Elastic anisotropy of core samples from the Taiwan Chelungpu Fault Drilling Project (TCDP): direct 3-D measurements and weak anisotropy approximations

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1 INTRODUCTION

The elastic anisotropy of a porous rock reflects the anisotropic attributes of both the solid matrix and pore space. Bedding in sedimentary rocks, cleavage in slates, preferred orientation of anisotropic minerals and anisotropic distribution of microcracks and pores can all contribute to elastic anisotropy, a seismic manifestation of which is shear wave splitting (Crampin 1981). Even though analysis of elastic wave propagation in an anisotropic medium can be highly complex, the investigation of this phenomenon has evolved to become a useful tool for deciphering rock physics attributes in a geologic formation, especially when the anisotropy is relatively weak which would allow the analysis to be simplified (Helbig & Thomsen 2005).

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Extensive laboratory studies have been conducted on the elastic or seismic anisotropy of different rock types (Lo et al. 1986; Hornby 1998). These studies show that seismic anisotropy at elevated pressures arises primarily from lattice preferred orientation (Kern 1993; Johnston & Christensen 1995), whereas at relatively low pressures it is usually controlled by the presence of an oriented system of open microcracks, which may develop as a result of rock forming processes or tectonic deformation. Indeed Nur & Simmons (1969) demonstrated in a seminal study the correspondence among seismic anisotropy, stress-induced cracking and orientations of the applied stress field.

In 1999 the Mw 7.6 Chi–Chi earthquake resulted in significant casualty and damage (Shin & Teng 2001). Its main rupture along the Chelungpu fault system was associated with surface break extending >100 km. The Taiwan Chelungpu-fault Drilling Project (TCDP) was undertaken to drill into this thrust fault, with the overall research goal to gain fundamental understanding of the physics
of earthquakes and faulting. Vertical drilling of Hole A at Takeng to a depth of 2 km through the northern portion of Chelungpu fault was completed in 2004. In a previous study, Louis et al. (2008) investigated the elastic and magnetic anisotropies of a total of 15 core samples retrieved from TCDP Hole A at depths ranging from 589 to 1412 m. They systematically characterized the anisotropies of $P$-wave velocity (APV) and magnetic susceptibility (AMS) as functions of depth and porosity. Microstructural observations were also conducted so as to establish possible correlations between the anisotropic properties and the petrofabrics induced by tectonic deformation and stress field.

The sonic velocity measurements were conducted following the protocol of Louis et al. (2004). For each available core a triplet of cylindrical plugs were drilled along three perpendicular directions. The velocity anisotropy of each plug was characterized by measuring the travel times for $P$-wave propagation across the mid-section along 8 diameters at an angular interval of 22.5°. For a weakly anisotropic rock, they demonstrated that data for the APV of the three orthogonal plugs can be approximated by a second-rank velocity tensor. Since the magnetic susceptibility is also described by a second rank tensor (Nye 1957), this provides a common basis for comparison of elastic and magnetic anisotropies as well as their relations to microstructure.

With reference to Thomsen’s (1986) and Tsvankin’s (1997) results for a weakly anisotropic rock, Louis et al. (2004) argued that the APV data of most sedimentary rocks can be approximated by a second rank tensor with an error of < 4 per cent. A primary objective of this study is to assess the validity of this approximation, by comparing with direct measurement of the 3-D anisotropy in a spherical sample under confinement using a specially designed apparatus at the Geophysical Institute of Prague. The apparatus has been used to acquire relatively comprehensive data on APV of samples from a variety of geological settings (Pros et al. 1998b; Vajdova et al. 1999; Pros et al. 2003; Machek et al. 2007; Vilhelm et al. 2008). Similar measurements on spheres have also been undertaken at Institut Français du Pétrole, with a focus on microcrack-induced anisotropy (Arts et al. 1994, Rasolofosaon et al. 2000) and its relation to permeability anisotropy (Rasolofosaon & Zinszner 2002).

However preparation of a spherical sample is quite cumbersome, and in contrast, the tensorial method of Louis et al. (2004) is relatively straightforward to implement, as long as core samples in three mutually orthogonal orientations are available and the anisotropy is relatively weak so that the approximation is valid. Our objective in this work is first to compare direct $P$-wave velocity anisotropy measurements on spheres to the prediction of the tensorial method of Louis et al. (2004), secondly to provide a microstructural interpretation for the observed pressure dependence of the measured properties which complements the work of Louis et al. (2008) on core samples from the same TCDP borehole.

2 SAMPLING AND METHOD

2.1 TCDP overview and core locations

The main rupture of the 1999 Chi–Chi earthquake was located along the Chelungpu fault system (Fig. 1a), with surface rupture extending over 100 km and uplifts of 8 m in some locations. The vertical drilling of TCDP Hole A in 2004 through the northern portion of Chelungpu fault at Takeng (near the city of Taichung) reached a depth of 2 km. Total recovery of the mostly continuous cores from TCDP Hole A was about 97 per cent (Song et al. 2007). At least three stratigraphic sequences oriented N15–30E could be identified from the TCDP core samples (Fig. 1b): the Cholan formation (late Pliocene to early Pleistocene), the Chinshui shale formation (Pliocene) down to 1712 m, and the Kueichulin formation (late Miocene to Pliocene) down to 1712 m, and the

![Figure 1](https://example.com/figure1.png)

**Figure 1.** (a) Map of Eastern Taiwan with the location and focal mechanism of the 1999 ChiChi devastating earthquake. The TCDP drill site is also indicated in the northern part of the fault zone. (b) Vertical cross section showing the geological formations involved in the Chelungpu thrusting. Again the TCDP drill hole reaching the major fault zone is shown.

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Cholan formation again at depths below 1712 m. The Cholan formation at the top is characterized by thick sandstone layers and alternating sandstone–siltstone–mudstone layers with weak to intense bioturbations. The sandstone is predominately made up of quartz and slate fragments, with some feldspar, sandstone quartzite fragments and clay matrix. The Chinsui formation is primarily made up of siltstone, with subsidiary thin layers of fine-grained sandstone, mudstone and alternating layers of sandstone and siltstone. The Kueichulin formation is made up of thick sandstone layers, with subsidiary thin layers of siltstone and shale. Structural and deformation features indicate the existence of several fracture zones in the Chinsui shale formation with one particular at 1111 m depth considered to be the plausible location that underwent significant slip during the Chi–Chi earthquake (Song et al. 2007).

The 15 core samples investigated by Louis et al. (2008) were retrieved from TCDP Hole A at depths ranging from 589 to 1412 m, including six sandstones and nine siltstones, on which the anisotropy of magnetic susceptibility and P-wave velocity was analysed. They concluded that the directions of anisotropy for magnetic susceptibility were consistent for both sandstones and siltstones, and could be related to the regional stress regime, whereas an additional contribution of anisotropy for the P-wave velocity was detected only in the sandstones, which could be related to the existence of an anisotropic crack distributions at the grain scale as revealed by quantitative characterization of three samples (#04, #09 and #14). For this study, we investigated the velocity anisotropy of three spherical sandstone from the following depths: 850 m (in the proximity of sample #04 in the Cholan formation), 1340 m and 1365 m (in the proximity of sample #09 in the Kueichulin formation).

2.2 3-D measurement on spherical samples

Velocity measurements on spherical dry samples were conducted using the experimental facilities at the Geophysical Institute of Prague described by Pros et al. (1998a). A schematic view of the experimental setup is shown in Fig. 2(a). It is based on the classical pulse transmission method using two ultrasonic transducers, one transmitter (T), one receiver (R), mounted on a frame which can rotate about two perpendicular axes (Fig. 2b) defining the angles \( \lambda \) (longitude) and \( \phi \) (latitude). This allows one to attach the two transducers at diametrically opposite locations in virtually any direction in space, except near the vertical direction where the sample is firmly clamped. Typically an angular step (\( \Delta \phi \) or \( \Delta \lambda \)) of 15° is imposed in the latitude and longitude, which would allow one to acquire a total of 132 independent measurements. Confinement of this experimental device inside a pressure vessel allows one to characterize the velocity anisotropy at confining pressures ranging up to 400 MPa. To prevent the confining fluid from penetrating into the rock, the spherical sample (50 mm in diameter) is encapsulated in a thin epoxy resin film (0.05 mm thickness). Both ultrasonic transducers have a resonance frequency of 2.5 MHz. A pulse generator is connected to the transmitter, and the signal recorded by the receiver is sent to a digital oscilloscope with a 100 MHz sampling rate. Only 40 \( \mu \)s of the transmitted signals are stored for the

Figure 2. Sphere method: (a) Schematic representation of the experimental system for measuring the P-wave velocity anisotropy on spheres under confining pressure. (b) Vertical and horizontal rotating frames where the ultrasonic transducers are implemented. (c) Stereonet corresponding to lower hemisphere equal area projection of directions defined on the sphere with angular steps of 15° in latitude and longitude. (a and b modified from Pros et al. 2003)
analysis, which is considered to be long enough for picking the first arrival.

To visualize the 3-D data set for the angular distribution of $P$ velocity at each pressure level, we will use a 2-D stereonet that corresponds to a lower hemisphere equal-area projection (Schmidt stereonet). The correspondence with the $(\lambda, \phi)$ coordinate system is illustrated in the inset schematic diagram in Fig. 2(c), and orientations for the 132 measurements have been labelled in the stereogram. The solid red circle corresponds to the upward vertical direction that is along the axis of the cylindrical core retrieved from the TCDP hole. The strike of the bedding plane (N105E horizontal direction) is indicated by the open circle. Accordingly the centre of the stereonet corresponds to the N105E horizontal direction.

2.3 Tensorial method on three orthogonally cored samples

The experimental configuration used by Louis et al. (2003, 2004) is illustrated in Fig. 3. It is necessary to have a good spatial distribution of measuring directions to correctly characterize the anisotropy, and this can be achieved by measuring the $P$-wave velocity across several diameters in three samples cored in mutually orthogonal directions from one single block (Fig. 3a). An optimized angular measuring scheme was proposed as shown in Fig. 3(b). As for the measurements on spheres, a classical setup for acoustic properties is used, including a pulse generator, two ultrasonic transducers with a resonance frequency of 0.5 MHz, and a digital oscilloscope connected to a PC for data acquisition and analysis. For each measurement the piezoelectric transmitter and receiver are located in opposite positions across a diameter on the circular surface of the sample, and the time of flight of the $P$ wave is measured. This is repeated eight times per sample after rotating it with an angular shift of 22.5°. The velocity profiles obtained for the three orthogonal samples are then corrected from slight variations in density from sample to sample (Louis et al. 2004), and the set of 24 measurements provides traveltimes along 21 independent orientations uniformly distributed in three orthogonal planes. Accounting for the errors associated with picking the first arrival and measuring the sample diameter, the standard error for the measurements is about 0.03 km s$^{-1}$ (Louis et al. 2003).

Louis et al. (2004) showed that most sedimentary rocks can be modelled as transversely isotropic rocks such that the $P$-wave velocity $V_P$ as a function of $\theta$, the angle between the axis of symmetry and the direction of propagation, can be fitted with an error of less than 4 per cent by the equation

$$V_P = V_0 (1 + \kappa \sin^2 \theta),$$  

where $V_0$ is the $P$-wave velocity in the direction perpendicular to the isotropic plane, and $\kappa$ is the anisotropy parameter in the tensor approach. With reference to Thomsen’s (1986) expression for weak anisotropy

$$V_P = V_0 (1 + \delta \sin^2 \theta + (\epsilon - \delta) \sin^2 \theta)$$  

(2)

this implies that the rock can be approximated as ‘elliptically anisotropic’, with the Thomsen anisotropy parameters $\epsilon$ and $\delta$ being equal such that $P$ wave fronts emanating from a point source are elliptical in shape (in any plane containing the symmetry axis of the transversely isotropic rock). The approximation in eq. (1) would also imply that the velocity can be described by a symmetric, second rank tensor $V_{ij}$, so that the $P$-wave velocity for propagation in the direction of unit vector $n$ is simply given by

$$V_P = V_{ij} n_i n_j.$$  

(3)

Since Thomsen’s analysis has been generalized by Tsvankin (1997) from a transversely isotropic to an orthotropic material, Louis et al. (2004) concluded that the $P$-wave anisotropy in any weakly anisotropic rock can generally be approximated by such a symmetric, second rank tensor. Accordingly, they proposed a methodology whereby the laboratory data of $P$-wave velocity in multiple directions are fitted to eq. (3) to find the ellipsoidal envelope of the representation quadric, as well as the directions and magnitudes of the three principal axes defined by the eigenvectors and eigenvalues of this symmetric tensor (Fig. 3c). Our elliptical anisotropy best fit given by the eigenvalues of a second rank symmetric tensor that produce the best fit with the measured data in the least-squares sense by no means represents the best fit in terms of the approximate stiffness tensor as compared to the true observed one, as rigorously derived by Sevostianov & Kachanov (2008).

The six independent tensor elements $V_{ij}$ are calculated from the 21 independent velocities measured on diameters with known orientations by a least-squares inversion method. Eigenvalues and eigenvectors are then obtained by diagonalization of the velocity tensor and can be represented in 3-D on an ellipsoid (Fig. 3c). By convention we will call $V_1$ the eigenvector with maximum magnitude, $V_2$ the eigenvector with intermediate magnitude and $V_3$ the eigenvector with minimum magnitude. The advantage of the tensorial approach is that, first, it is simpler than estimating the elastic fourth rank tensor from which the velocities must theoretically be derived.
(Mavko et al. 1998), and secondly, the velocity tensor can conveniently be compared to any other second rank tensor representing other physical properties like permeability, electrical conductivity or magnetic susceptibility (e.g. David et al. 2007). To represent the 3-D orientation of the velocity eigenvectors, one can use a 2-D stereonet like for the sphere measurements.

3 RESULTS

We first present and discuss the measurements obtained on the spherical sample in terms of anisotropy and compare those results to the prediction of the tensorial approximation. Then we focus on the pressure dependence of P-wave velocity and anisotropy up to 200 MPa also obtained in the same experiments on the spherical samples.

3.1 Velocity anisotropy measured on spherical samples

We plotted in Fig. 4 the results of the sphere measurements for the three sandstone samples from cores retrieved at depths 850, 1365 and 1394 m, at six different confining pressures: 5, 10 and 20 MPa on Fig. 4(a); 50, 100 and 200 MPa on Fig. 4(b). For each pressure, the stereonets on the left correspond to the raw velocity measurements in the 132 different directions investigated in the sphere analysis,
which correspond to an angular sampling of 15° in latitude and longitude. As explained above the directions are represented on an equal area lower hemisphere projection. The colour scale next to the stereonet indicates the range of velocities measured in each sample. The range of measured velocities is quite large, from 1900 m s⁻¹ at low pressure up to 4600 m s⁻¹ at the highest pressure. Note that the colour scale is kept constant and systematically adjusted to the extreme velocity values, therefore the velocity scale for a given sample is not exactly the same from one pressure to the other. The solid squares show the location of the maximum velocity measured $V_{\text{max}}$, and the solid circle where the minimum velocity $V_{\text{min}}$ was found.

The measurements were made on the intersections of the 15° × 15° angular grid, and the stereonet is made up of 60 quadrilateral elements that surround a central, circular element (Fig. 2c). To each quadrilateral element we assigned a constant velocity equal to the arithmetic mean of the velocities measured along the four adjacent nodes. The centre of the projection, which corresponds to the direction along which the sample is clamped, was attributed the average velocity of the 12 surrounding nodes. After each element was assigned a value, a light Gaussian smoothing was finally applied to facilitate observation. This operation did not affect the colour scale extrema since the filter radius was smaller than the half-length of the elements.

On the stereonets, for the spherical sample data acquired at the lowest pressure (5 MPa), the solid red circle marks upward vertical direction and the open circle marks the strike of the bedding plane (N15E horizontal direction). As can be seen there is a good agreement for our three samples between the location of minimum velocities on the stereonet and the position of the bedding strike, in agreement with the series of room pressure measurements of Louis et al. (2008) on many TCDP sandstone samples using the tensor method.

Let us next describe briefly the evolution of the directions of maximum and minimum measured velocity. If we look at the colour patterns, it is clear that for both samples at 1365 m and 1394 m the overall velocity distribution is pretty the same for all the pressure range and the position of maximum and minimum velocities (solid symbols) do not change significantly with increasing pressure. A slightly different picture is obtained for the sample at 850 m where at low pressure the dominant colour on the stereonet is blue, then evolves towards dominant red tones at higher pressure. This change in colour is associated with a rotation of the velocity maximum. For the three samples, the orientation of the minimum velocity direction remains stable, except for sample at 850 m and 20 MPa confining pressure where the minimum rotates by about 60° in azimuth, although the location of the larger low velocity sector remains consistent with the ones at other pressures. For samples at 1365 m and 1394 m, the maximum velocity direction also remains fairly stable across the pressure range. In the sample at 850 m, two distinct high velocity zones are sequentially observed upon pressurization with a transition at 50 MPa. As discussed later, this observation suggests the participation of several sources of anisotropy, of which one at least is pressure dependent. Note also that, as they result from measurements, the directions of maximum and minimum velocities are not necessarily orthogonal, in contrast with the principal directions implicitly assumed in the tensor analysis.

### 3.2 Prediction of second rank tensor approximation

The velocity versus orientation data set obtained on the spherical samples was used as input data for the tensor method described above. The velocity tensor defined in eq. (3) is built, and the eigenvalues and eigenvectors are calculated using a least-squares scheme (Louis et al. 2004). The maximum and minimum velocity values obtained with the tensor approximation are given in Table 1 for comparison with the direct measurements. We also provide for each pressure step a residual value that corresponds to the average of the differences between direct measurements and values recalculated from the best-fitting tensor. In Fig. 4 the directions of the three eigenvectors, necessarily orthogonal, are plotted as open symbols on the same stereonets as the measured velocities for comparison. The open square correspond to the eigenvector with maximum amplitude $V_1$, the open circle to the one with the smaller amplitude $V_2$ and the open triangle to the intermediate eigenvector $V_3$. Looking at the spatial location of these eigenvectors with respect to the measured maxima and minima velocities, we can see that in general there is a good agreement between the orientation of measured and predicted velocity extrema. On the average, the discrepancy is of the order of the width of the angular sectors, say +/− 15° which is rather good. To go further in the comparison between measured and predicted values, the stereonet plotted on the right next to the measured data represents for each pressure the predicted velocity data field calculated at each location where a measurement was done on the spheres. Therefore if the tensor approach provides a valid description of the actual velocity anisotropy, the distribution of colours in adjacent stereonets should be similar (although the velocity scale might slightly change as mentioned above). In general we observe a very good agreement between the prediction of the tensorial approach and the measured values, for the three different samples, at all the pressures: this gives us some confidence in our assumption that velocity anisotropy in rocks can be approximated by a second-rank tensor.

The results from the tensor approximation may also be compared with what was obtained by Louis et al. (2008) at virtually ambient pressure condition in the same samples. Fig. 5 is a stereoplot taken from Louis et al. (2008) showing in grey the eigenvectors obtained for all of their sandstone samples in the geographic reference. We overlay on the same figure the maximum and minimum velocity directions obtained from the direct measurements on spheres (solid symbols) at 5 MPa, and the directions corresponding to the three eigenvectors obtained with the tensor approximation (open symbols). As compared to Fig. 4, the directions have been rotated to fit the geographic reference. First, the location for the minimum velocity direction is very well constrained overall, with an N15° subhorizontal orientation parallel to the strike of the bedding. For the maximum and intermediate velocity directions, two observations can be made. First, the maximum velocity directions generally scatter within the plane perpendicular to the average minimum velocity direction, which is standard for the tensor-derived vectors (open squares) but not forced as far as the direct measurements are concerned (solid squares). Secondly, while the tensor-derived maximum and intermediate velocity directions are a good match for samples at 850 m, a poorer match is observed between maximum and intermediate velocity directions for the two other samples (1365 m and 1394 m) as compared to the results of Louis et al. (2008). This discrepancy, which will be addressed in the discussion section, can be associated with the two independent observations that (1) there is a 5 MPa difference in confining pressure between the two data sets, and (2) as can be seen in Fig. 4(a), the plane within which intermediate and maximum velocity values are located is nearly isotropic, and therefore unlikely to result in well constrained positions for the eigenvectors. It is worth mentioning here that this nearly isotropic plane matches exactly the orientation...
of the network of parallel microcracks observed by Louis et al. (2008).

### 3.3 Pressure dependence of velocity and anisotropy

Let us now focus on the pressure dependence of P-wave velocity in the tested rock samples. Again we will compare the measured data and the results predicted by the tensor approximation, focusing on the maximum and minimum velocities. In Fig. 6(a) we plotted for each sample the evolution of the maximum ($V_{\text{max}}$) and minimum ($V_{\text{min}}$) velocities measured on the spheres (solid squares and circles, respectively) as well as the best-fit ($V_1$) and minimum ($V_3$) eigenvalues estimated by the tensor approximation (see also Table 1). The initial value at low pressure is compared to the one found by Louis et al. (2008) on three orthogonal cylindrical samples (red circle): whereas a good agreement is found for the minimum velocities measured on the spheres, or the maximum and minimum velocities, and the magnitudes retrieved by the tensor approach. In Fig. 6(b) we plotted the P-wave anisotropy parameter $A$ defined as

$$A(\text{per cent}) = 100 \left( \frac{\max(V_P) - \min(V_P)}{\max(V_P) + \min(V_P)} \right)$$

(4)

where $\max(V_P)$ and $\min(V_P)$ represent either the maximum and minimum velocities measured on the spheres, or the maximum and minimum eigenvalues estimated by the tensor approximation (see also Table 1). The initial value at low pressure is compared to the one found by Louis et al. (2008) on three orthogonal cylindrical samples (red circle): whereas a good agreement is found for the sample at 850 m, there is a discrepancy of $+/-$15 per cent in absolute values for the other samples. As pressure increases, the P-wave anisotropy decreases sharply from an initial value close to 30–35 per cent, then flattens to become almost constant at a value

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<th>Sample (MPa)</th>
<th>$V_{\text{max}}$ (m s$^{-1}$)</th>
<th>$V_{\text{min}}$ (m s$^{-1}$)</th>
<th>Total anisotropy (per cent)</th>
<th>$V_1$ (m s$^{-1}$)</th>
<th>$V_2$ (m s$^{-1}$)</th>
<th>$V_3$ (m s$^{-1}$)</th>
<th>Total anisotropy (per cent)</th>
<th>$P_j$</th>
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Note: the grey-shaded lines contain data plotted in the equal area projections (common pressures).
of about 10 per cent for pressures higher that 20 MPa. There is therefore a large change in anisotropy by a factor 3, and a residual anisotropy at the highest pressure, which persists after unloading. Comparing the measured and the predicted anisotropy, we can see in Fig. 6(b) that the predicted results are systematically lower than the measured data, with an almost constant shift by about 5 per cent in absolute value. Tentatively we explain this discrepancy by the fact that the values predicted by the tensor analysis are smoother than the raw data set which is sensitive to local heterogeneities: milder contrasts between extreme values should then be expected from the tensor approximation, leading to a reduced anisotropy. However the shape of the anisotropy decreasing trend is preserved and compares satisfactorily with the measured data.

In Fig. 7 we analysed in more details our predicted directional results using the so-called $P_j/T_j$ plot which compares the corrected degree of anisotropy $P_j$ to the shape parameter $T_j$ (Jelinek 1981), two parameters which depend on the velocity eigenvalues, and which definition can be found in our previous work (Louis et al. 2008). Typically, $P_j$ is a measure of the sphericity of the ellipsoid (with $P_j \geq 1$, with $P_j = 1$ for a perfect sphere), and $T_j$ is the shape parameter taking negative values ($-1 < T_j < 0$) when the fabric is elongated in one direction and positive values ($0 < T_j < 1$) when the fabric is planar (Jelinek 1981). When $T_j = 0$, the ellipsoid is said ‘triaxial’, meaning that the ratio between maximum and intermediate eigenvalues is identical to the one between intermediate and minimum eigenvalues. The use of these parameters, which is fairly extensive in magnetic susceptibility studies (e.g. Tarling & Hrouda 1993; Borradaile & Henry 1997), may inform on the amount, shape and/or distribution of the elements responsible for the measured signal, which can be in some cases compared with the finite strain ellipsoid. Starting from the initial values at room conditions (solid symbols) obtained on core samples by Louis et al. (2008), the anisotropy parameter $P_j$ decreases while the shape parameter $T_j$ globally increases when the confining pressure is raised up to 200 MPa. The decrease in $P_j$ basically repeats what was shown in Fig. 6(b), with the difference that here the anisotropy is calculated using the three eigenvalues as opposed to the maximum and minimum ones only. The global increase in $T_j$ shows that for the three samples, the $P$-wave velocity fabric becomes more planar as pressure increases, which suggests changes in the relative contributions of the microstructural sources of anisotropy, and more specifically an enhanced contribution of the set of parallel microcracks observed by Louis et al. (2008). A closer look at the evolution of $T_j$ with pressure shows that a decrease seems to occur at higher pressure steps, which at least for the sample at 850 m seems to reflect the displacement of the maximum velocity zone in Figs 4(a) and (b), although irrecoverable mechanical deformation past 50 MPa cannot be ruled out.

4 DISCUSSION AND CONCLUSION

We have shown that the data set of $P$-wave velocity measured in 132 independent directions on dry spherical samples machined in TCDP samples cored at three different depths can reasonably well be described by the tensorial method proposed by Louis et al. (2003, 2004). Furthermore, the directions of anisotropy are in good agreement with the previous results obtained by Louis et al. (2008) on a larger set of core samples. Let us first discuss the validity of the tensorial method for describing $P$-wave velocity anisotropy, then we intend to give an interpretation of our results in terms of microstructure on the basis of theoretical models for the pressure dependence of seismic properties.

4.1 VALIDITY OF THE TENSORIAL APPROACH FOR DESCRIBING $P$-WAVE VELOCITY ANISOTROPY

As mentioned earlier, the rationale for using a simple tensorial method for $P$-wave velocity anisotropy, such as presented in Louis et al. (2003, 2004) on various core samples, is the possibility of describing efficiently the full 3-D velocity anisotropy using a limited number of measurements (as low as 6, minimum required for calculating a second-order symmetric tensor), hence of incorporating the richness of $P$-wave velocity data into structural studies along with other tools such as the anisotropy of magnetic susceptibility. Although such an approach would obviously prove to be wrong in the case of a monocrystal, we demonstrate here that a geologically processed aggregate does present anisotropy characteristics, which are necessarily related to a certain extent to experienced stresses and strains.

In our comparison between thorough $P$-wave velocity measurements on spheres and their tensorial best fit, we showed that overall we were able to recover a large portion of the original information. First the locations of the maximum and minimum velocity directions were preserved through the tensorial analysis (Fig. 4). Secondly, the general picture of the velocity magnitudes were also very well reproduced, including the anisotropies and their evolution across pressure steps, despite a slight misfit explained by a smoothing effect of the tensorial approach on the extreme measurements (Figs 4 and 6). Finally, in terms of geological significance, we showed that the results obtained from the spheres are virtually identical to the ones obtained with the tensorial method by Louis et al. (2008), with a very well constrained location for the minimum velocity direction, and a slight mismatch between the other principal directions which might be related to pressure effects (Fig. 5). It is useful to recall here that,
Figure 6. (a) Evolution of maximum and minimum $P$-wave velocities measured on spheres (solid symbols), and maximum and minimum eigenvectors derived from tensor analysis (open symbols with confining pressure. (b) Evolution of measured and derived $P$-wave anisotropies versus confining pressure. The value obtained by Louis et al. (2008) for TCDP samples at similar depths is plotted in red for comparison.

Figure 7. $T_j$ versus $P_j$ plot in the three samples showing the evolution of the shape of the velocity tensor ellipsoid.

although the maximum and minimum velocity directions obtained on the spheres were not geometrically constrained, the maximum velocity directions were observed in Fig. 5 to lay within the plane perpendicular to the minimum velocity direction, along with the ones obtained using the tensor method.

An intrinsic limitation of our approach is the case where two high (respectively low) velocity zones coexist, a situation that cannot be accounted for by a second-order symmetric tensor. However, among all the sample/pressure configurations studied here (Figs 4a and b), only one might correspond to this limiting case (sample at...
850 m and 50 MPa), an interpretation of which is provided in the following.

4.2 Pressure dependence of P-wave velocity and inferred crack densities

The most remarkable observation made when mapping the 3-D velocity anisotropies in Figs 4(a) and (b) is that, for every sample, the direction of the minimum value remains very stable across all pressure steps, and is in geometrical agreement with the network of subparallel microcracks already described by Louis et al. (2008) in the sandstone samples, as was checked in Fig. 5 by rotating the directions obtained at 5 MPa from both the directly measured extremum and the recalculated eigenvalues. This observation implies that the ‘velocity (or elastic) fabric’ corresponding to the network of parallel microcracks persists at high confinement, which might appear counter-intuitive since microcracks are generally expected to close early in the deformation process. The possibility that the anisotropy signal is influenced by lattice preferred orientation of constituent minerals (like phyllosilicates) is not ruled out. Even more surprising is the fact that the value of the shape parameter $T_j$ globally increases with pressure, suggesting that the crack effect is enhanced throughout the process as the fabric becomes more planar (Fig. 7). The persistence of the microcrack-related velocity fabric may be explained by the recent findings of Humbert (Fig. 7). The persistence of the microcrack-related velocity fabric may be explained by the recent findings of Humbert et al. (2011). In that study, the authors employed a series of magnetic anisotropy measurement techniques to investigate a possible magnetic signature of the same TCDP sandstone microcracks. They were able to propose that (1) the vertical microcracks had been coated by a late generation of magnetite grains, and that (2) these microcracks however were not sealed, and therefore were likely to remain open at depth. If the microcracks are coated by neocrystallizations of micron-sized hard cubic crystals, it becomes much more difficult for the applied pressure to achieve complete closure, allowing the persistence of a high compliance direction normal to the plane of the microcracks.

Our data set on directional P-wave velocities measured under increasing pressure permits to estimate the so-called crack density tensor and its variation with pressure. Two different approaches were used, both of them based on the work of Sayers & Kachanov (1995). First, we applied a numerical inversion scheme described by Fortin et al. (2011), secondly we calculated the analytical solution proposed by Wong & Zhu (2007). To do so several assumptions had to be made to make the calculations easier. First, we assume that the velocity distribution in space is transversely isotropic with a symmetry axis corresponding to the direction of minimum velocity (which is close to the bedding strike). This assumption is not supported by the results of Louis et al. (2008) who found a ‘triaxial’ distribution of clustered eigenvectors (Fig. 5): however if we look at the distribution of maximum and intermediate principal axes derived from this study, we observe that these axes are scattered in a plane roughly orthogonal to the bedding strike, in agreement with what would be observed in the case of transversely isotropic symmetry. Given this assumption it is possible to calculate the elastic parameters $C_{11}^{\text{exp}}$ and $C_{33}^{\text{exp}}$ (in the simplified Voigt notation) from the measured maximum and minimum P-wave velocities at each pressure step ($C_{11}^{\text{exp}} = \rho V_{\text{max}}^2$, and $C_{33}^{\text{exp}} = \rho V_{\text{min}}^2$). The second assumption is that there are two populations of cracks in the rock, one considered to be isotropic with crack density $\rho_{\text{iso}}$, and a second one made of parallel cracks oriented perpendicular to the bedding strike as shown by the study of Louis et al. (2008), with a crack density $\rho_{\text{cr}}$ (‘$\nu$’ is for vertical as the crack planes are subvertical).

The first method is based on the numerical scheme proposed by Fortin et al. (2011). The compliance tensor of the sandstone can be written as: $S_i^{\text{el}} = S_i^{\text{fr}} + \Delta S_i^{\text{cr}}$, where $S_i^{\text{fr}}$ is the compliance tensor of the crack free solid matrix, and $\Delta S_i^{\text{cr}}$ is the additional compliance resulting from the presence of cracks. Here, we will restrict our attention to the case when the rock matrix is isotropic, so that its elastic stiffness can be specified in terms of the Young’s modulus $E$ and Poisson’s ratio $\nu$. To model the additional compliance, $\Delta S_{ijkl}$, we consider the case of a medium containing circular cracks. Thus, a crack density $\rho_c$ can be defined as $\rho_c = \frac{1}{V} \sum_i c_i^2$, where $c_i$ is the radius of the $i$th crack and $N$ is the total number of cracks embedded in the representative volume $V$ (Bristow 1960, Walsh 1965). In sandstones, microcracks are the result of imperfectly bonded interfaces at grain boundaries. These cracks may not be circular, but can be considered in a first approximation as flat cracks (Schoenberg 1980). In this case, the distribution of cracks of irregular shapes can be replaced by an equivalent distribution of circular cracks. Thus, the crack density as defined above is an effective density of the equivalent distribution of the circular cracks (Sevostianov & Kachanov 2002, Guéguen & Kachanov 2011). This parameter, $\rho_c$, is adequate for the isotropic case of randomly oriented cracks but cannot be used for other distributions of crack orientation. Following the work of Kachanov (1980), the scalar crack density, $\rho_c$, can be generalized to a second crack density tensor, $\alpha$, defined as: $\alpha = \frac{1}{r} \sum (c^2 \mathbf{n} \mathbf{n}^T)$, where $\mathbf{n}$ is the unit vector normal to a crack, and $\mathbf{nn}$ is the dyadic product. The linear invariant $\alpha_{kk} = \rho$ so that $\alpha$ is a natural tensorial generalization of $\rho$.

In the case, where contributions of cracks are evaluated in the non-interaction approximation (NIA)—without accounting for the interaction between cracks—the additional compliance, $\Delta S_{ijkl}$, due to multiple circular cracks in arbitrary orientational distribution is given by Kachanov (1980), as

$$\Delta S_{ijkl} = h \left( \frac{1}{4} (\delta_{ij} \alpha_{kl} + \delta_{ik} \alpha_{jl} + \delta_{il} \alpha_{jk} + \delta_{jl} \alpha_{ik}) - \frac{1}{2} \beta_{ijkl} \right),$$

where $\delta_{ij}$ is the Kronecker delta and $h$, a scalar defined as $h = \frac{321 - \nu^2}{4(1 - \nu^2)}$. Eq. (5) shows that the additional compliance is expressed as a function of the second-order crack density tensor $\alpha$, but also as a function of a fourth rank tensor $\beta$, defined as $\beta = \frac{1}{r} \sum (c^2 \mathbf{n} \mathbf{nn})$. However as discussed by Sayers & Kachanov (1995) and Guéguen & Kachanov (2011), the fourth rank tensor will be small and in most rocks Poisson’s ratio $\nu < 2$, thus neglecting the $\beta$-term and retaining $\alpha$ as the sole crack density parameter usually constitutes a good approximation, especially in the case of a dry rock.

If we focus on a transversely isotropic rock (with symmetry axis $x_3$) associated with an axisymmetric distribution of microcracks embedded in an isotropic elastic matrix, then the symmetry condition necessarily requires that $\alpha_{11} = \alpha_{22}$. Using the Voigt notation the non-vanishing components of the elastic compliance of a cracked rock are given by

$$S_{11} = S_{22} = \frac{1}{E_o} + h \alpha_{11},$$

$$S_{33} = \frac{1}{E_o} + h \alpha_{33},$$

$$S_{44} = \frac{1}{G_o} + h (\alpha_{33} + \alpha_{44}).$$
\[
S_{66} = \frac{1}{G_o} + 2h\alpha_{11}, \tag{6d}
\]
\[
S_{12} = S_{13} = S_{23} = -\frac{\nu_0}{E_o}, \tag{6e}
\]

where \(G_o\) is the shear modulus of the crack-free matrix. The elastic stiffness of a cracked rock \(C_{11}\) and \(C_{33}\) in the Voigt notation, are deduced using \(C_{ij} = (S_{ij})^{-1}\) and
\[
C_{11} = (S_{11}^2 - S_{12}S_{13})/D \tag{7a}
\]
\[
C_{33} = (S_{31}^2 - S_{12}S_{13})/D \tag{7b}
\]

with \(D = S_{12}^2(2S_{11} + S_{33}) - S_{12}S_{23} - 2S_{13}^2\). The eqs. (6a)–(6e) are valid, in the case of transversely isotropic symmetry for any arbitrary distributions of cracks. For a medium containing randomly oriented cracks, the elastic compliance can be simplified using \(\alpha_{11} = \alpha_{22} = \alpha_{33}\). However, as it can be seen from Fig. 6, P-wave velocity anisotropy exists with a minimum value in the direction of the symmetry axis \(x_3\). This implies \(\alpha_{11} \neq \alpha_{33}\) and \(\alpha_{33} \geq \alpha_{11}\). Thus, to interpret the second crack density tensor in terms of microstructural attributes, it is possible to rewrite \(\alpha\) as:
\[
\alpha = (\alpha_{11} I + (\alpha_{33} - \alpha_{11}) e_3 e_3), \tag{8}
\]

where \(I\) is the unit tensor, and \(e_3\) the unit vector along the \(x_3\) axis. The difference \(\rho_v = \alpha_{33} - \alpha_{11}\) characterizes the crack density of parallel cracks with normal along \((Ox_3)\), whereas the tensor \(\alpha I\) characterizes cracks randomly oriented with a crack density \(\rho_{iso} = 3\alpha_{11}\). As a consequence, the elastic compliance of a cracked rock (eqs 6a–6e) can be rewritten as a function of these two crack densities, \(\rho_{iso}\) and \(\rho_v\), using \(\alpha_{11} = \frac{\rho_v}{\rho_{iso}}\) and \(\alpha_{33} = \frac{\rho_v}{\rho_{iso}} + \rho_v\).

Finally, the theoretical prediction of the effective medium model provided by eqs (7) and (6) in terms of effective stiffness \(C_{11}\) and \(C_{33}\) are compared to the elastic stiffness \(C_{11}^{exp}\) and \(C_{33}^{exp}\) obtained from the elastic wave velocities measurements, and the distance between them is defined by a least-square function \(F\) given by
\[
F = (C_{11}^{exp} - C_{11})^2 + (C_{33}^{exp} - C_{33})^2 \tag{10a}
\]

that needs to be minimized at each loading stage with respect to the unit vectors \(\rho_{iso}\) and \(\rho_v\).

In the second method we used Wong & Zhu’s (2007) analytical solution for weakly anisotropic cracked rocks, which was also based on Kachanov’s (1980) formulation. In particular, their eq. (10a) gives the following expression for one of Thomsen’s (1986) anisotropy parameters
\[
\epsilon = \frac{C_{11} - C_{33}}{2C_{33}} = \frac{C_{11}}{E_o h} \left(\alpha_{33} - \alpha_{11}\right) \left(1 + E_o h \alpha_{11}\right) \frac{2}{\left(1 + E_o h \alpha_{11}\right)^2 - v_o^2} \tag{10b}
\]

It should be noted that our two crack density parameters \(\rho_{iso} = 3\alpha_{11}\) and \(\rho_v = \alpha_{33} - \alpha_{11}\) here correspond to the parameters \(a\) and \(b\) defined by Wong & Zhu (2007). Since we typically have \(v_o^2 \leq 1\), the above can be approximated by:
\[
\epsilon \approx \frac{C_{11} - C_{33}}{2C_{33}} \approx \frac{C_{11}}{E_o h} \frac{\rho_v}{2 \left(1 + E_o h \rho_{iso}/3\right)^2} \tag{10c}
\]

Substituting eq. (6) into eq. (7b), we can also obtain:
\[
C_{33} = \frac{(1 - v_o + E_o h \rho_{iso}/3)}{(1 - v_o + E_o h \rho_{iso}/3)} (1 - v_o - E_o h \rho_{iso}/3 - 2v_o^2) \approx \frac{1}{(1 + E_o h \rho_{iso}/3 + \rho_v)}. \tag{9}
\]

Solving the above simultaneous equations, we arrive at these analytic expressions for the two crack density parameters (and corresponding ratio) in terms of the measured velocities (and corresponding elastic stiffnesses and Thomsen parameter \(\epsilon\))
\[
\rho_{iso} \approx \frac{3}{E_o h} \left[\frac{E_o/C_{33}}{(1 + 2\epsilon) - 1}\right] \tag{10a}
\]
\[
\rho_v \approx \frac{2}{E_o h} \frac{\epsilon}{1 + 2\epsilon} \frac{E_o}{C_{33}} \tag{10b}
\]
\[
\rho_{iso} \approx \frac{2}{3} \left[\frac{(E_o/C_{33})}{(E_o/C_{33}) - (1 + 2\epsilon)} \right]. \tag{10c}
\]

It is of interest to note that this ratio is independent of the Poisson’s ratio, and the crack anisotropy is manifested by coupled effects on the normalized stiffness and Thomsen’s parameter.

In Fig. 8(a) we plotted the results for the numerical model, and in Fig. 8(b) the results given by the analytical solution, for the pressure dependence of both crack densities. For the calculation we took \(E_o = 51\) GPa, \(v_o = 0.2\) and a bulk density of 2240 kg m\(^{-3}\) (Louis et al. 2008). The input data for the inversion were the minimum and maximum measured P-wave velocities. The first striking observation is that both methods give almost the same results, which in a sense validates the numerical approach. The only visible difference is at the highest confining pressure where the analytical solution predicts smaller values for the crack density of the isotropic population. This will have some consequences as discussed below. The second striking observation is that at low pressure, the crack densities reach relatively high values: this is in agreement with the microstructural analysis conducted by Louis et al. (2008) who observed that most of the grains in their sandstone samples were cracked (see their fig. 12 in Louis et al. 2008). As expected both crack densities \(\rho_{iso}\) and \(\rho_v\) decrease significantly with increasing pressure, but the amplitude of the decrease is not the same for both distributions. Indeed, the isotropic crack density has a sharper decrease compared to the anisotropic one which becomes almost constant after applying 50 MPa confining pressure. The inserted graphs in Fig. 9 represent the evolution of the crack density ratio \(\rho_v/\rho_{iso}\), as a function of confining pressure. The small discrepancy at high pressure in crack density for the isotropic distributions especially in samples at 1394 m and 1365 m has a significant impact on the crack density ratio and the shape of the curves are slightly different. Nevertheless the important result is that in both cases the crack density ratio increases with pressure, providing quantitative estimates on the contrasting behaviours of vertical and randomly oriented sets of microcracks: as the pressure is increasing, the influence of crack anisotropy becomes relatively more important. Consequently we expect that the elastic fabric becomes more and more planar. Nevertheless reminding that the measurements have been done on dry samples, one should be cautious in extrapolating the results to rocks under in situ conditions where fluids are present in cracks and pores.

### 4.3 Evidence for higher order of anisotropy

The analysis in Section 4.2 was done assuming a transversely isotropic symmetry for the elastic fabric. Actually this assumption is not in complete agreement with our data set as three independent clustered velocity axes are found. In fact, it is possible that additional anisotropic features are present in the tested samples. The increase in the value of \(T_j\), at least up to 50 MPa of applied pressure, may in fact be associated with the transition from one (or a combination
of anisotropy source(s) to another. In Section 3.3, we pointed out the tendency for \( T_j \) to also decrease slightly for the highest pressure steps, and suggested for the sample at 850 m and 50 MPa that this behaviour might reflect a displacement of the maximum velocity sector in Figs 4(a) and (b). To obtain such a substitution, it is necessary that (1) several sources of anisotropy are present and that (2) these sources exhibit different pressure sensitivities. Louis et al. (2008) interpreted the orientation of the eigenvectors as resulting from a combination of vertical microcracks anisotropy and bedding anisotropy. The maximum velocity would be found at the geometric intersection between these two features (i.e. within both the plane of microcracks and the bedding plane). The concept of the maximum velocity direction lying parallel to the intersection direction of two fabric elements has also been described by Healy et al. (2009) and Lloyd et al. (2009) with combinations of cracks and ductile fabric elements. We saw in the present work that the velocity anisotropy deviates from this pattern as pressure rises. Indeed, although the minimum velocity tends to remain in the same location, the maximum velocity travels within the plane transverse to the minimum direction and is not systematically found at the intersection between the microcracks and the bedding plane (Fig. 5). In an early comparison between elastic and magnetic anisotropies in rock samples from the Bohemian Massif in Czech Republic, Hrouda et al. (1993) inferred the presence in a set of undeformed samples of bed parallel cracks potentially formed during erosion and uplift. Alternately, such microcracks may also simply form as collateral to the coring process. We suggest that the anisotropy associated with the bedding in our sandstone samples be due to intergranular microcracks

Figure 8. Crack densities estimated using the Sayers & Kachanov (1995) scheme as a function of applied confining pressure. Inserted is a plot of the crack density ratio between the isotropic and the anisotropic crack distributions. (a) Predictions of the numerical scheme proposed by Fortin et al. (2011) (b) Predictions derived from the analytical solution of Wong & Zhu (2007).

Figure 9. Conceptual model showing two different contributions to the rock anisotropy associated with cracks either vertical or parallel to the bedding plane.
that can close more readily than the partially filled vertical ones. Such bed parallel microcracks possibly related to cleavage planes in phylllosilicates would be difficult to identify in the microstructures as opposed to intragranular ones. This scheme, which refines the one proposed by Louis et al. (2008) by adding a pressure sensitive component to the velocity anisotropy, is illustrated in Fig. 9. In the absence of confining stress, both sources of anisotropy (bed parallel and vertical microcracks) cause the maximum P-wave velocity to be observed along the intersection between the two planes. As the pressure rises, the bedding parallel cracks readily close, causing the maximum velocity direction to be released from the intersection direction and scattered into the plane transverse to the minimum velocity direction.

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