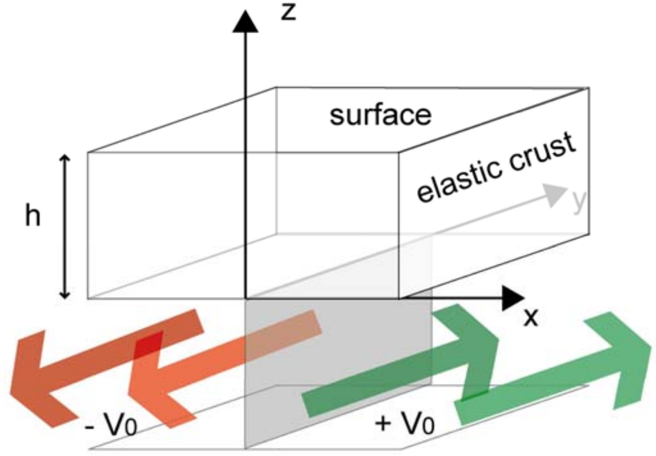
$$\text{(y)} \quad d\sigma_{xy}/dx + \cancel{d\sigma_{yy}/dy} + d\sigma_{yz}/dz = 0$$

$$\text{(z)} \quad d\sigma_{xz}/dx + \cancel{d\sigma_{yz}/dy} + \cancel{d\sigma_{zz}/dz} = 0$$

• Derivation of eq. 1 with x and eq. 3 give : $d^2\sigma_{xx}/dx^2 = 0$

• equation 2 becomes : $d\sigma_{xy}/dx + d\sigma_{yz}/dz = 0$

Mathematical formulation



viscous flow in the astenosphere

relations between

stress (σ) and displacement vector (U)

$$\sigma_{xy} = 2G\,\varepsilon_{xy} = 2G\left[\partial U_x/\partial y + \partial U_y/\partial x\right].^1/_2$$

$$\sigma_{yz} = 2G\,\varepsilon_{yz} = 2G\left[\partial U_z/\partial y + \partial U_y/\partial z\right].^1/_2$$

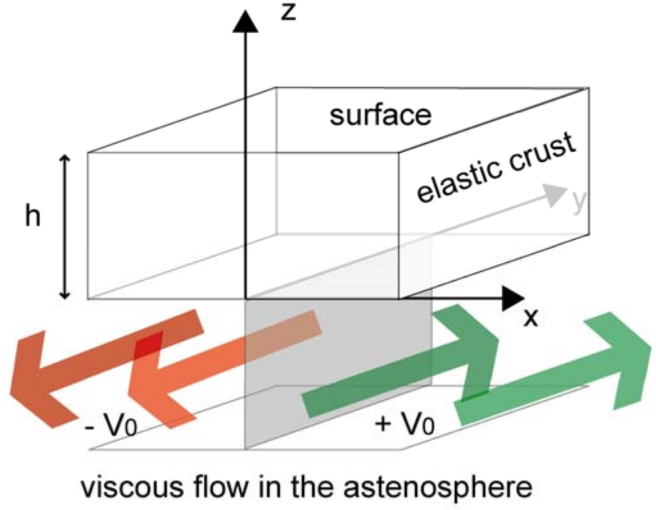
Using $\partial\sigma_{xy}/\partial x + \partial\sigma_{yz}/\partial z = 0$ we obtain :

$$\partial/\partial x\left[\cancel{\partial U_x/\partial y} + \partial U_y/\partial x\right] + \partial/\partial z\left[\cancel{\partial U_z/\partial y} + \partial U_y/\partial z\right] = 0$$

↳ $$\partial^2 U_y/\partial x^2 + \partial^2 U_y/\partial z^2 = 0$$

Mathematical formulation

$$d^2U_y / dx^2 + d^2U_y / dz^2 = 0$$



What is U_y, function of x and z, solution of this equation ?

Guess : $U_y = K \arctan g\,(^x/_z)$ works fine !

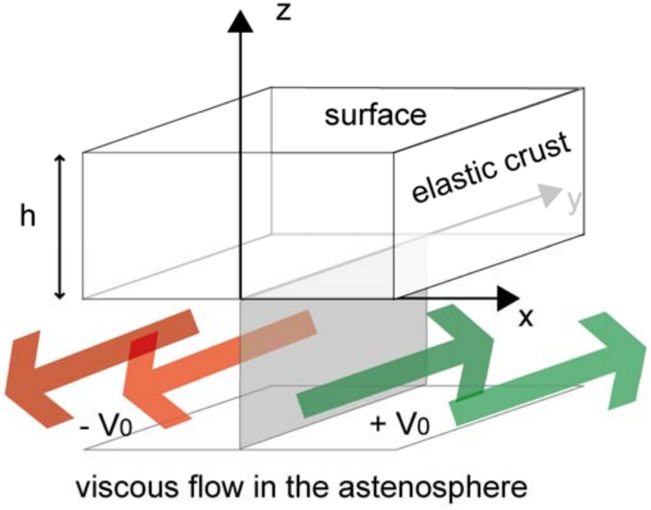
Nb. $\quad ^{d\arctan(\alpha)}/_{d\alpha} = {}^1/_{(1+\alpha 2\,)}$

$$dU_y/dx = {}^K/_{z(1+x^2/z^2)} \quad => \quad d^2U_y/dx^2 = -2Kxz / (z^2+x^2)$$

$$dU_y/dz = {}^{-Kx}/_{z^2\,(1+x^2/z^2)} \quad => \quad d^2U_y/dz^2 = 2Kxz / (x^2+z^2)$$

Mathematical formulation

$$U_y = K \arctan g\,(^x/_z)$$



Boundary condition at the base of the crust (z=0)

$$U_y \quad = K \,.\, \Pi/2 \quad \text{if } x > 0 \qquad = K \,.\, -\Pi/2 \quad \text{if } x < 0$$

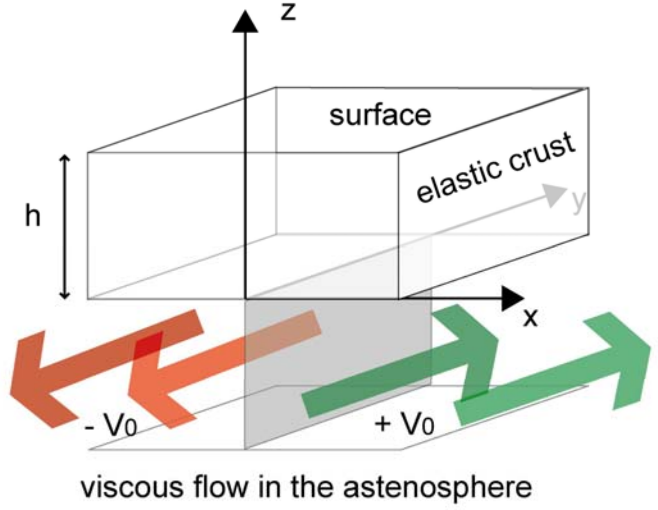
And also :

$$U_y \quad = +V_0 \quad \text{if } x > 0 \qquad = -V_0 \quad \text{if } x < 0$$

$$\Rightarrow \quad K = 2\,.\,^{V_0}/_{\Pi}$$

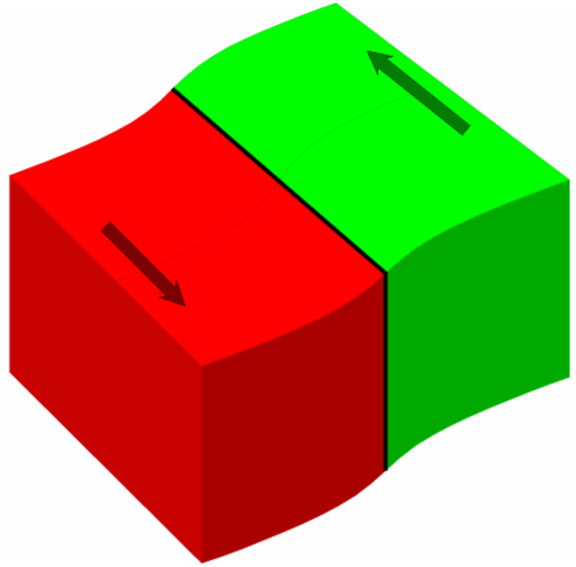
Mathematical formulation

$$U_y = K \arctan g\,(x/z)$$



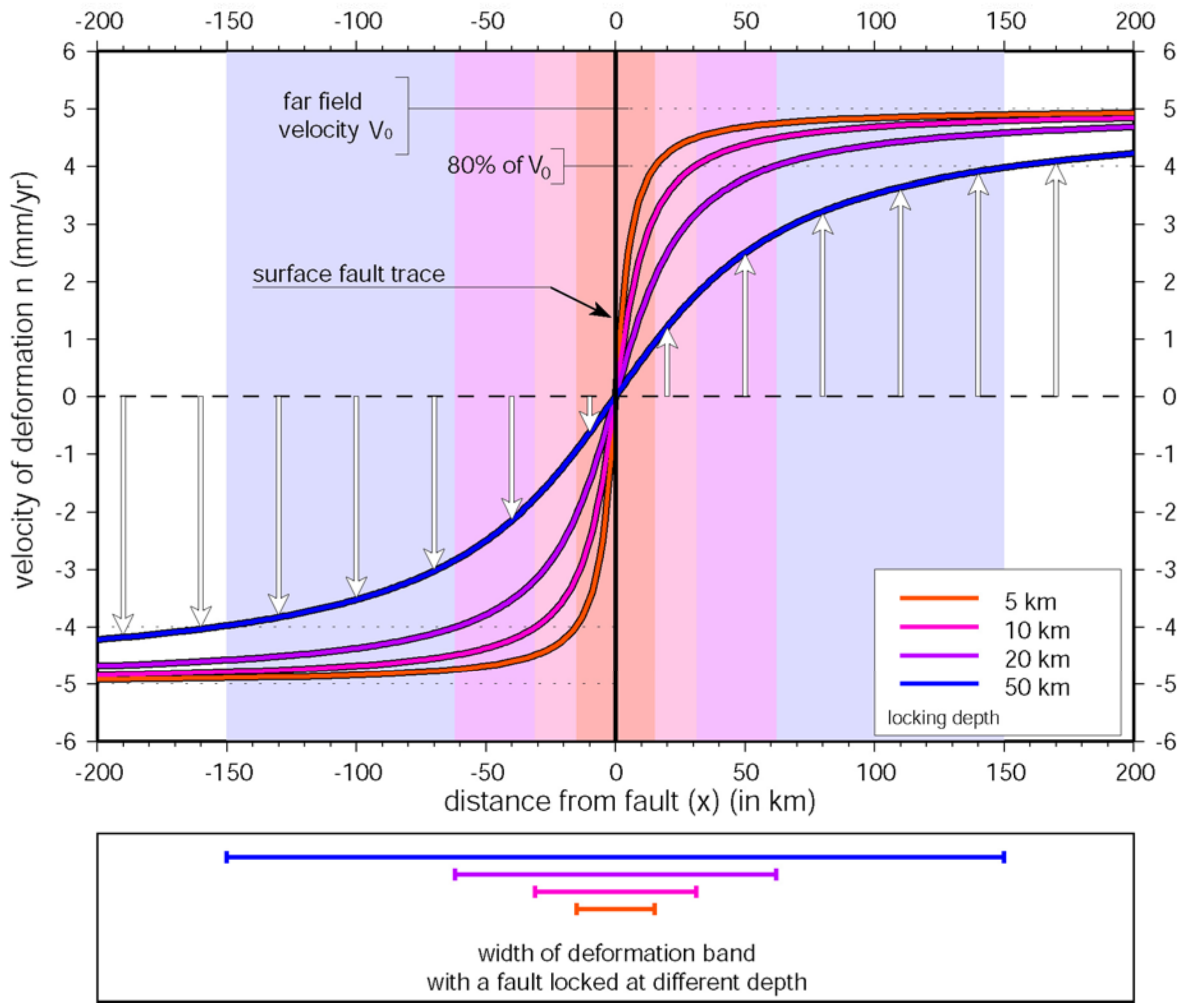
at the surface (z=h)

$$U_y = 2 \cdot V_0 / \pi \arctan g\,(x/h)$$

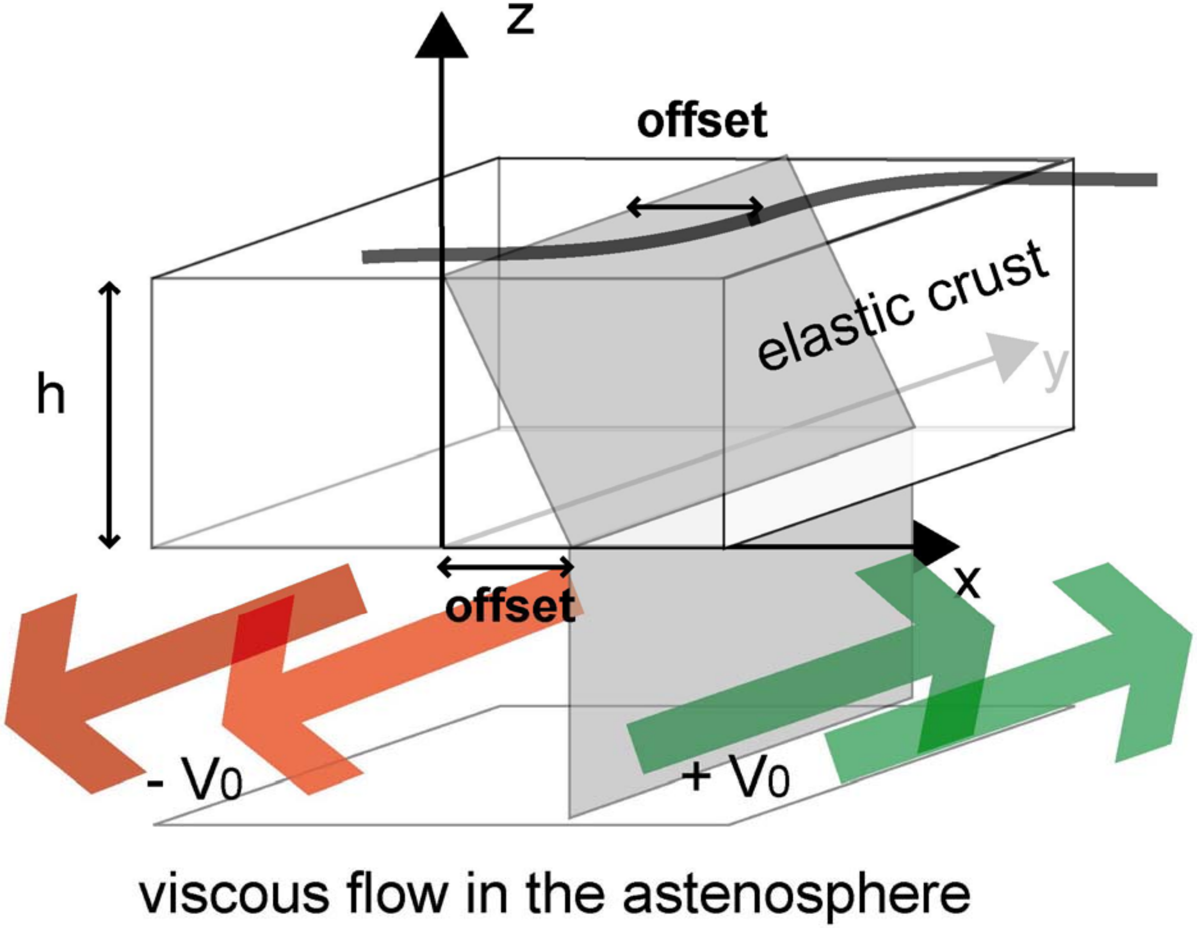


Arctang profiles

$$U_y = 2 \cdot {}^{V_0}/_{\Pi} \arctan g ({}^{x}/_{h})$$

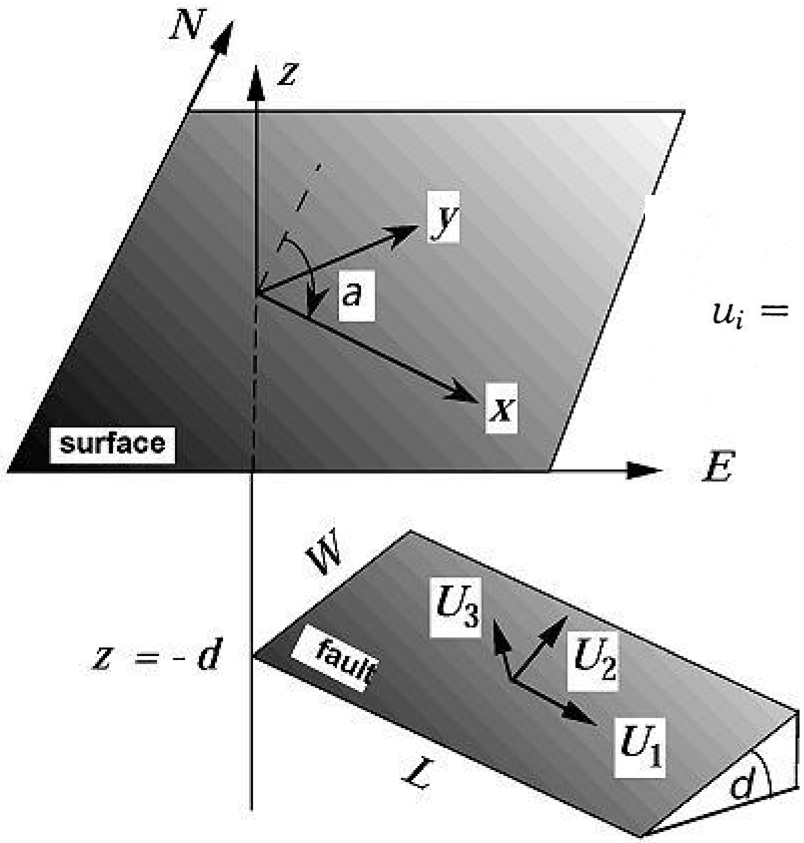


deeping fault



Elastic dislocation (Okada, 1985)

Surface deformation due to shear and tensile faults in a half space, BSSA vol75, n°4, 1135-1154, 1985.



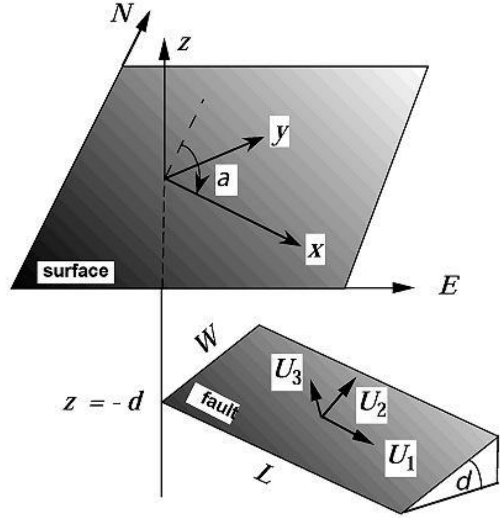
The displacement field $u_i(x_1,x_2,x_3)$ due to a dislocation $\Delta u_j(\xi_1,\xi_2,\xi_3)$ across a surface Σ in an isotropic medium is given by :

$$u_i = \frac{1}{F}\int\int_\Sigma \Delta u_j \left[\lambda\delta_{jk}\frac{\partial u_i^n}{\partial \xi_n} + \mu\left(\frac{\partial u_i^j}{\partial \xi_k} + \frac{\partial u_i^k}{\partial \xi_j}\right)\right]\nu_k \, d\Sigma$$

Where δ_{jk} is the Kronecker delta, λ and μ are Lamé's parameters, ν_k is the direction cosine of the normal to the surface element $d\Sigma$.

u_i^j is the ith component of the displacement at (x_1,x_2,x_3) due to the jth direction point force of magnitude F at (ξ_1,ξ_2,ξ_3)

Elastic dislocation (Okada, 1985)



(1) displacements

For strike-slip

$$\begin{cases} u_x^{\,0} = -\dfrac{U_1}{2\pi}\left[\dfrac{3x^2q}{R^5} + I_1^{\,0}\sin\delta\right]\Delta\Sigma \\ u_y^{\,0} = -\dfrac{U_1}{2\pi}\left[\dfrac{3xyq}{R^5} + I_2^{\,0}\sin\delta\right]\Delta\Sigma \\ u_z^{\,0} = -\dfrac{U_1}{2\pi}\left[\dfrac{3xdq}{R^5} + I_4^{\,0}\sin\delta\right]\Delta\Sigma. \end{cases}$$

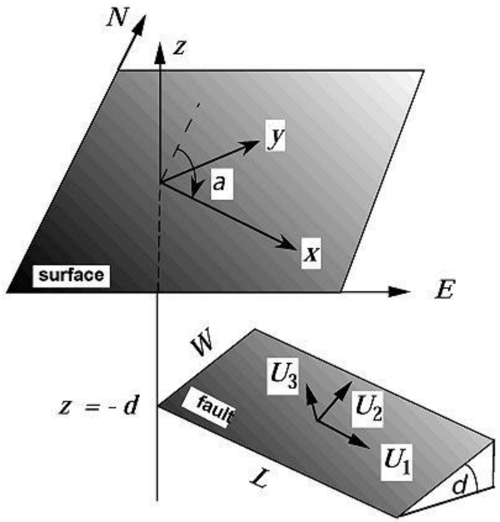
For dip-slip

$$\begin{cases} u_x^{\,0} = -\dfrac{U_2}{2\pi}\left[\dfrac{3xpq}{R^5} - I_3^{\,0}\sin\delta\cos\delta\right]\Delta\Sigma \\ u_y^{\,0} = -\dfrac{U_2}{2\pi}\left[\dfrac{3ypq}{R^5} - I_1^{\,0}\sin\delta\cos\delta\right]\Delta\Sigma \\ u_z^{\,0} = -\dfrac{U_2}{2\pi}\left[\dfrac{3dpq}{R^5} - I_5^{\,0}\sin\delta\cos\delta\right]\Delta\Sigma. \end{cases}$$

For tensile fault

$$\begin{cases} u_x^{\,0} = \dfrac{U_3}{2\pi}\left[\dfrac{3xq^2}{R^5} - I_3^{\,0}\sin^2\delta\right]\Delta\Sigma \\ u_y^{\,0} = \dfrac{U_3}{2\pi}\left[\dfrac{3yq^2}{R^5} - I_1^{\,0}\sin^2\delta\right]\Delta\Sigma \\ u_z^{\,0} = \dfrac{U_3}{2\pi}\left[\dfrac{3dq^2}{R^5} - I_5^{\,0}\sin^2\delta\right]\Delta\Sigma \end{cases}$$

Elastic dislocation (Okada, 1985)

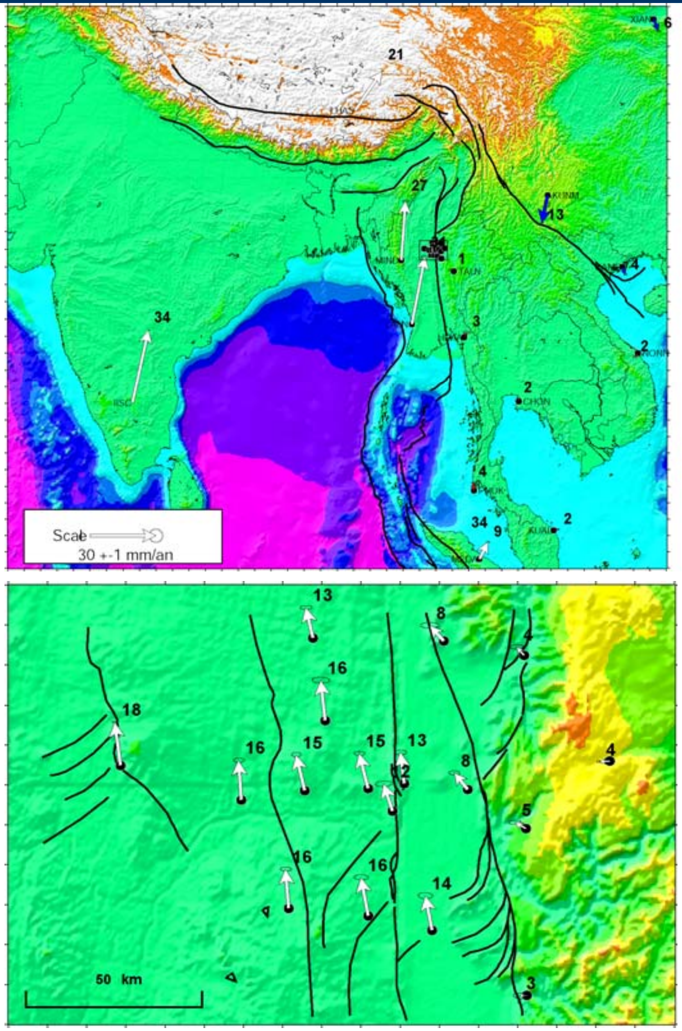


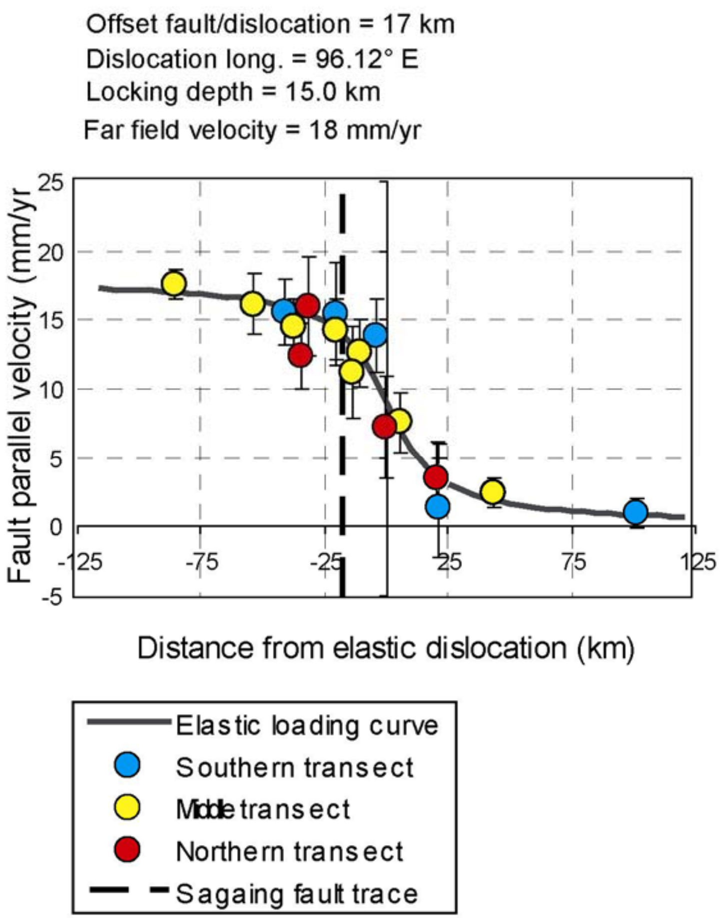
Where :

$$
\begin{cases}
I_1^0 = \dfrac{\mu}{\lambda + \mu} y \left[\dfrac{1}{R(R+d)^2} - x^2 \dfrac{3R+d}{R^3(R+d)^3} \right] \\[2ex]
I_2^0 = \dfrac{\mu}{\lambda + \mu} x \left[\dfrac{1}{R(R+d)^2} - y^2 \dfrac{3R+d}{R^3(R+d)^3} \right] \\[2ex]
I_3^0 = \dfrac{\mu}{\lambda + \mu} \left[\dfrac{x}{R^3} \right] - I_2^0 \\[2ex]
I_4^0 = \dfrac{\mu}{\lambda + \mu} \left[\qquad - xy \dfrac{2R+d}{R^3(R+d)^2} \right] \\[2ex]
I_5^0 = \dfrac{\mu}{\lambda + \mu} \left[\dfrac{1}{R(R+d)} - x^2 \dfrac{2R+d}{R^3(R+d)^2} \right]
\end{cases}
$$

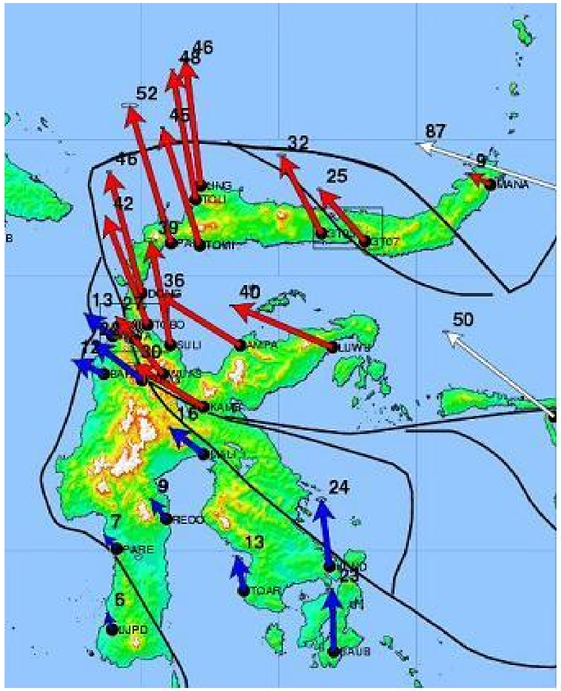
$$
\begin{cases}
p = y \cos \delta + d \sin \delta \\
q = y \sin \delta - d \cos \delta \\
R^2 = x^2 + y^2 + d^2 = x^2 + p^2 + q^2.
\end{cases}
$$

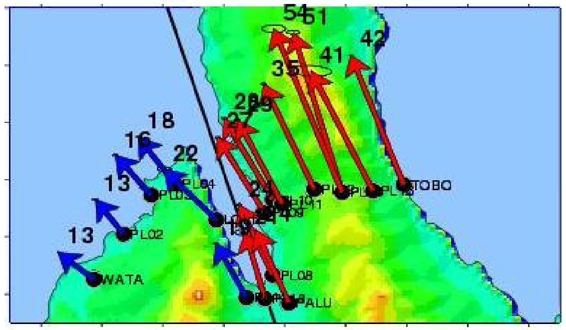
Sagaing Fault, Myanmar

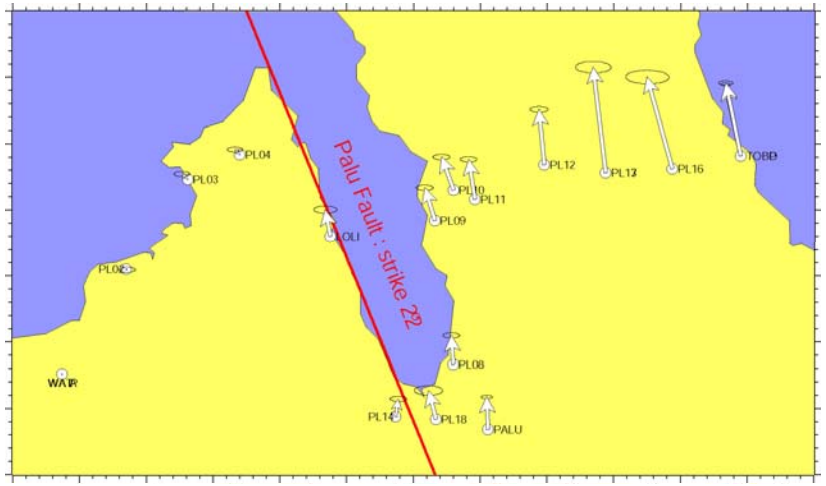


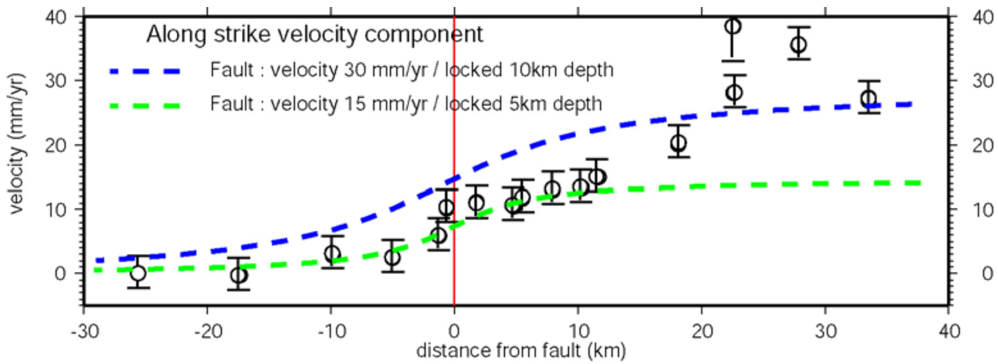


Palu Fault, Sulawesi

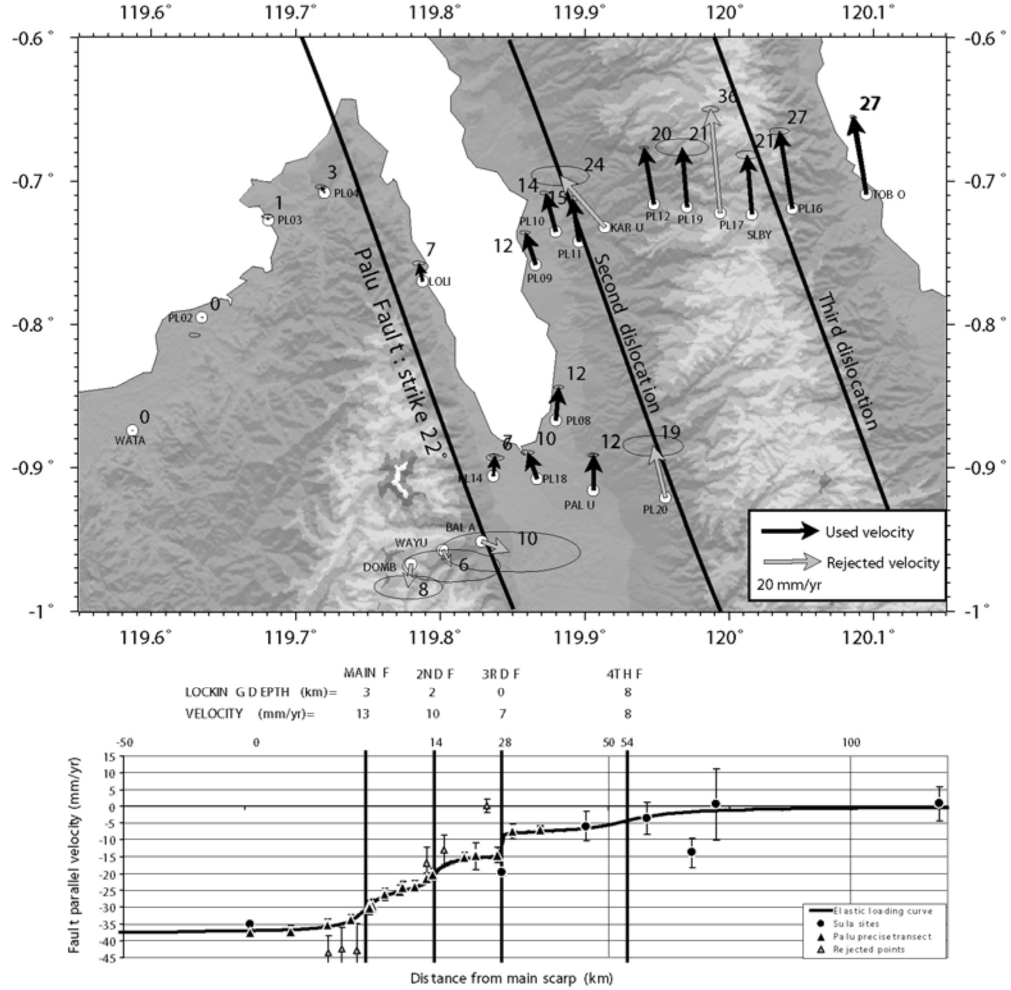




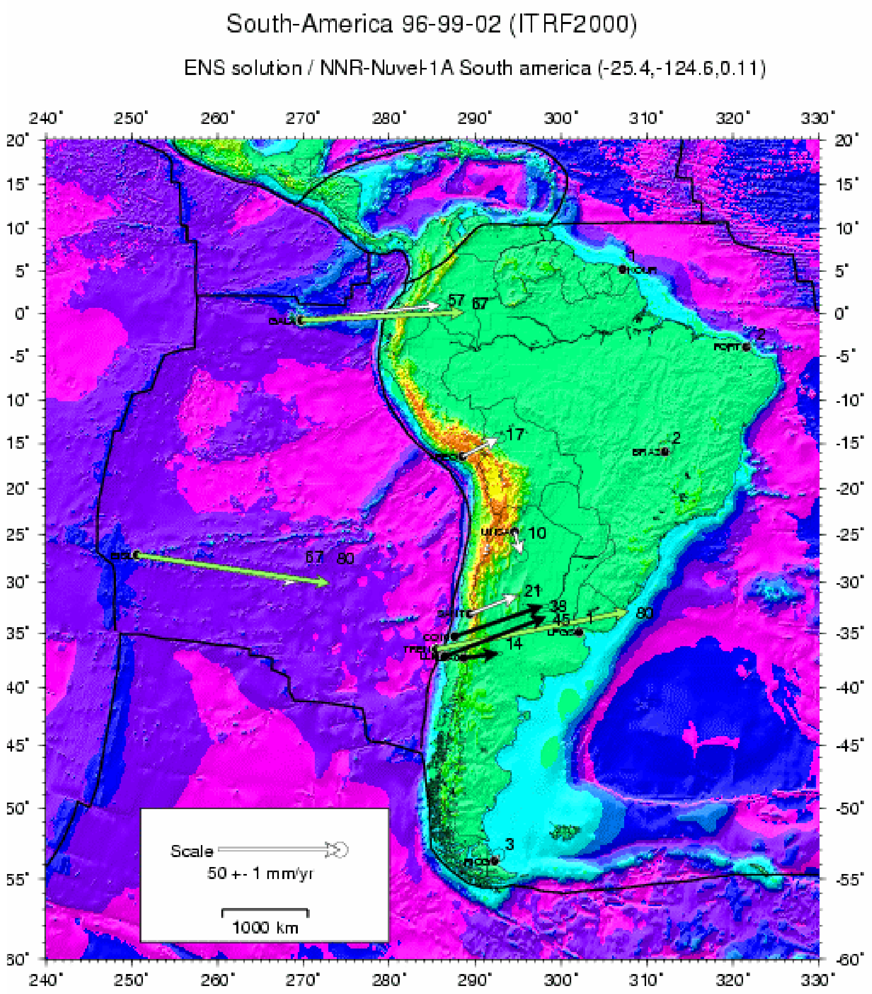


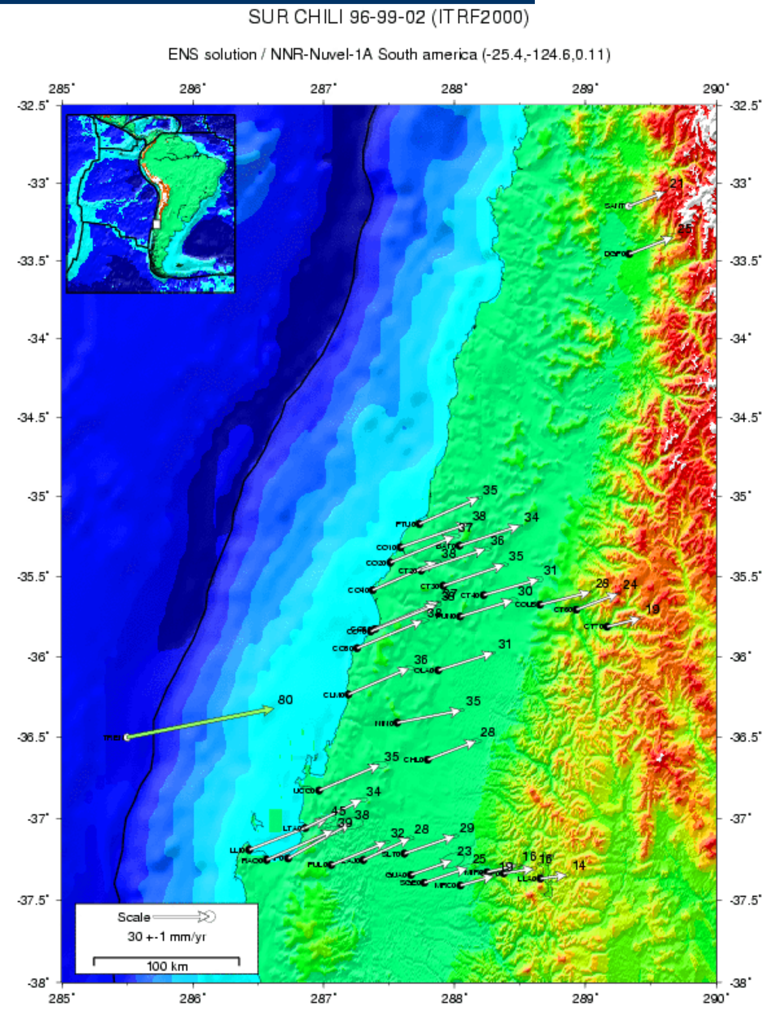


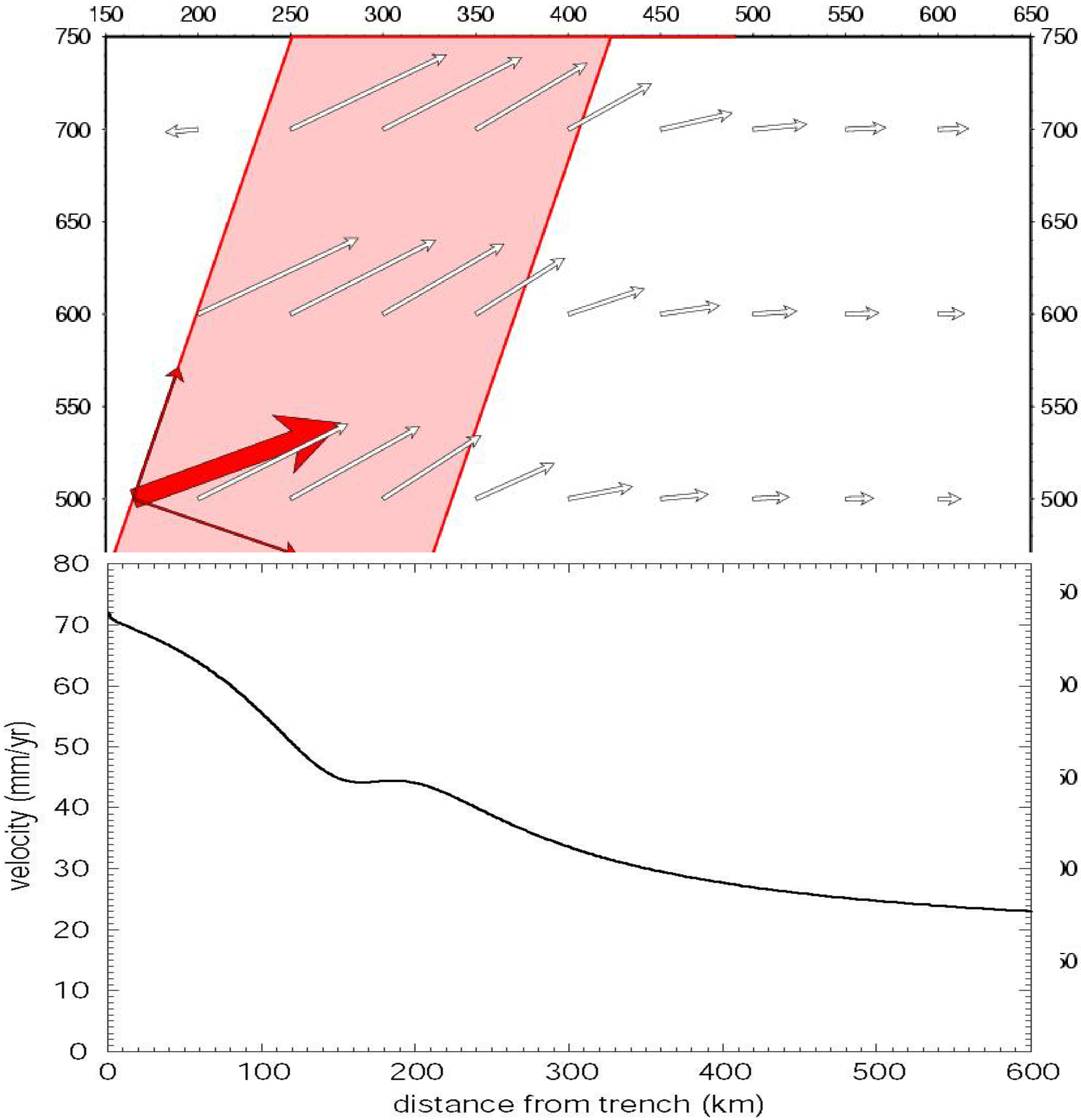
Getting more complex



Subduction in south america

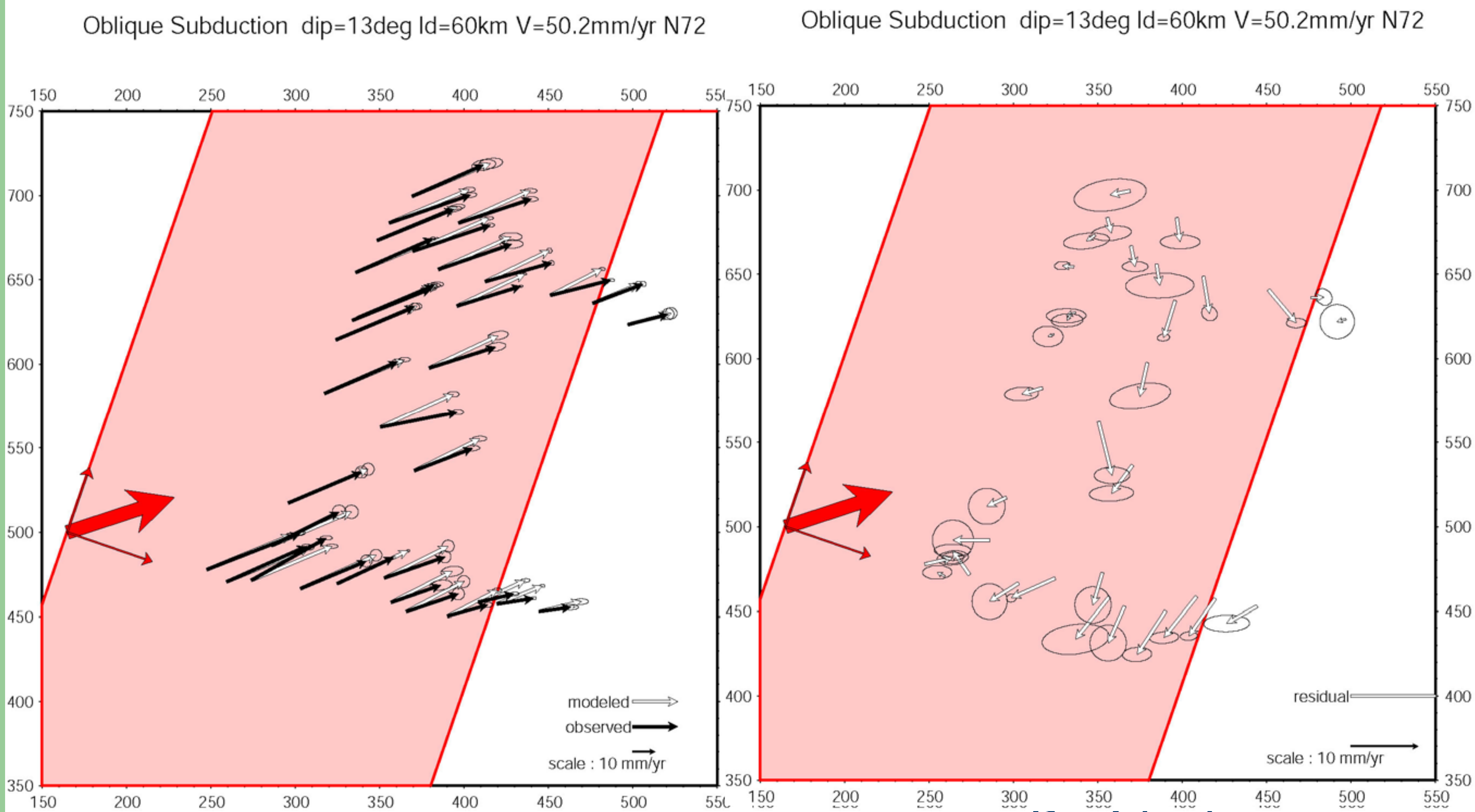






150 200 250 300 350 400 450 500 550 600 650
750
700
650
600
550
500
80
70
60
50
40
30
20
10
0
velocity (mm/yr)
0 100 200 300 400 500 600
distance from trench (km)

Subduction parameter adjustments



Model and data

Residuals