GPS Noise, Noise Models and Sources

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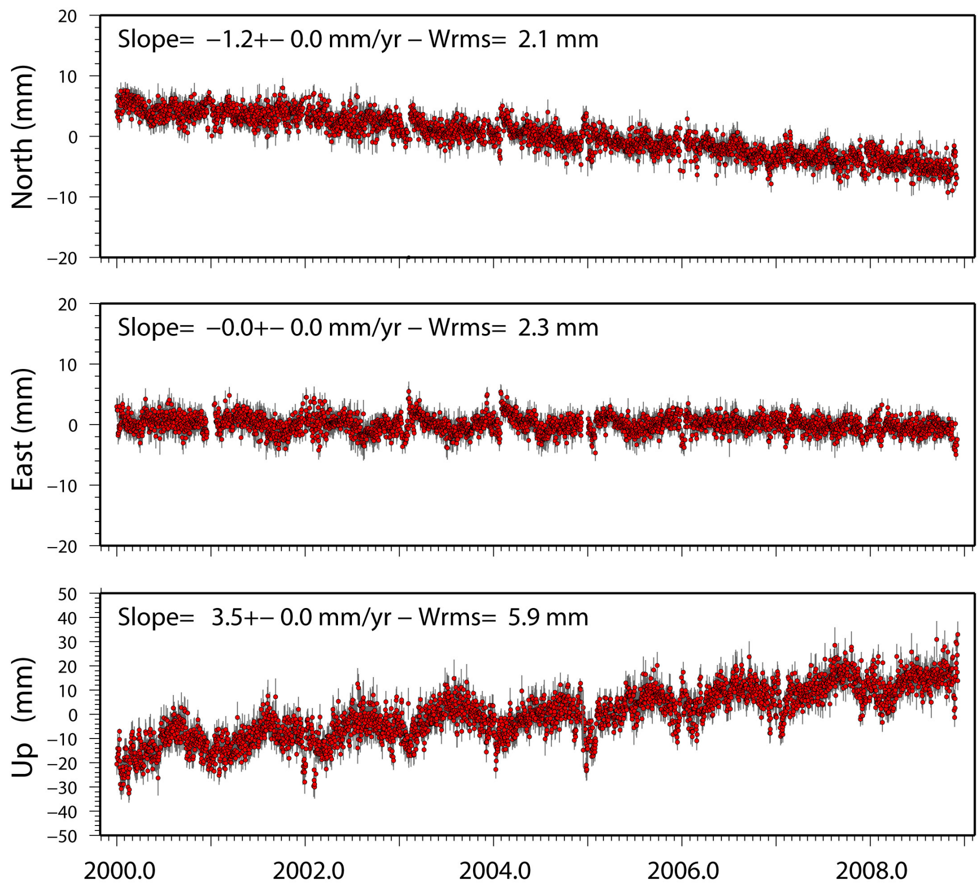
Position time series



- Time series = successive estimates of site positions + formal errors
- How to quantify the precision of the position from repeated measurements?
- Use scatter of data with respect to mean – for instance, "mean" = linear displacement
- Data scatter can therefore be computed using:

$$wrms = \sqrt{\frac{N}{N-2}\frac{\sum_{i=1}^{N}\frac{(y_i-(a+bt_i))^2}{\sigma_i^2}}{\sum_{i=1}^{N}\frac{1}{\sigma_i^2}}}$$

= repeatability (in mm)

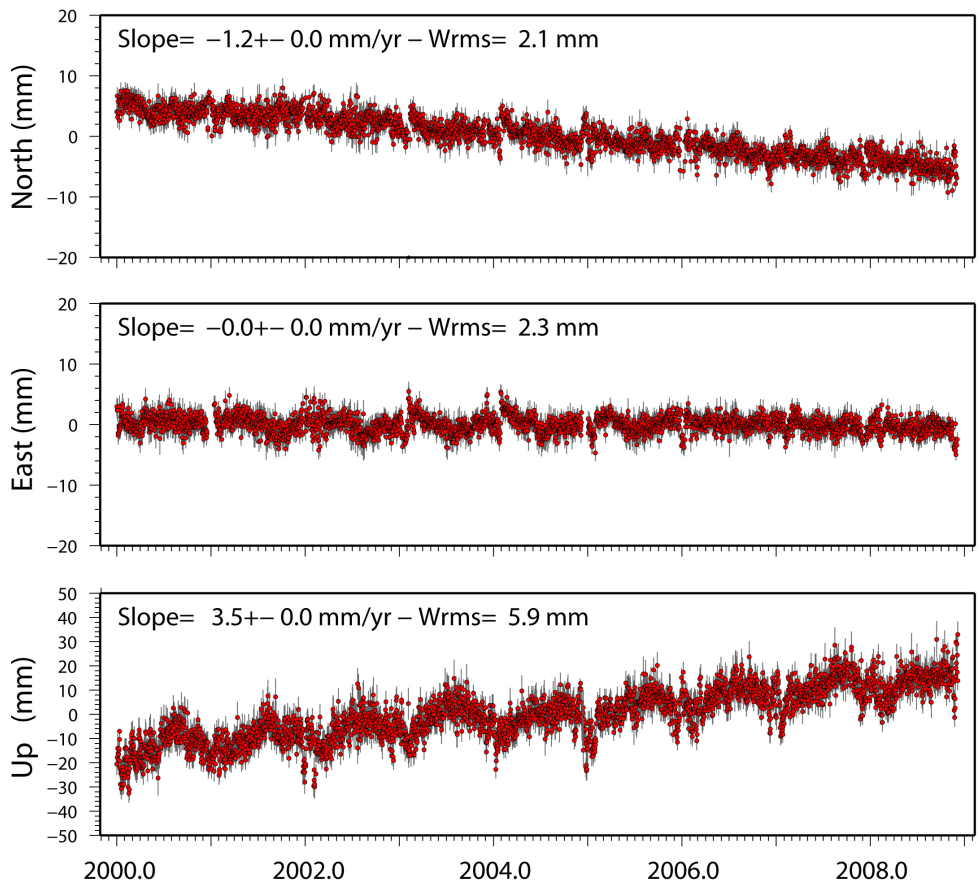
y_i and σ_i = position and associated formal error

a, b = parameters of best-fit straight line through the data

N = number of data points

2 = number of model parameters (intersect and slope here)

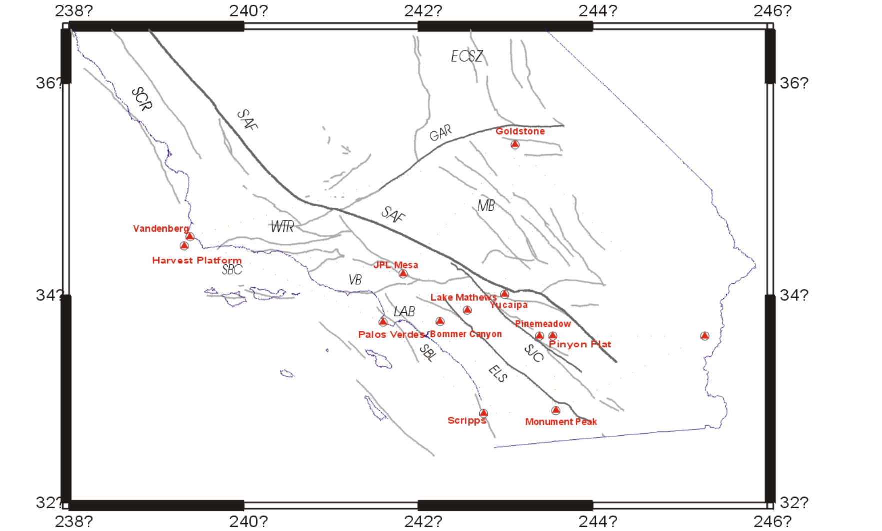
Position time series



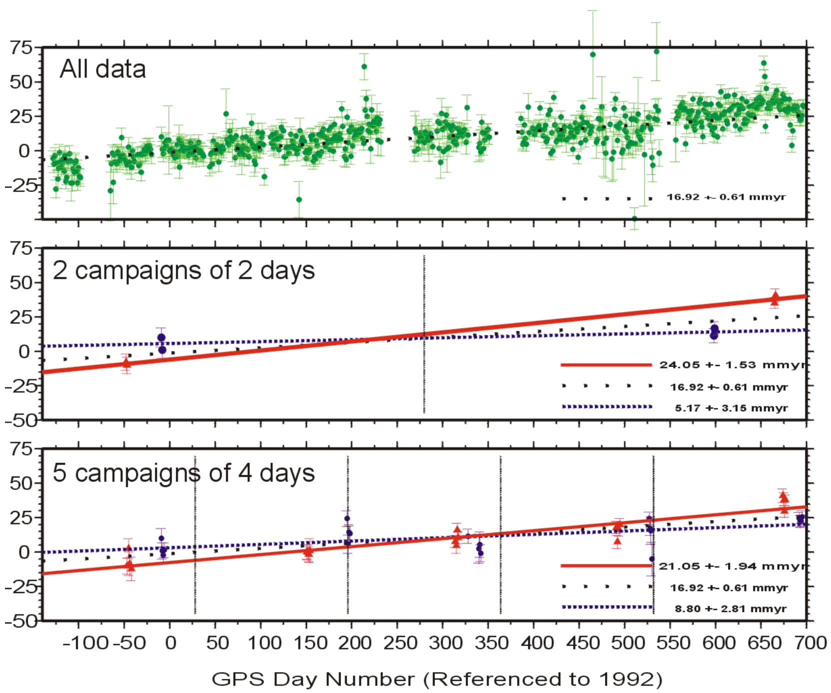
- Fit a straight line to time series using least squares and get:
  - Velocity
  - Associated uncertainty

- Since least squares are used, then:
  - ⇒ Position errors are assumed to be independent from day to day.
  - ⇒ Velocity uncertainty decreases as 1/sqrt(number of samples)
  - ⇒ Velocity uncertainty is NOT a function of time (duration of measurements)

- Example of daily GPS positions at Algonquin:
  - S-ward + up motion = GIA
  - Seasonal on vertical
  - Formal velocity uncertainties = 0.0 mm/yr?



Permanent GPS Geodetic Array (PGGA) - Southern California



Relative Position JPL1 to PIN1 (mm)
East component

From Wdowinski and Bock, 1995

Continuous versus episodic measurements

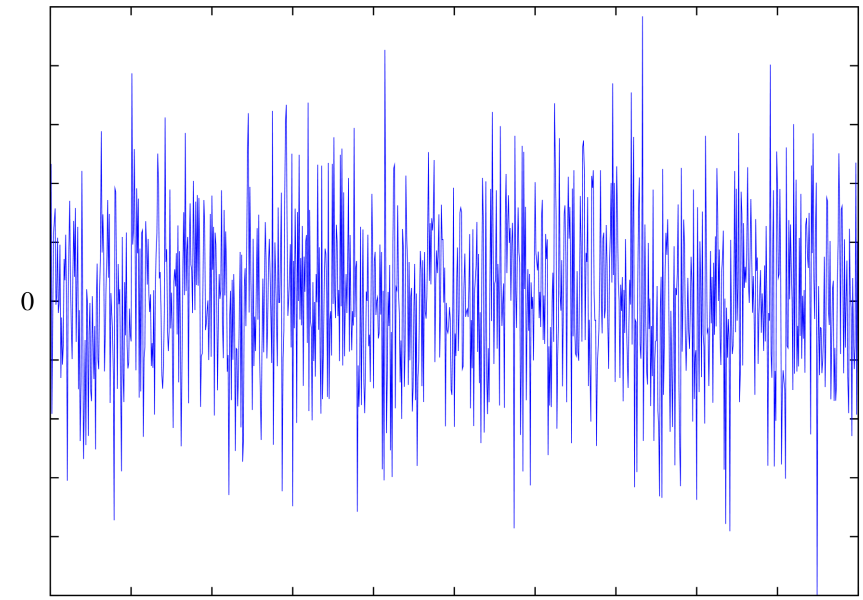
- 150 km baseline across the San Jacinto active fault, 2.5 years of continuous GPS observations
- Linear trend ⇒ fault slip rate: 16.9 ± 0.6 mm/yr
- Comparison with “simulated campaigns”:
  - Difference with continuous up to 10 mm/yr…
  - 10 mm/yr >> uncertainties estimates
  - Long period fluctuations in the continuous time series
- What noise model?

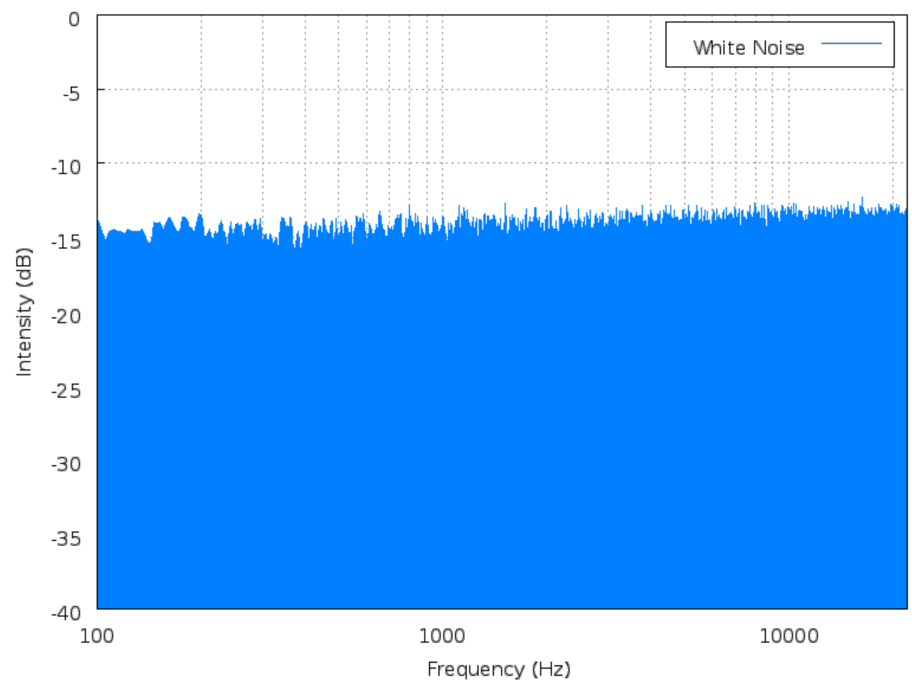
White noise

- Random signal with samples uncorrelated in time: *e.g.,* position error at time t does not depend on error at t+dt
- Statistical properties:
  - Zero mean
  - Covariance matrix:

$$R_{ww} = \sigma^2 I$$

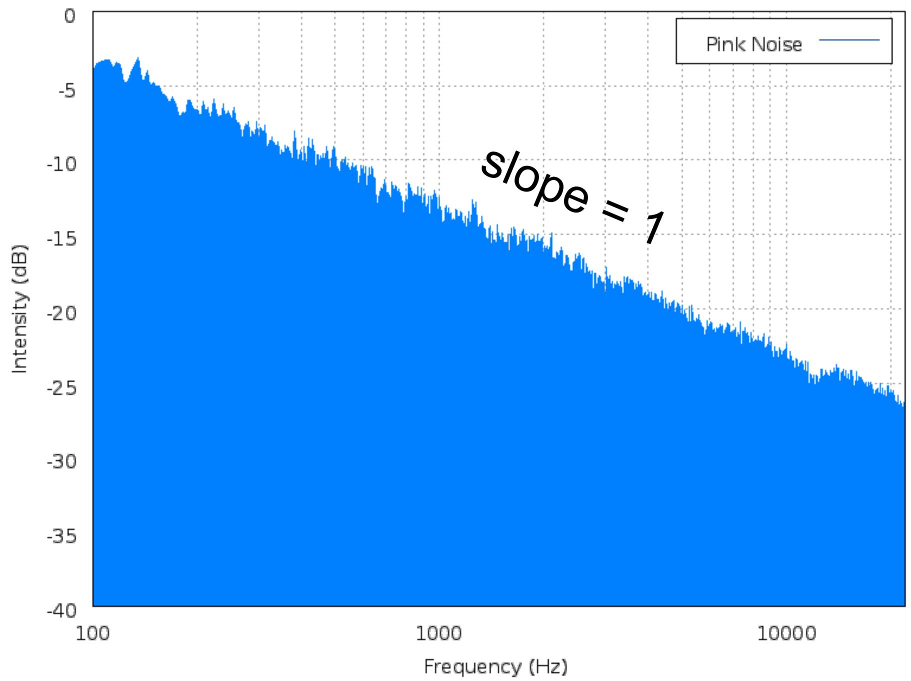
- Equal power (= amount of energy) in all frequency bands

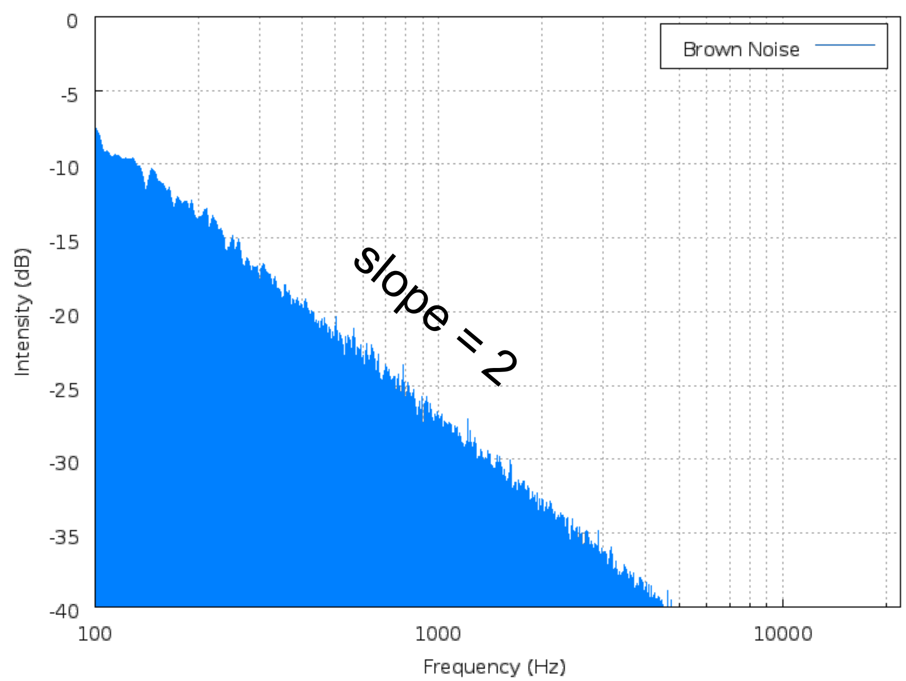




Colored noise

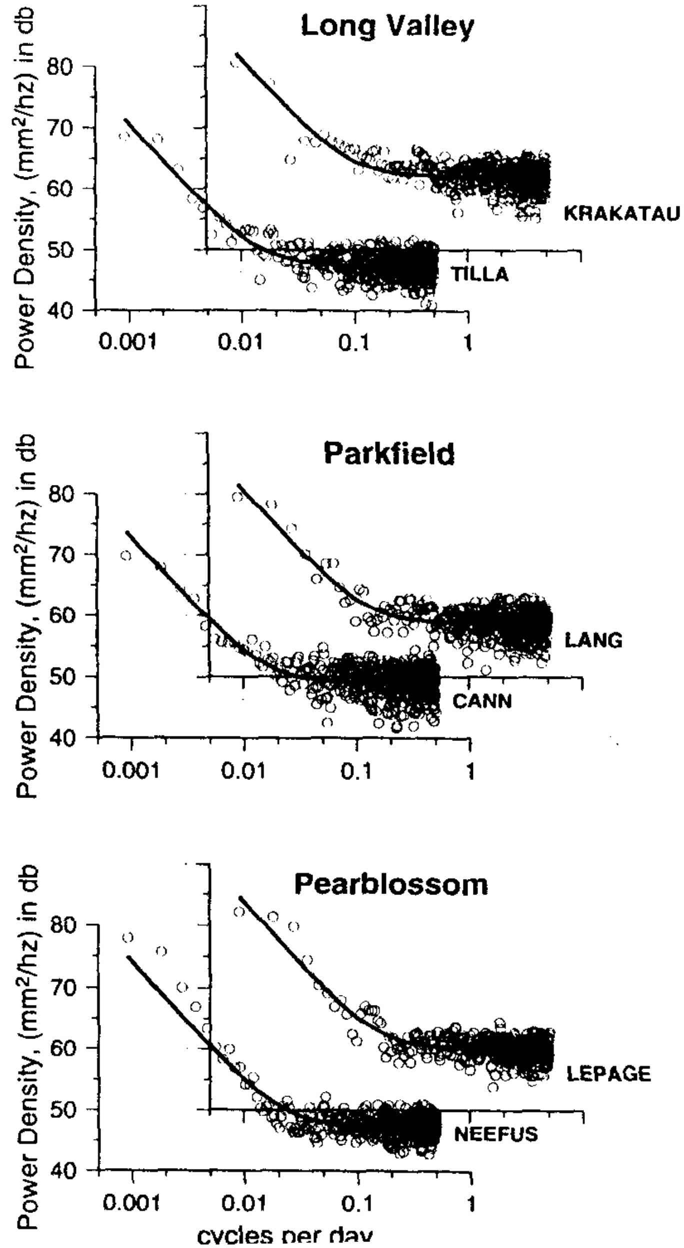
- Random signal with samples correlated in time
  - Least-squares condition fails
  - Estimated uncertainties not representative of true errors
- Two common types of colored noise:
  - Pink noise = flicker noise
  - Brown noise = red noise = random-walk noise = drunkward's walk
- Both have more power at lower frequencies:
  - Flicker: power proportional to $1/f$
  - Random walk: power proportional to $1/f^2$





White and colored noise in EDM measurements

- Example of 2-color electronic distance meter (EDM) measurements in California:
  - Baseline length = 5-10 km
  - 15 years of observations
- Spectral analysis of time series:
  - Flat spectrum at high frequencies => white noise
  - Red spectrum at low frequencies => colored noise, spectral index of 2 (= random walk)
- Cause of colored noise? Proposed to be “monument noise” because its amplitude is correlated with bedrock type and monument type.



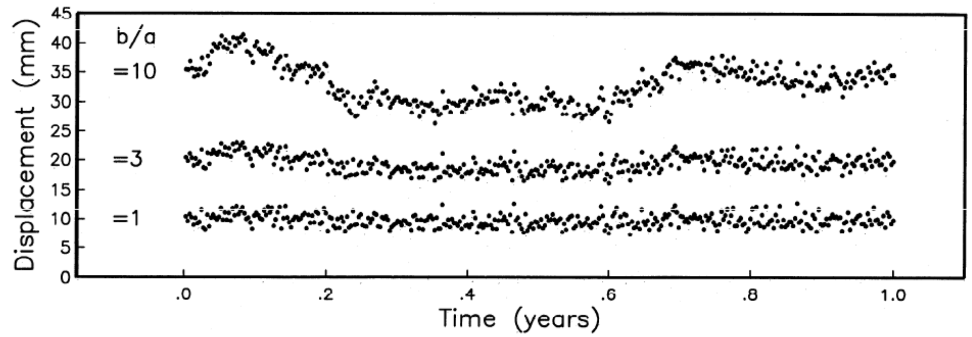
Langbein et al. (1993)

Impact on velocity uncertainties

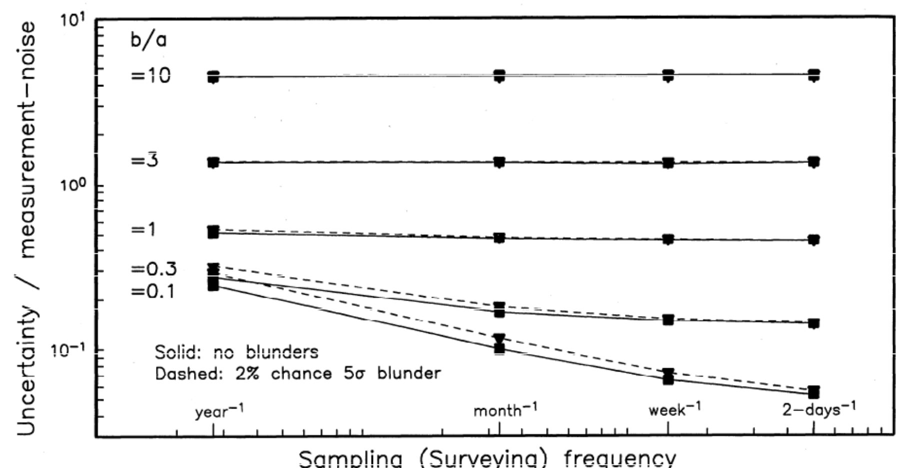
- Compute synthetic time series assuming slope + white noise + random walk noise:

$$x(t) = x_o + rt + a\alpha(t) + b_\kappa \beta(t)$$

  - x_0 = x-axis intersect
  - r = velocity (constant)
  - a and b_k = magnitude of the white and colored noise, respectively
  - α and β = uncorrelated random variables



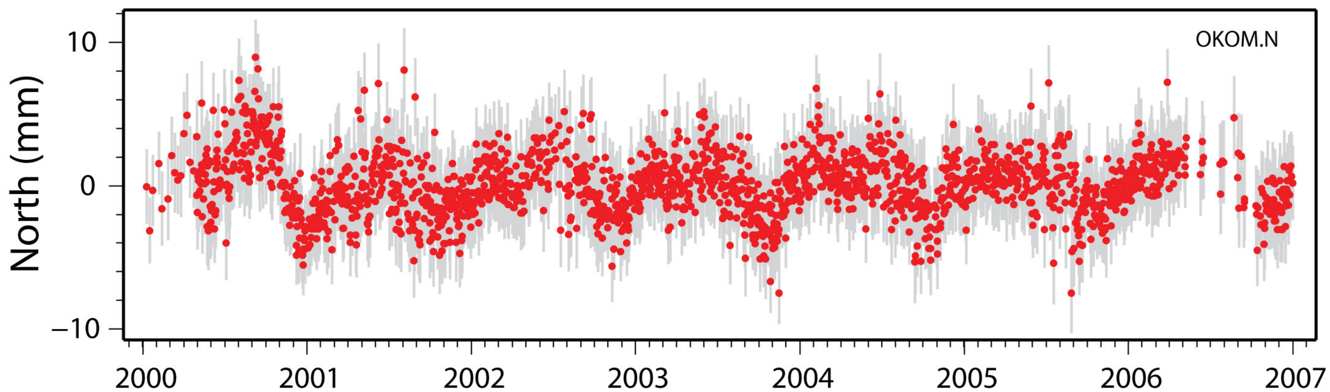
Synthetic time series, 1 year of daily positions: b/a = rwn/wn relative amplitude – correlated noise can be "hidden in the time series"



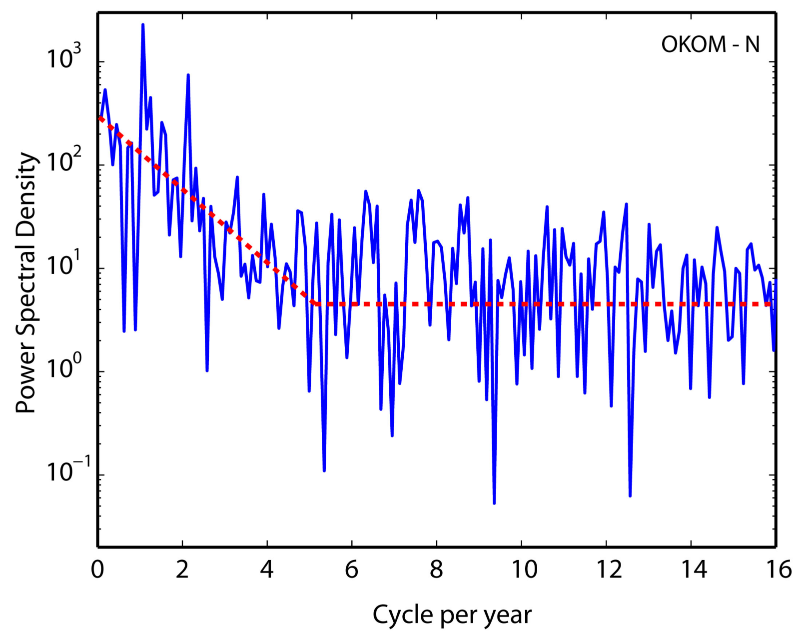
Rate uncertainty as a function of sampling frequency for 5 years of measurements:
⇒ Little gain with continuous measurements!
⇒ Emphasis on monuments to reduce rwn

Johnson and Agnew (1995)

White and colored noise in GPS measurements



- Site OKOM (US midcontinent)
- Power spectrum of position time series:
  - White noise above 5 cycles per year
  - Colored noise at lower frequencies
  - Spectral density = 1 => flicker noise
- Case of most GPS sites
- Origin unclear.



Impact on velocity uncertainties

Mao et al. (1999) came up with an empirical model combining white, flicker, and random walk noise, where the velocity uncertainty σ_r is given by:

$$\sigma_r \cong \left(\frac{12\,\sigma_w^2}{g\,T^3} + \frac{a\,\sigma_f^2}{g^b T^2} + \frac{\sigma_{rw}^2}{T} \right)^{1/2}$$

T = time span

g = number of measurements per year

σ_w = magnitude of white noise

σ_f = magnitude of flicker noise

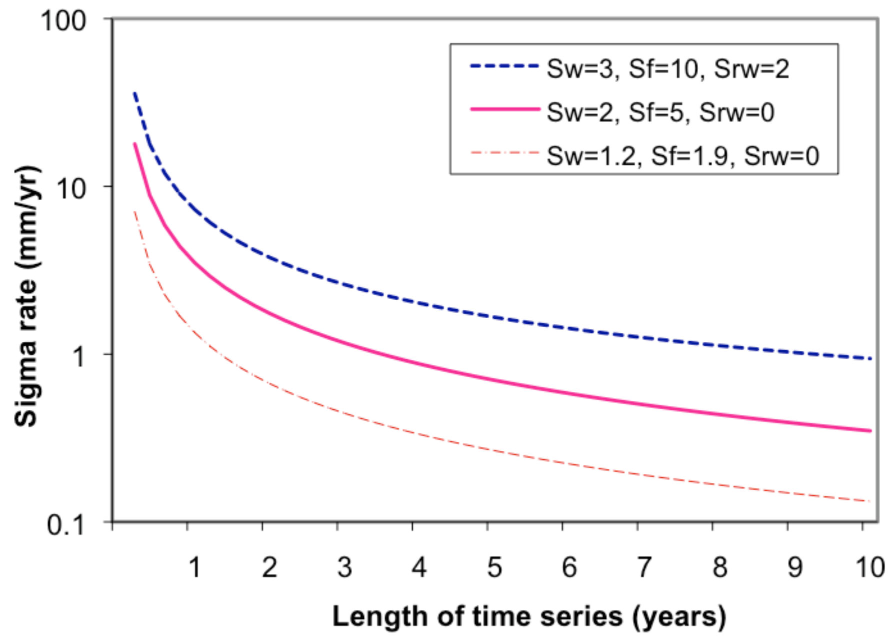
σ_{rw} = magnitude of random walk noise

a,b = empirical constants

⇒ Uncertainty depends on time span

⇒ 1 mm/yr uncertainty can be reached in 1 year in the best case, in 10 years in the worst…

Example from GPS network in the Western Alps, France (Calais et al., 2000)



Mean values for white and flicker noise

Maximum amplitude of white and flicker noise, plus 2 mm/√yr of random walk noise (Langbein and Johnson, 1997).

Minimum amplitude of white and flicker noise of the REGAL time series.

GPS noise analysis

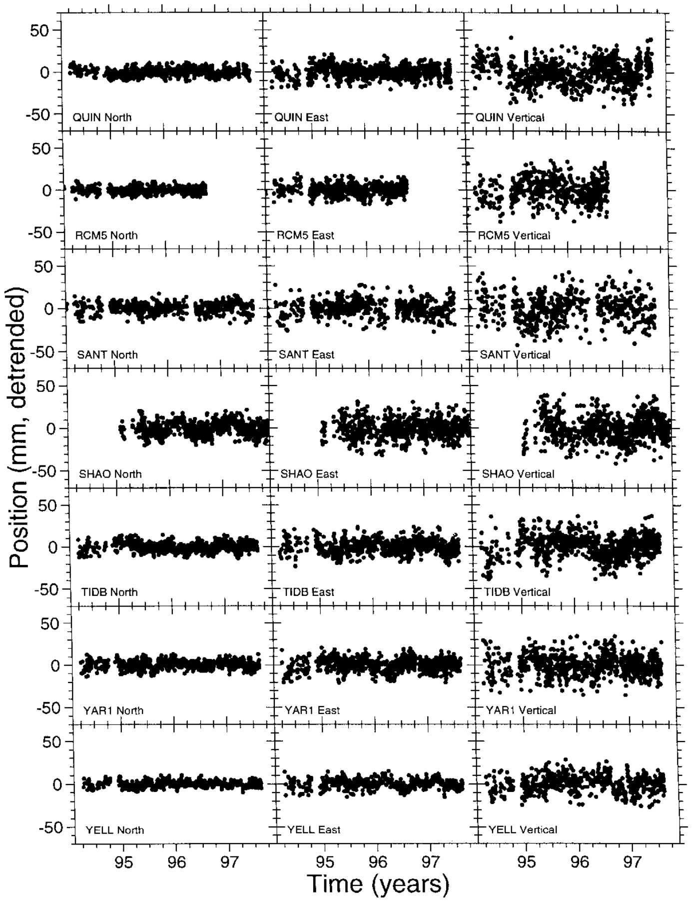
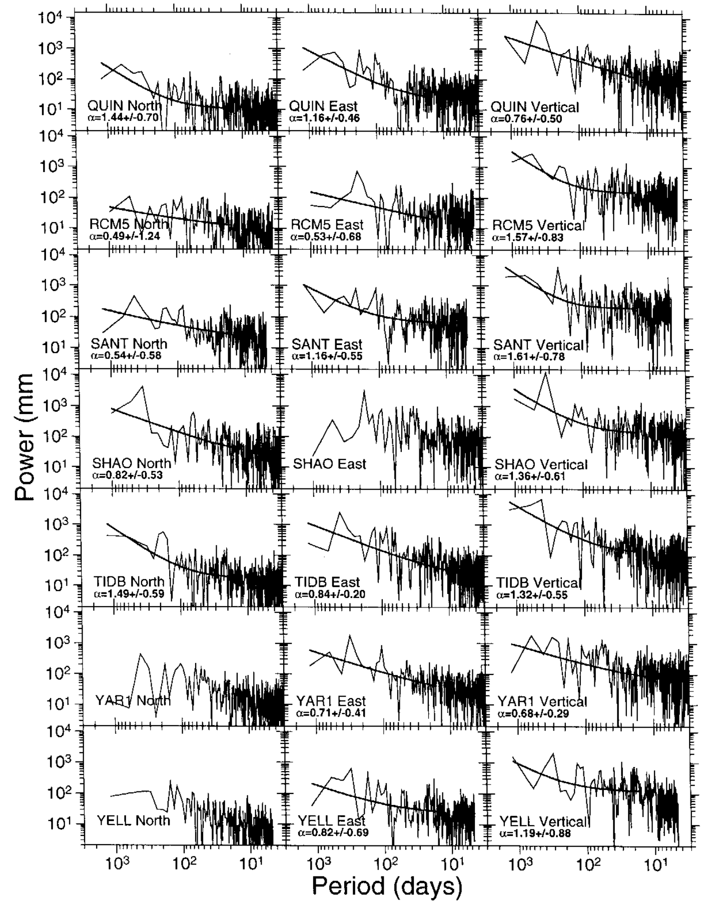


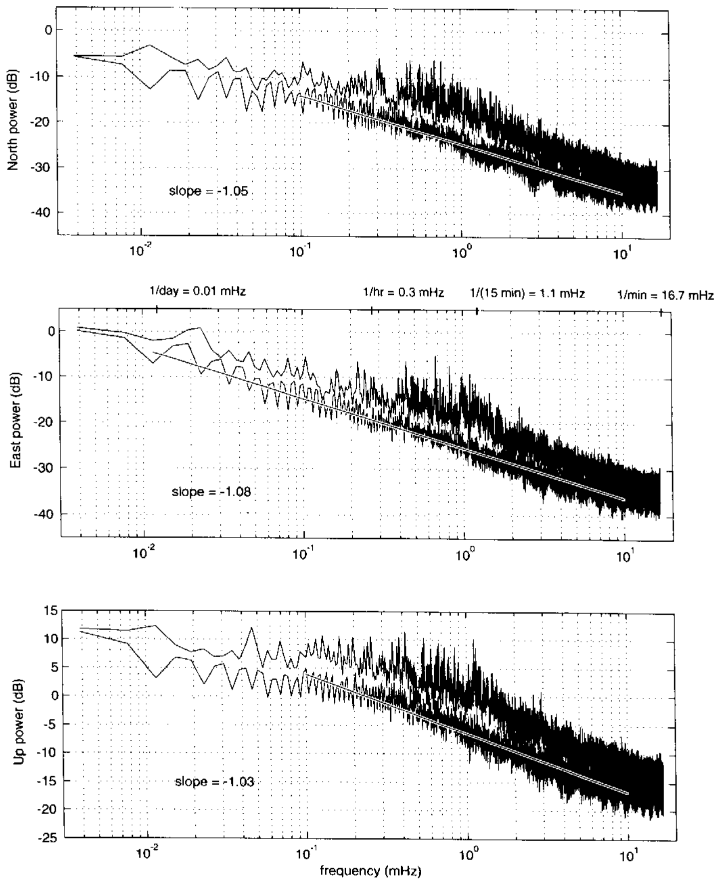
Figure 3. (continued)



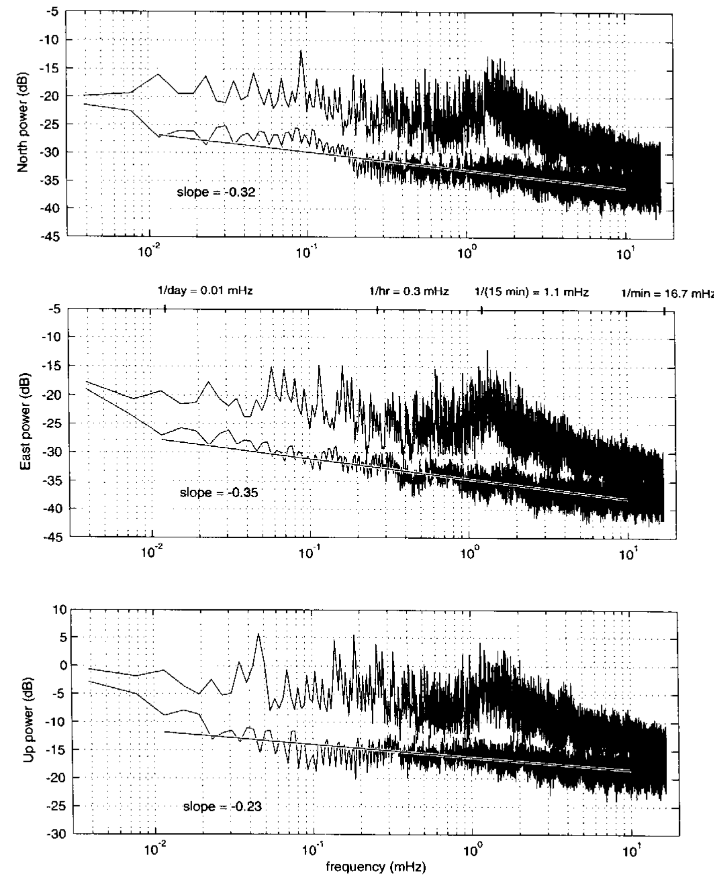
- Regional analysis: Zhang *et al*. (1997), GAMIT, double differences
- Global analysis: Mao *et al.* (1997), GIPSY, point positioning
- Spectral indices for GPS time series range from 0.74 to 1.02 (Mao et al., 1997)
- Dependence in latitude: tropical stations have larger white noise $\Rightarrow$ Troposphere?

GPS noise sources

PIN1-ROCH, 14 km



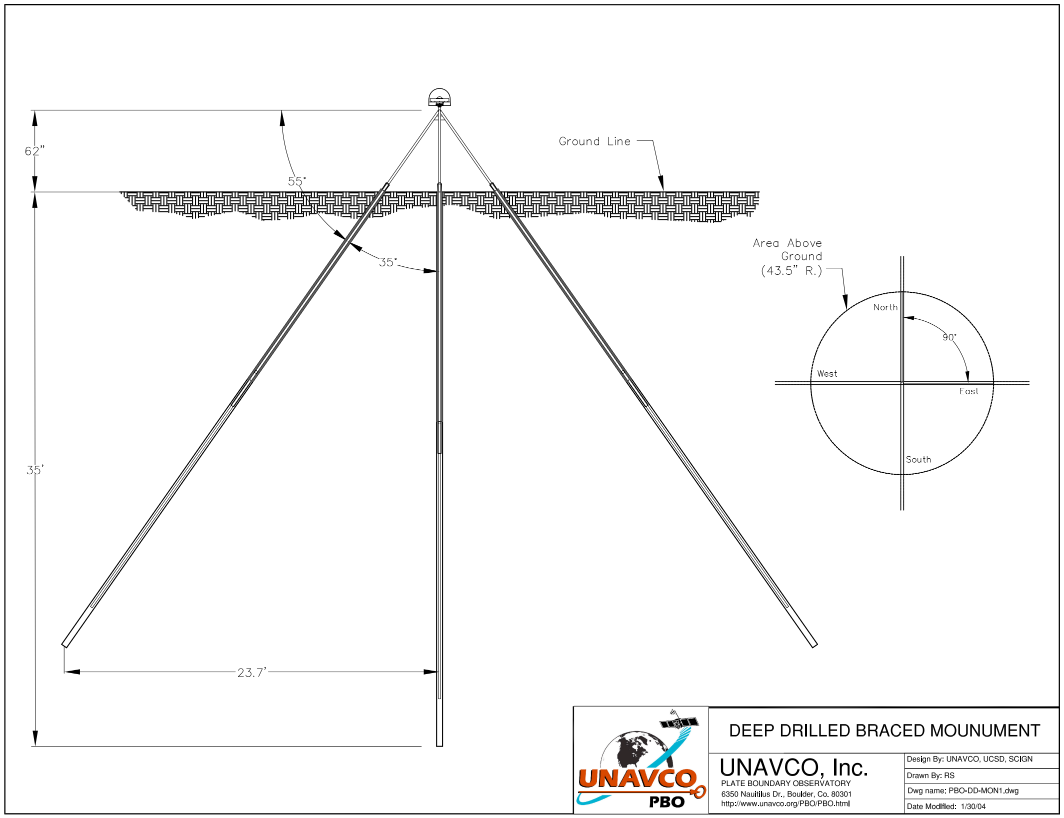
PIN1-PIN2, 50 m

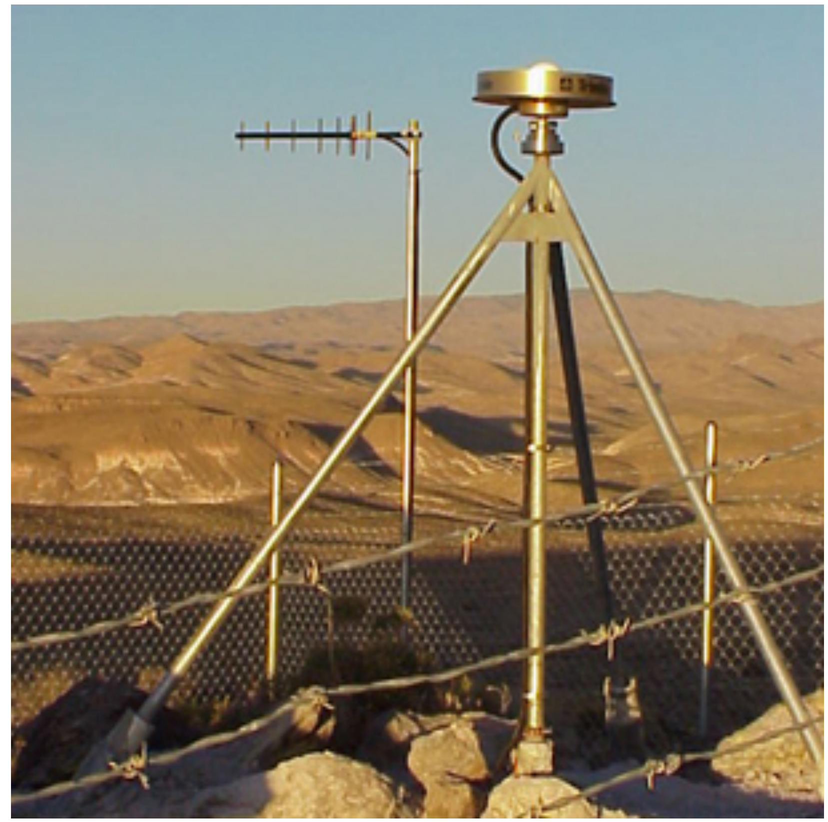


Bock *et al.*, 2000

- Kinematic analysis of GPS baselines (50 m, 14 km, 37 km)
- 1 sec. sampling rate
- Epoch-by-epoch processing, no time-correlation introduced
- No flicker noise on very short baseline => tropospheric origin?

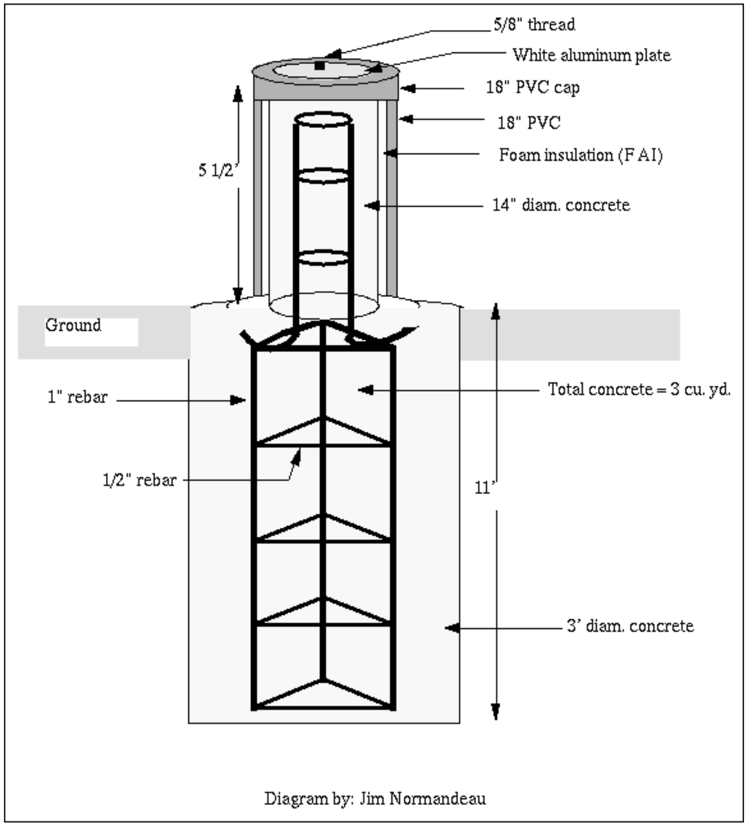
GPS monuments





- High stability and longevity
- Can be installed in unconsolidated materials
- Labor and tool intensive (requires a driling rig and crew)
- Expensive and time intensive
- May not be able to install in some remote locations...
- Large construction disturbance footprint

GPS monuments



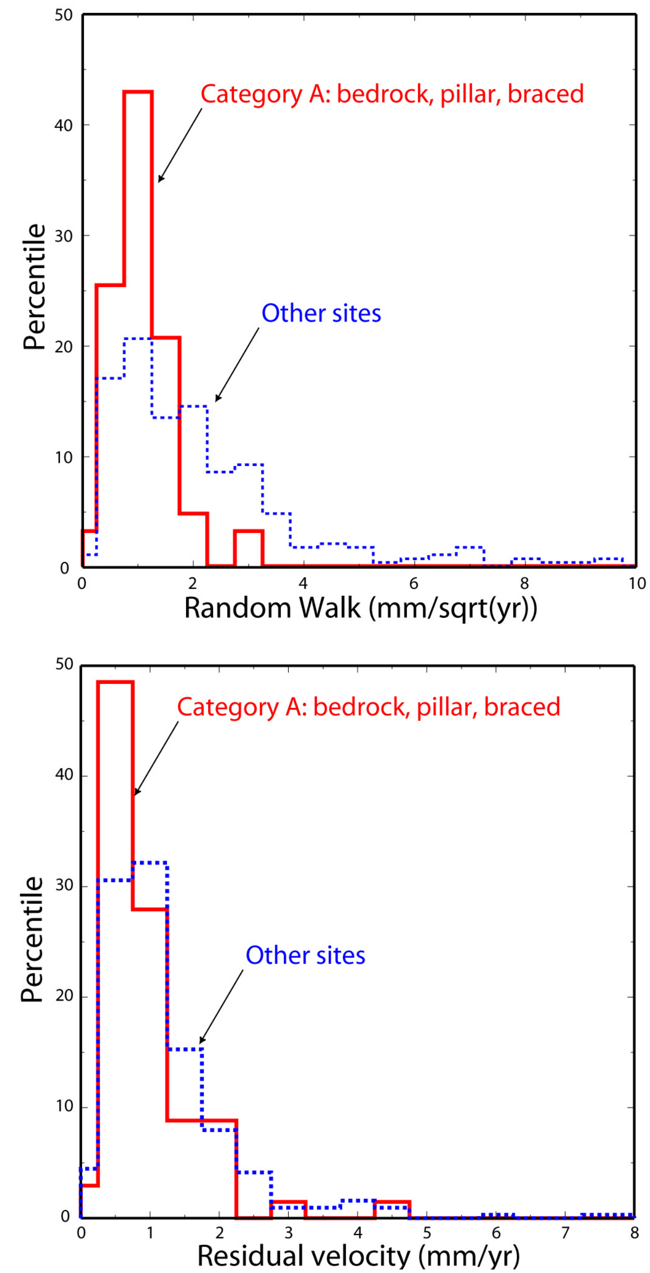
- Can be very inexpensive
- Materials and tools required are widely available
- Easy to construct
- Can be installed upon bedrock or in unconsolidated material
- Concrete can degrade over time through freeze-thaw action
- Weight of concrete mass can settle in certain unconsolidated materials over time

GPS monuments

They come in all flavors…!

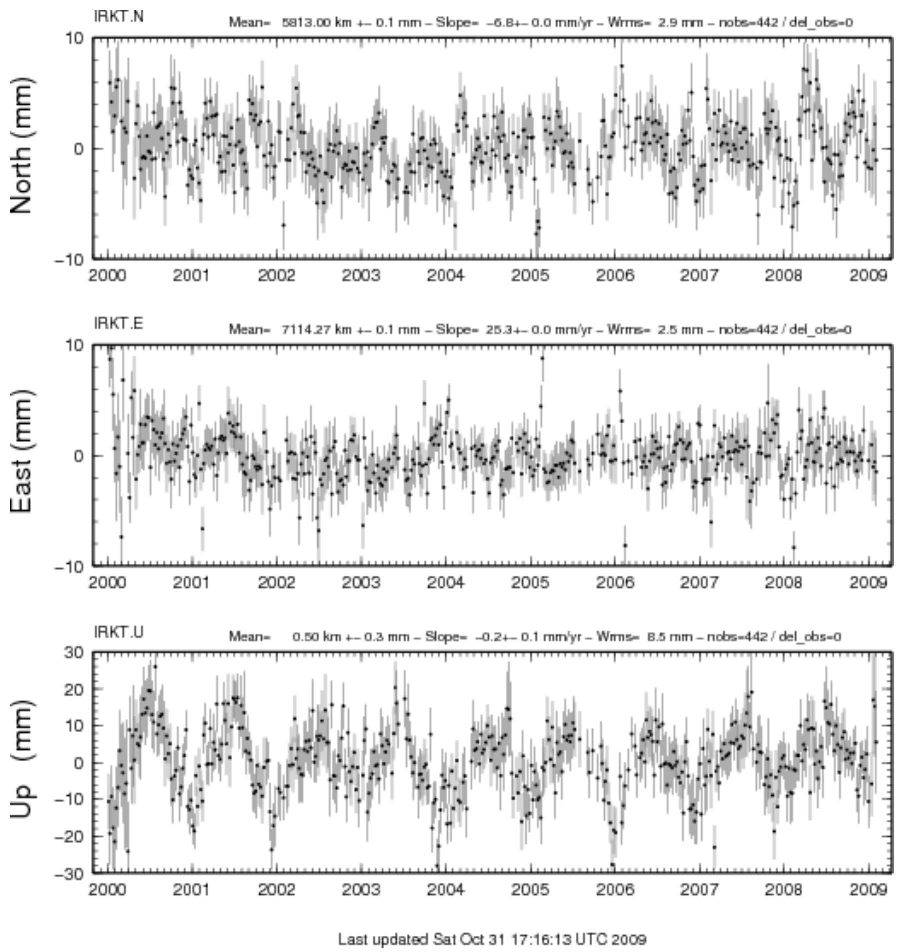
GPS monument noise

- Due to rainfall, freeze-thaw cycles, rock and soil weathering effects, etc.
- Common types of monuments:
  - Pillars
  - Deep-drilled braced ($$$)
- Williams et al. (2004): deep braced monuments outperform all other monument types
- Beavan (2005) and Langbein (2006): no clear difference between deep braced and pillars…
- Noise in time series may not be monument instability…

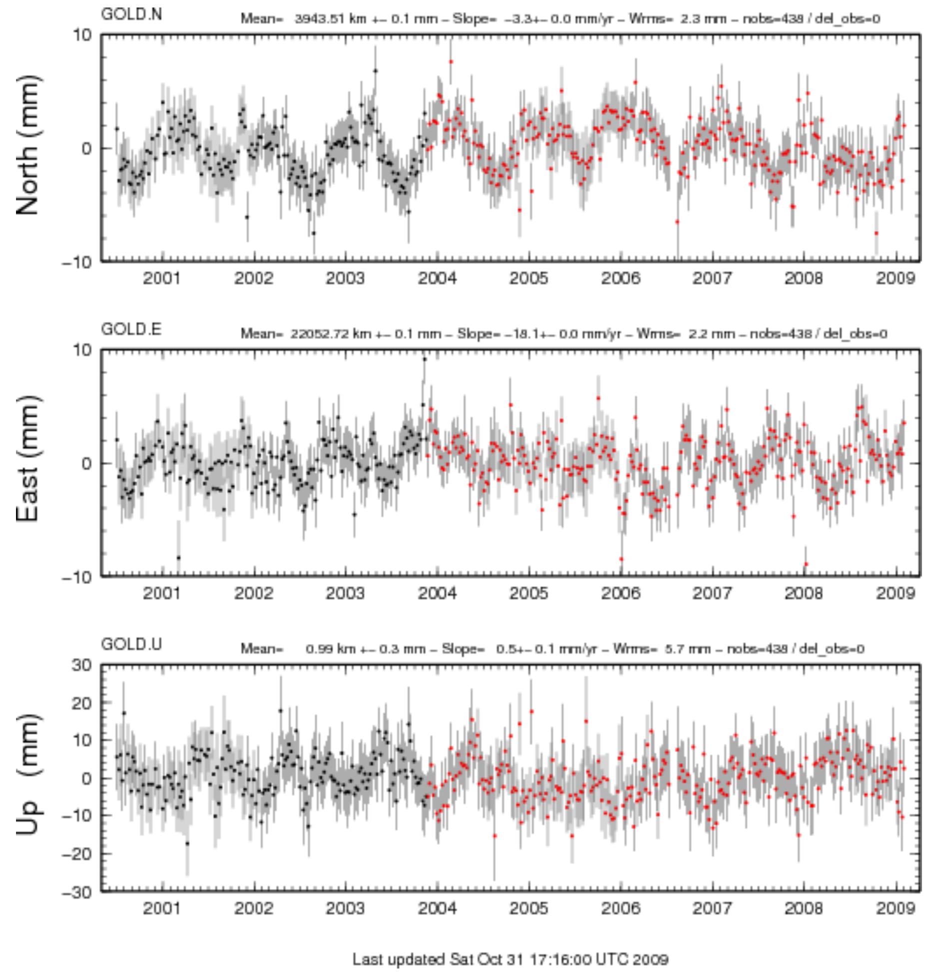


CORS sites in CEUS: best-monumented sites have lower colored noise and (slightly) smaller residual velocity (w.r.t. rigid plate model) – Calais et al., 2006

Seasonal signals



Irkutsk, Siberia



Goldstone, CA

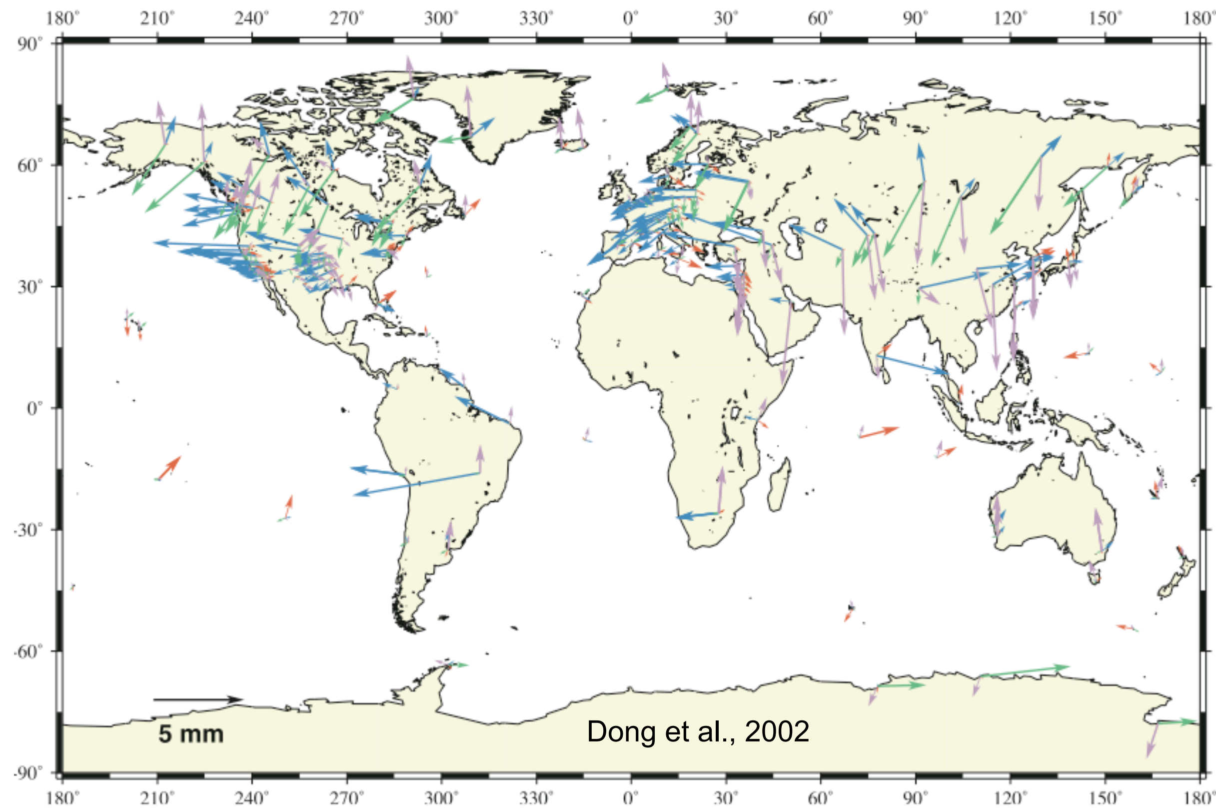
Seasonal signals

- Annual and semi-annual signals can be described by:

$$x = A\cos(\omega t + \varphi)$$

A = amplitude, φ = phase, ω = $2\pi/T$ (T = period, 1 for annual)

- Result from:
  - –Pole tide and ocean loading (largest, usually corrected in GPS analysis)
  - –Mass loading (usually not corrected in GPS analysis):
    - Atmospheric pressure
    - Snow and soil moisture
    - Non-tidal ocean water

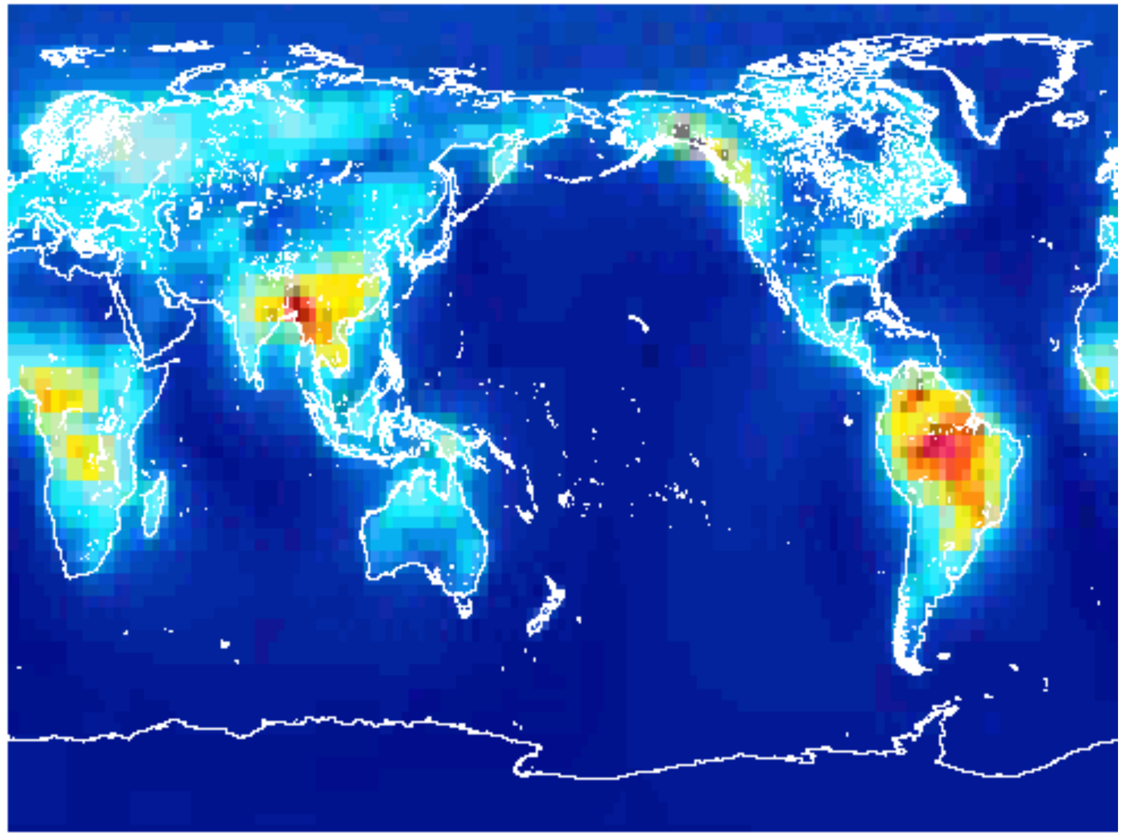


Vertical annual terms: Purple = atmosphere, Red = non tidal ocean, Green = snow, Blue = soil moisture. Arrows represent amplitudes. Phases are counted counterclockwise from east. Ellipses are 95% confidence.

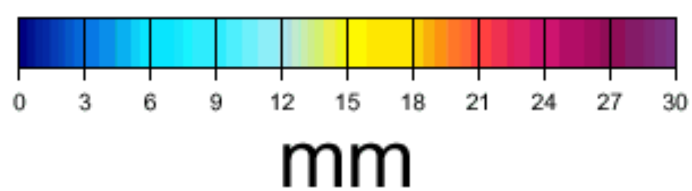
Continental water loading

- Water cycle + climatological model
- Load = model storage output = snow, ground water, soil water
- 1°x1° grid
- Load dominated by annual signal
- Vertical displacement range caused by total stored water/ snow (max-min, 1994 to 1998)
- On average: 9-15 mm over continents
- Max: monsoon and tropical continental areas
- Horizontal: 5 mm max.

Seasonal peak-to-peak vertical displacements due to hydrological loading

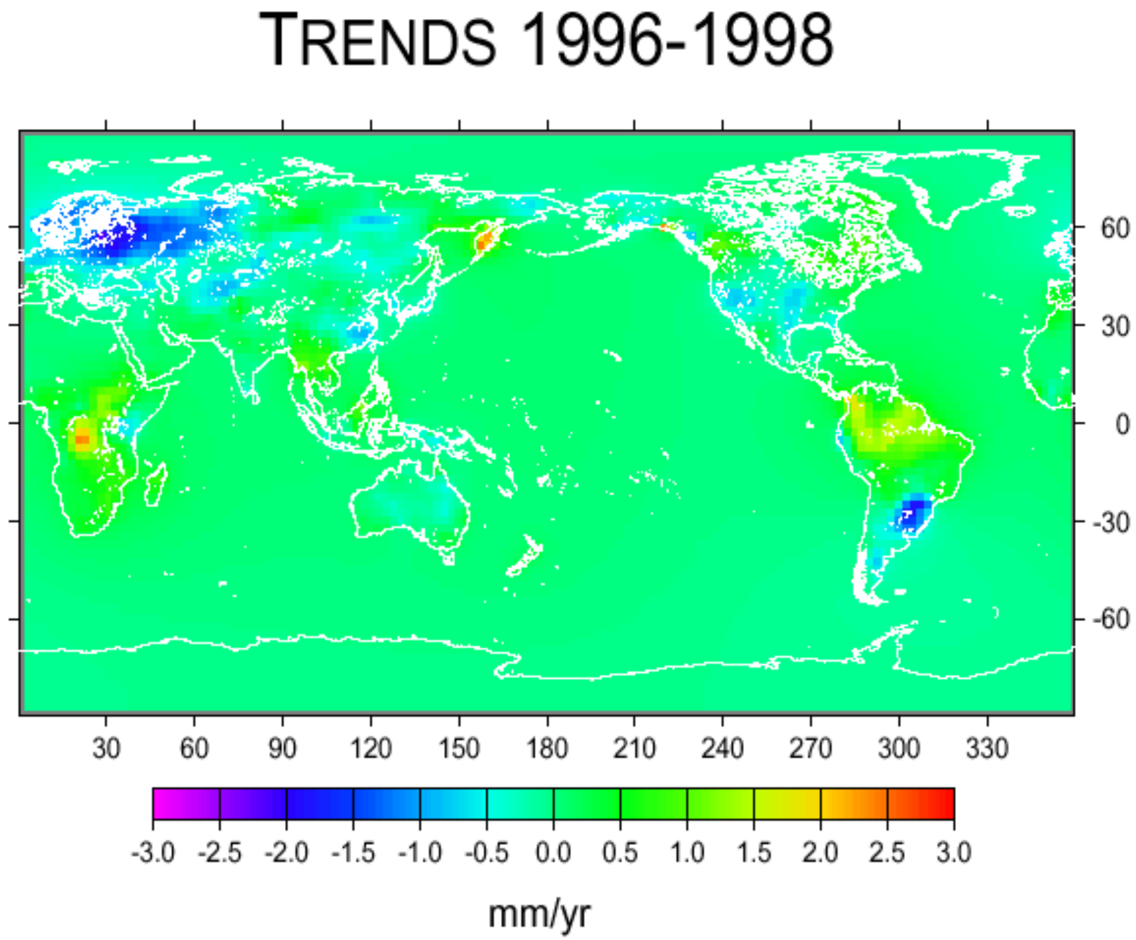


VanDam et al., 2001



Continental water loading

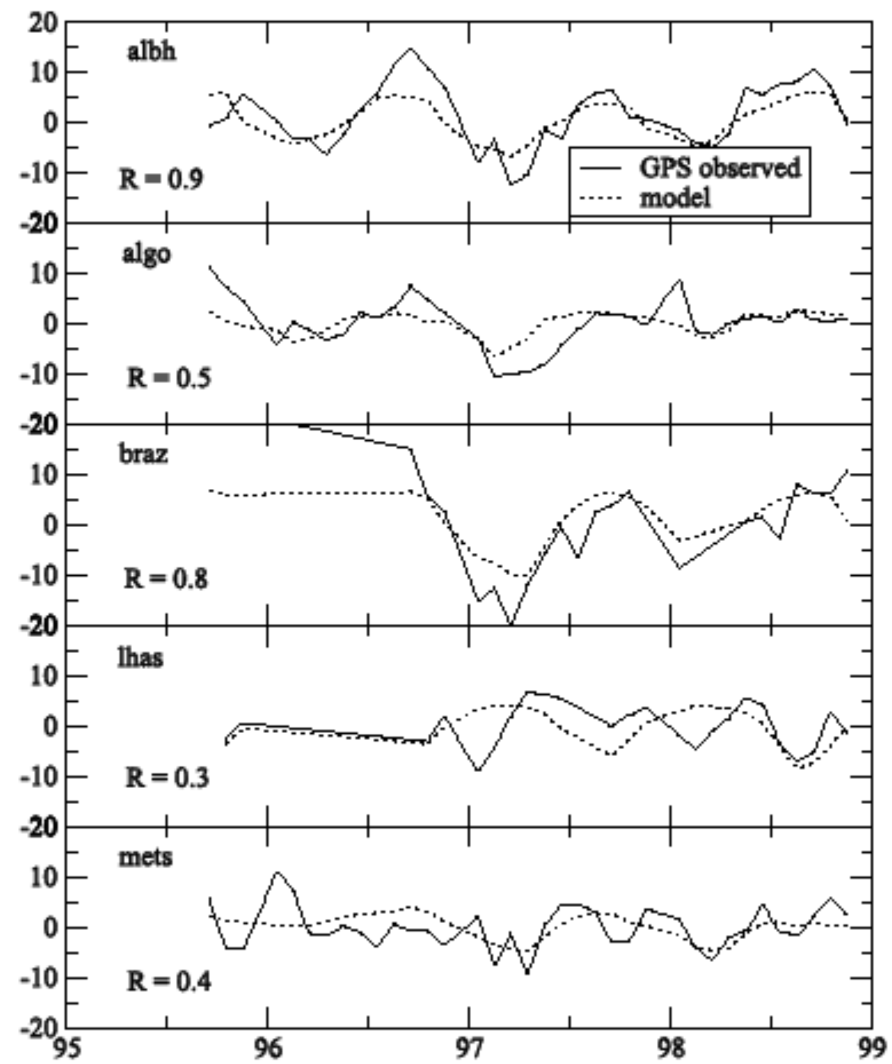
- Effect on secular trends for vertical displacement
- Small effect in most regions (<0.5 mm/yr)
- 20 years => < 0.3 mm/yr at all sites



VanDam et al., 2001

Continental water loading

- Monthly averages of vertical component
- Atmospheric loading removed
- Annual signal, 2-3 cm
- Hydrological loading does not explain fully GPS height residuals
- Goal: validate models
- Amplitude varies from year to year



VanDam et al., 2001

Continental water loading

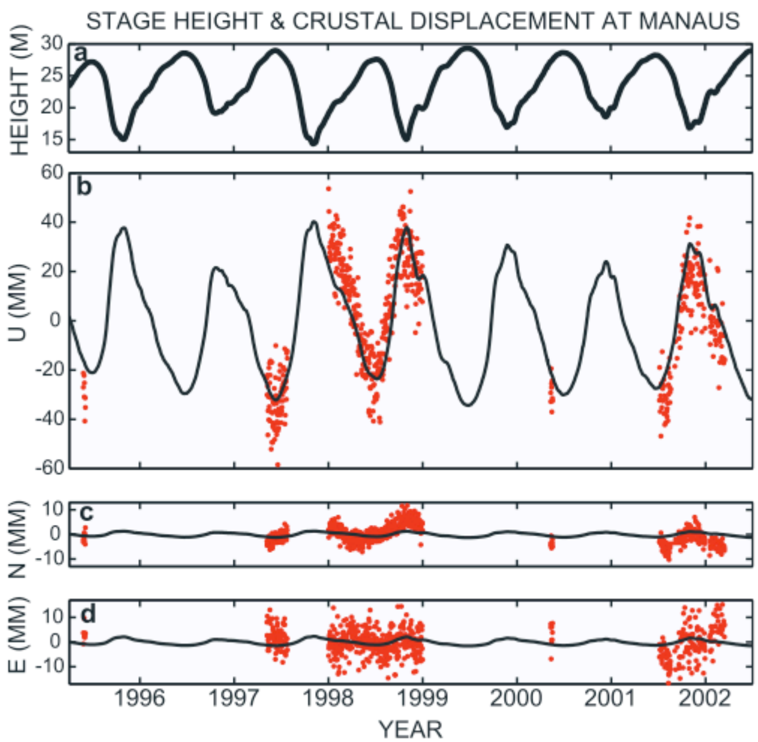
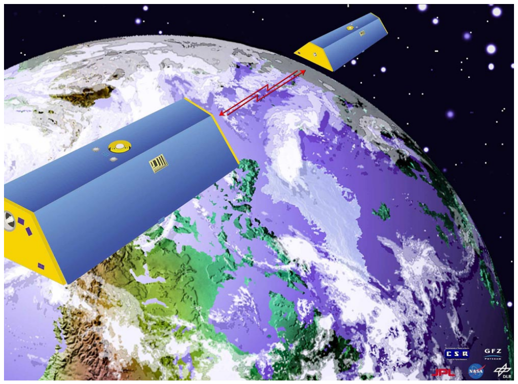


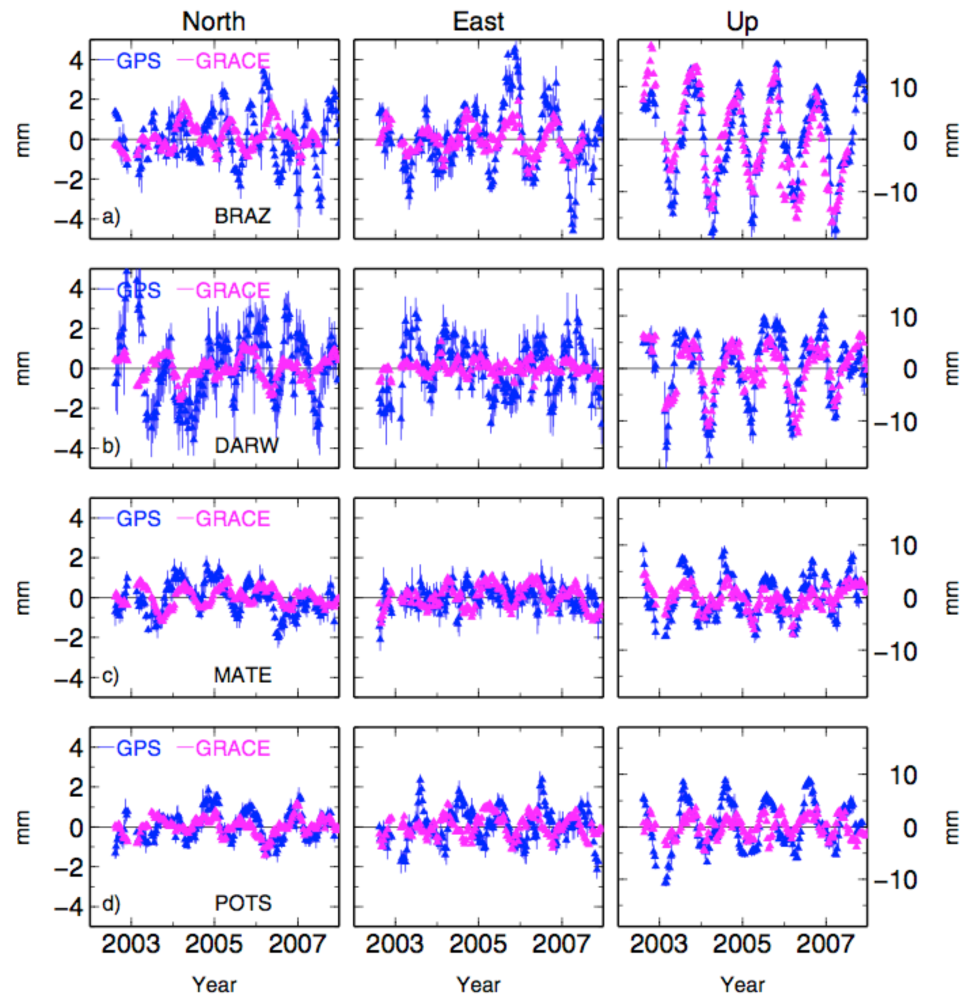
Figure 1. (a) Stage height time series H(t) observed in Manaus, (b) daily solutions for the upwards component of displacement U(t) at GPS station MANA (red dots), and the model prediction (solid curve), (c) and (d) geodetic measurements (red dots) and model predictions (solid curves) for the north and east components of displacement.

- Extreme example: Manaus, Brasil (Bevis et al., GRL 2005)
- Seasonal signal up to 10 cm peak-to-peak on the vertical
- Explained by elastic response (flexure) of lithosphere caused by fluctuations in the mass of the Amazon River system.
- Can be used to infer elastic properties of the lithosphere…

Continental water loading



- GRACE (Gravity Recovery and Climate Experiment) mission
  - Derive equivalent water mass from temporal gravity change
  - Compute equivalent flexure of Earth's surface
  - Compare with GPS observations
- Good correlation between GPS and GRACE:
  - In vertical and horizontal
  - GRACE sees only long wavelength features
- Some differences too: local hydrology or other local causes?

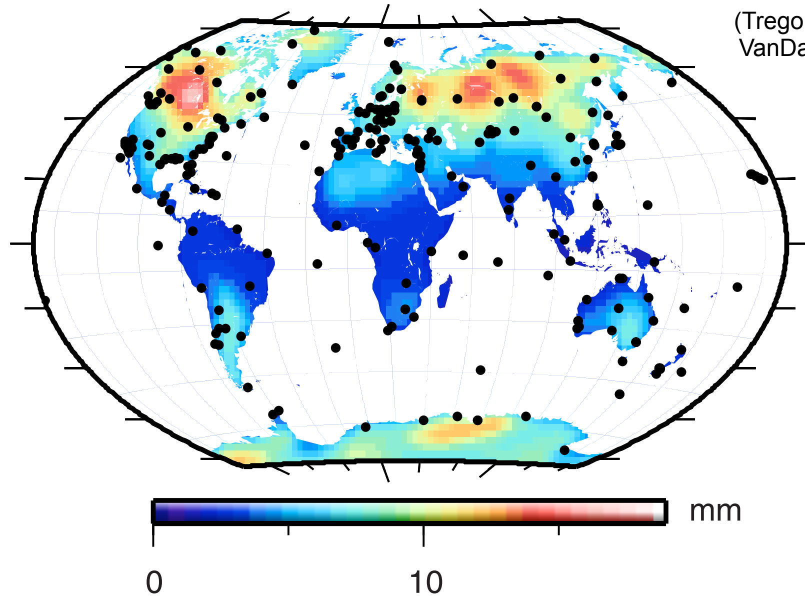


Tregoning et al., GRL 2009

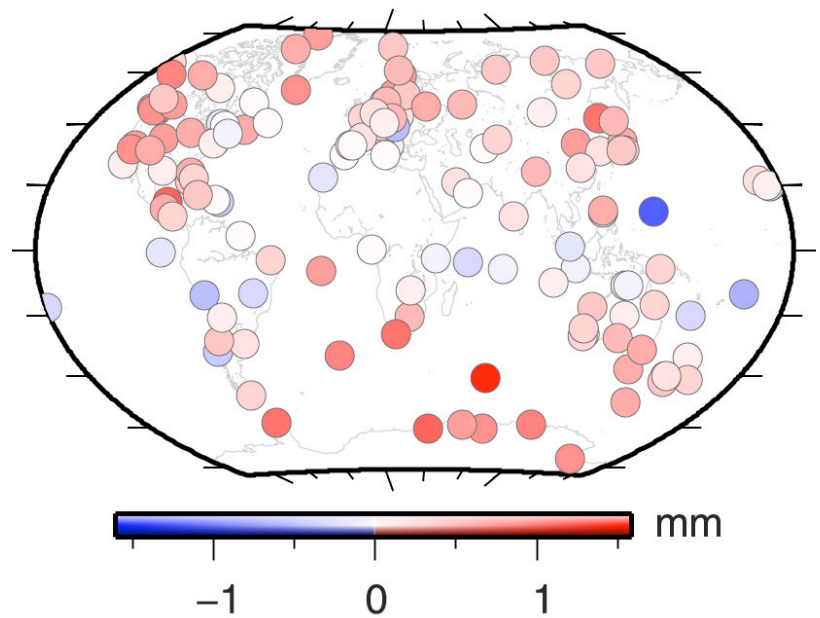
Atmospheric pressure loading

- Atmospheric loading affects positions at various temporal scale:
  - Seasonal – can be taken care of after processing phase data (*e.g.*, time series filtering)
  - Subdaily – must be accounted for during phase data processing
- Sub-daily effects can reach 2 cm, combination of:
  - Tidal effects
  - Non-tidal effects
- Accounting for sub-daily effects (from models) during processing improves height for 70% of stations
- Problem: IGS analysis centers do not model atmospheric loading

(Tregoning and VanDam et al., 2005)



Sub-daily peak-to-peak vertical displacements due to atmospheric loading



Reduction in WRMS for station heights when applying a tidal correction at the observation level.

Effect of seasonal signals on velocity?

- Blewitt and Lavallée (2002)
- Annual signals can significantly bias estimation of site velocities
- Annual and semiannual sinusoidal signals should be estimated simultaneously with site velocity and initial position at least for the first 4.5 years
- Velocity bias unacceptably large below 2.5 years
- **“We recommend that 2.5 years be adopted as a standard minimum data span for velocity solutions intended for tectonic interpretation or reference frame production and that we be skeptical of geophysical interpretations of velocities derived using shorter data spans. “**

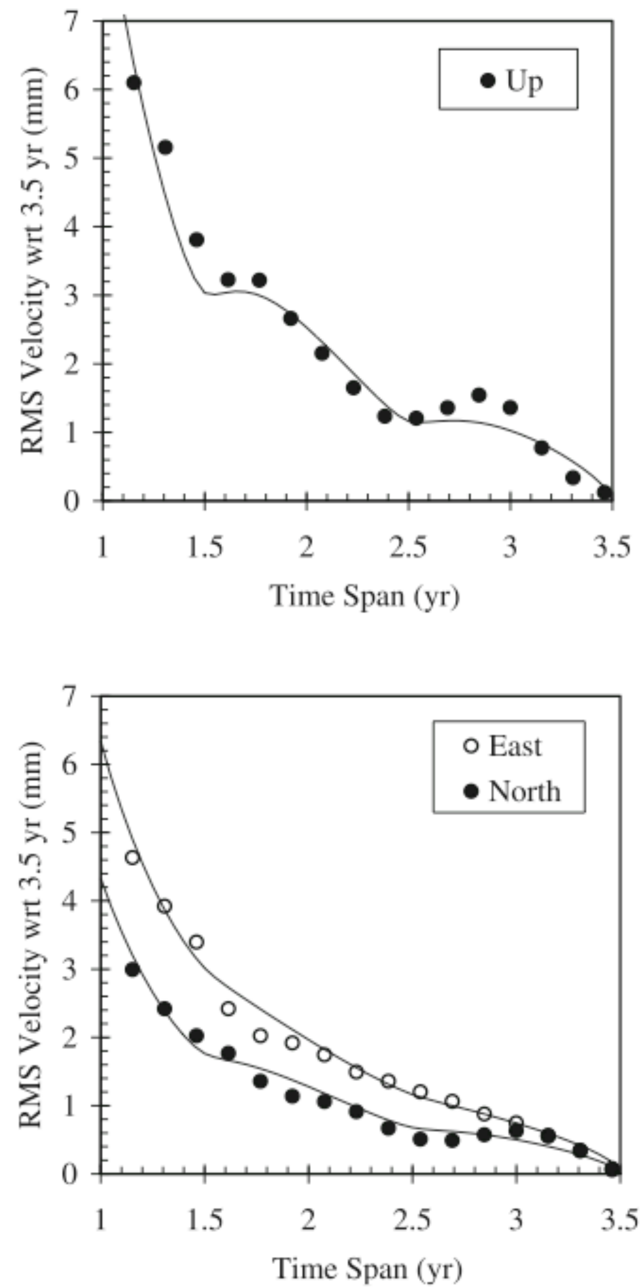
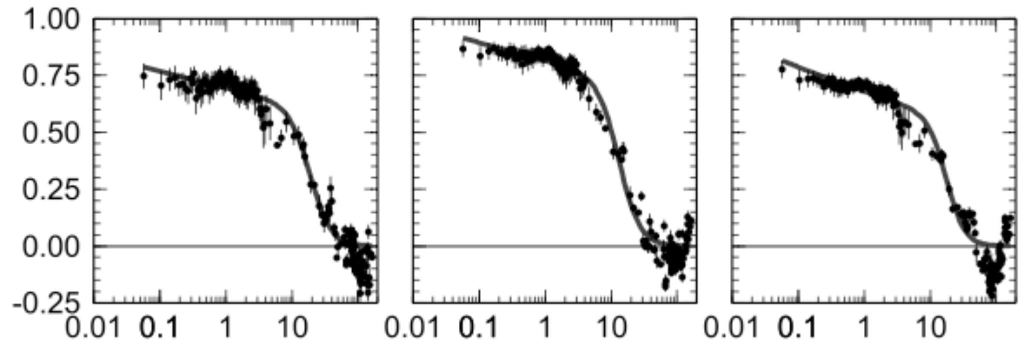


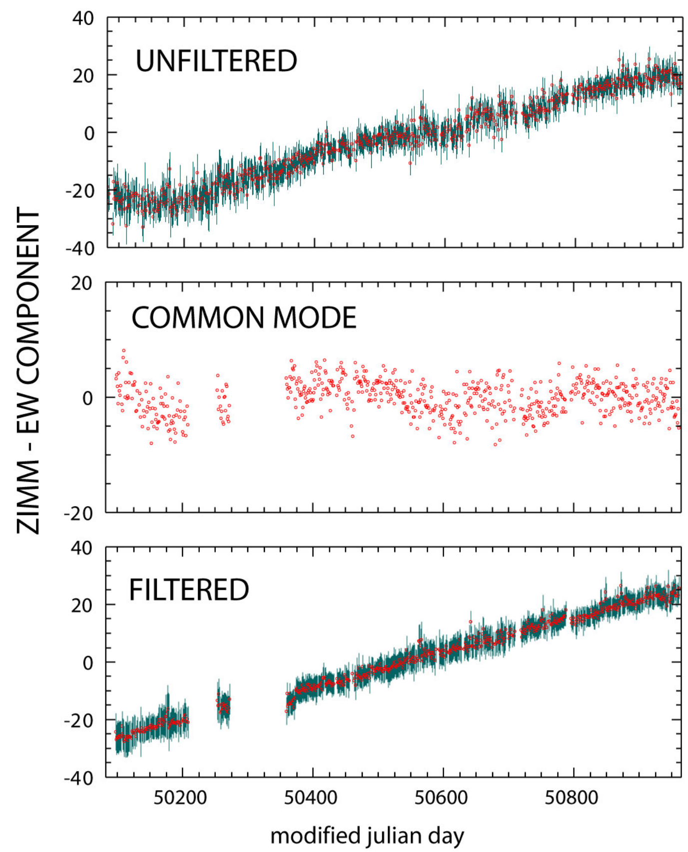
Figure 7. Root-mean-square velocity bias for data spans with respect to 3.5-year data span for the (top) vertical component and (bottom) horizontal components. Real data are plotted. Curves represent models for each component.

Common-mode noise

- Regional GPS network show errors that are correlated among sites
- Results from unmodeled mass loading, orbits, reference frame, etc.
- Possible approach:
  - Stack time series from many sites to emphasize common elements and remove random noise => common mode noise time series
  - Subtract common mode noise from individual time series
- Warning:
  - This can remove signal as well…
  - This does not help understand the cause of the “common noise”

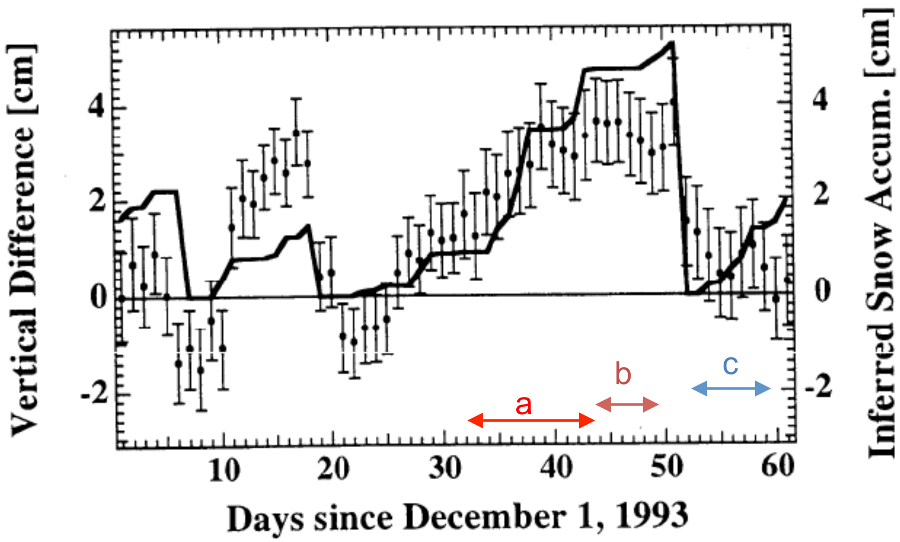


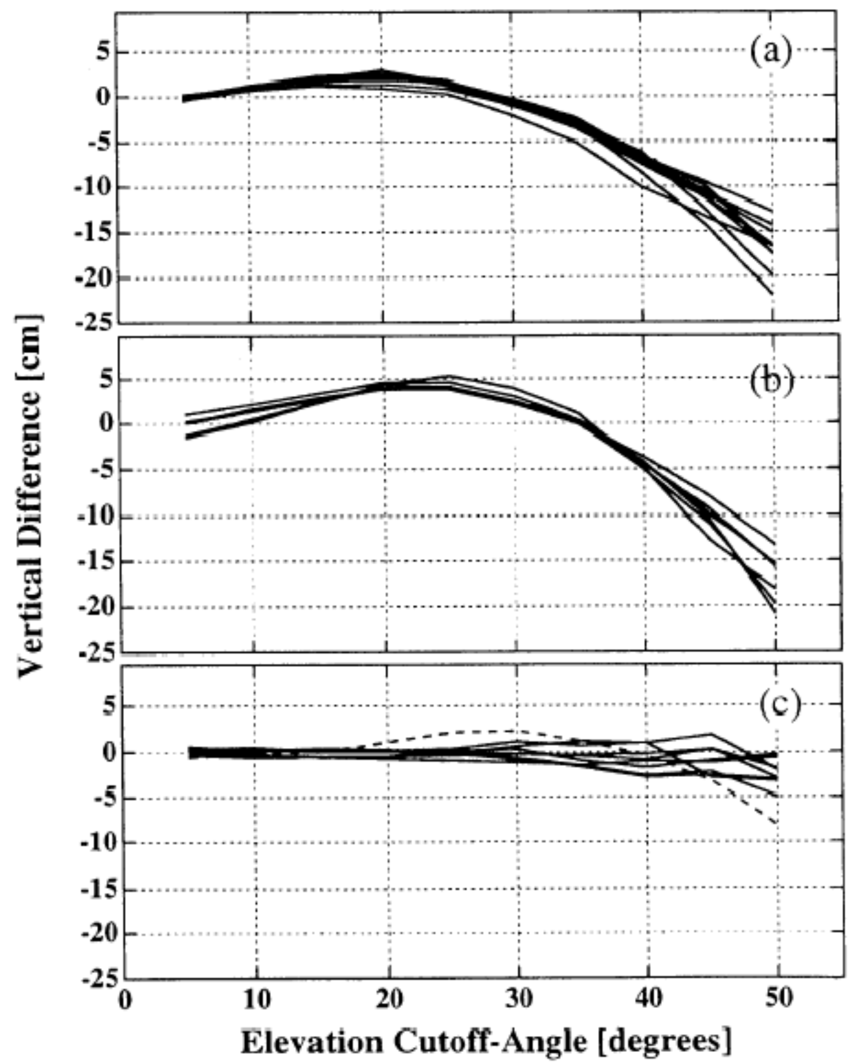
Correlation coefficient between position time series as a function of site separation in angular distance (Williams et al., 2004)



Removal of common-mode noise at site ZIMM (Switzerland; Calais et al. 1999)

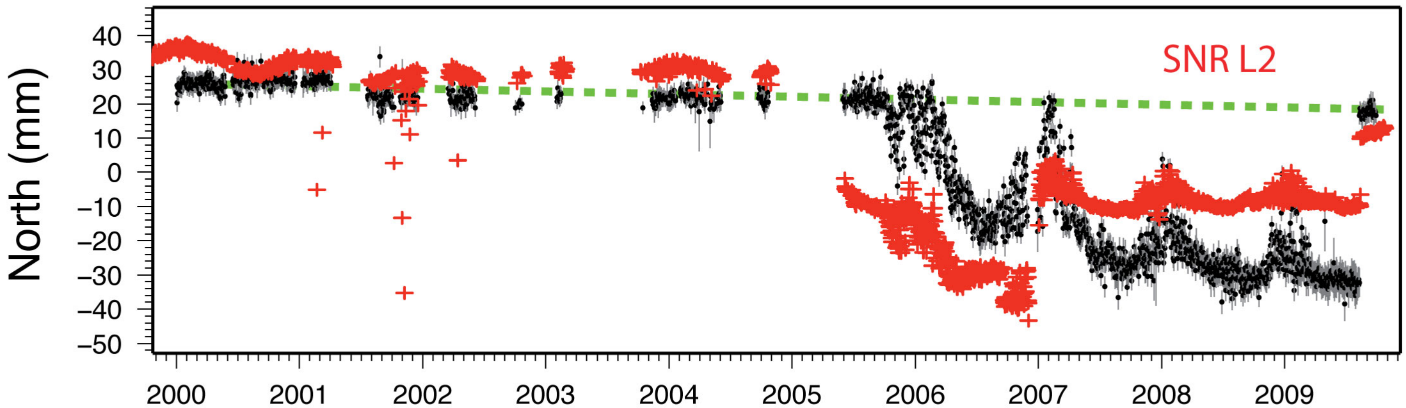
Snow on antenna





- Effect of snow on GPS antenna radome: Jaldehag *et al.*, 1996
- Elevation angle dependency strong when snow on radome: local refraction effects

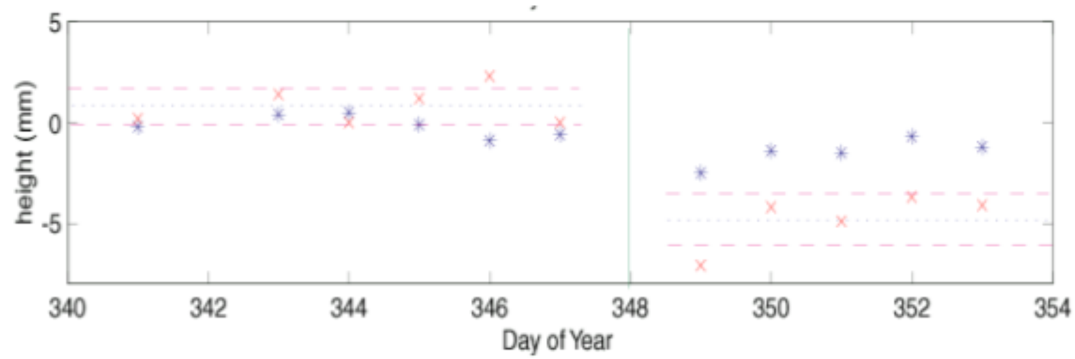
Antenna problems



A failing antenna (site RLAP)

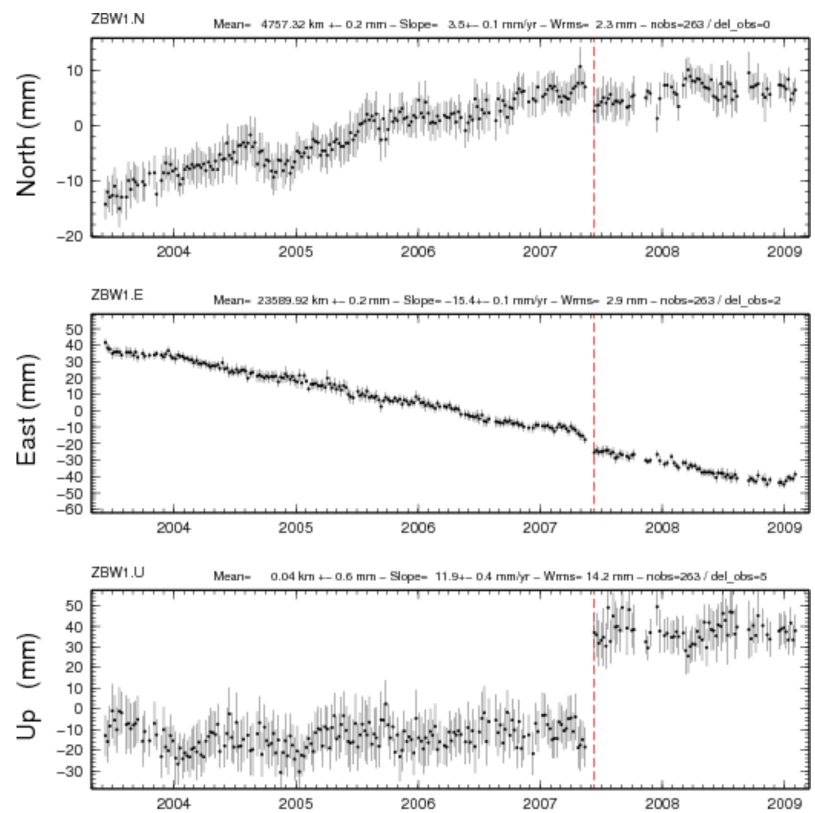
A shattered radome (site NLIB)

Equipment issues



Effect of a radome installation on height (http://facility.unavco.org/science_tech/dev_test/testing/domereport.pdf)

- Change of antenna, radome, receiver, etc…
- Result in systematic errors, often (but not always) offset in position
- Critically important to:
  - Document all changes in site log…!
  - Use proper site information when processing data…!



Effect of an antenna change from NOV_WAAS_600 to MPL_WAAS_2225NW not accounted for in the data processing

Noise sources and models

- Formal errors largely underestimate true errors because they assume errors are uncorellated in time.
- GPS velocity error estimates must account for colored noise (= temporally correlated noise), which can be estimated from position time series.
- This colored “noise” actually includes “signals”: mass loading, troposphere, monument motion, etc.
  - Current limitation for long-term velocity determination.
  - But source of data for research in hydrology…