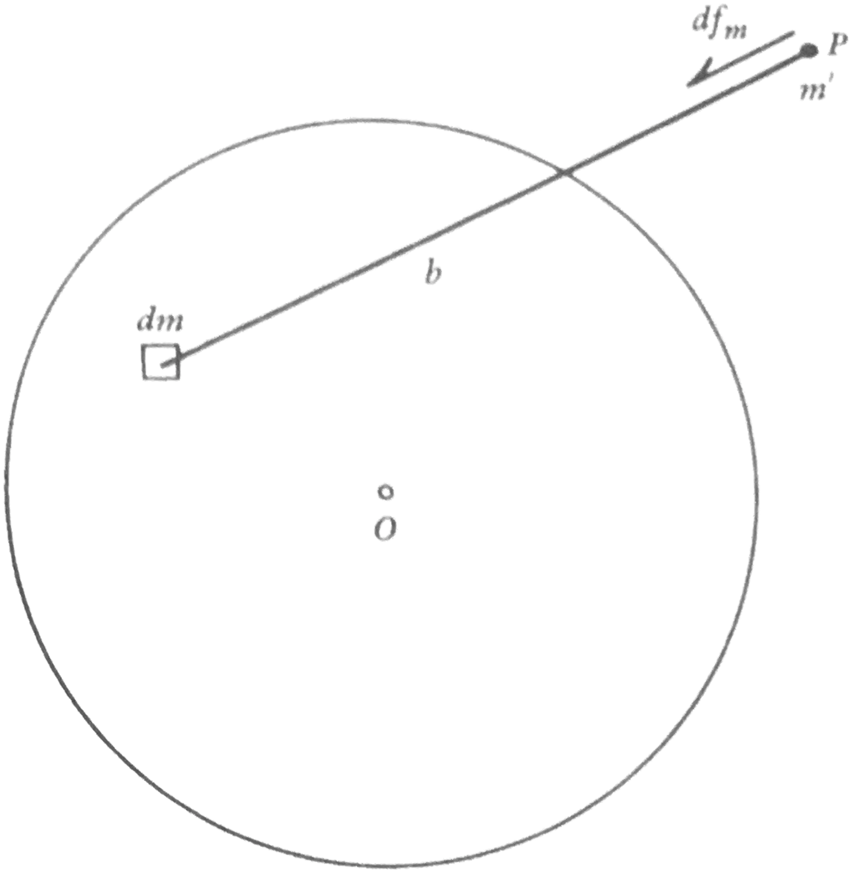


Figure 5.1 Force on a mass m' due to the gravitational attraction of an infinitesimal element of mass dm in the Earth.

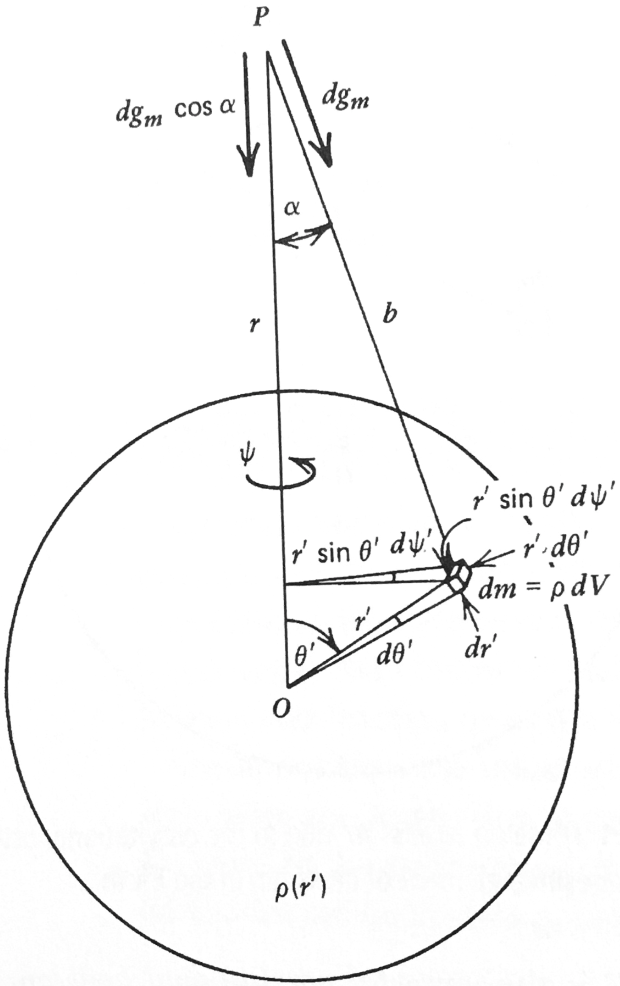
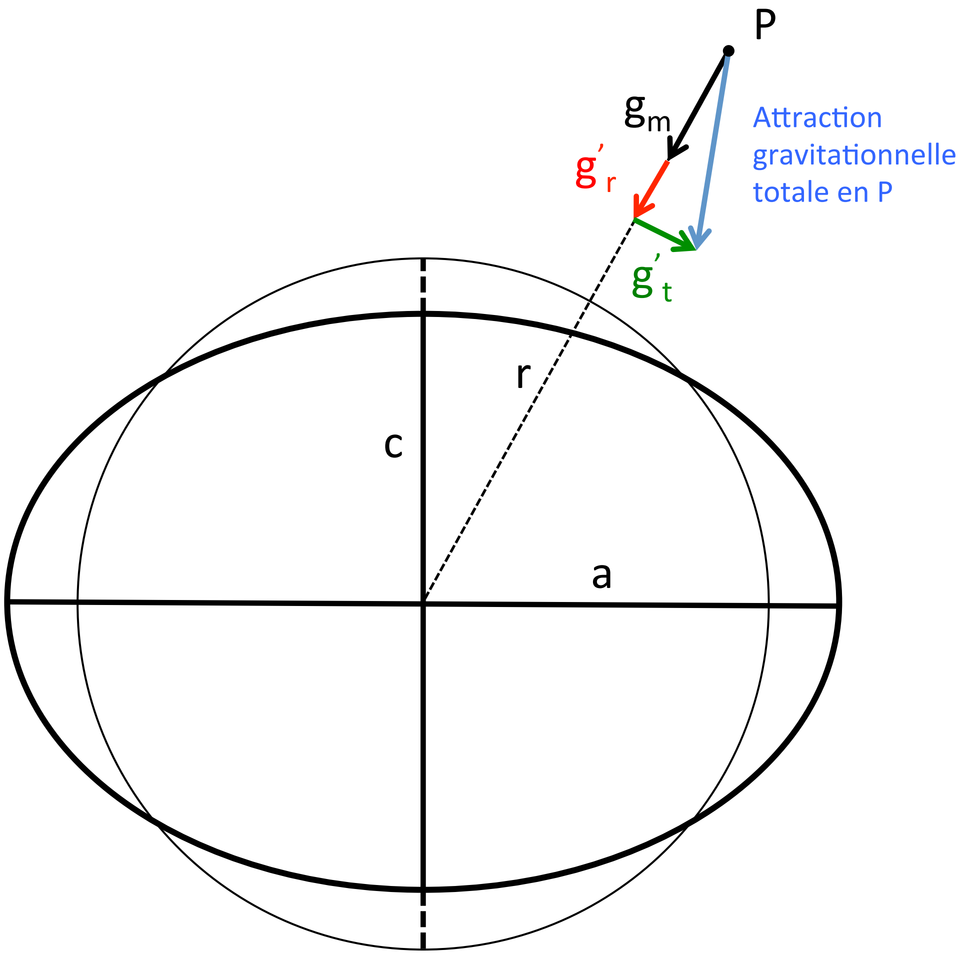


Figure 5.2 Geometry for the calculation of the gravitational acceleration at a point outside a spherically symmetric mass distribution.



Attraction gravitationnelle totale en P

Avec: $g_m = \frac{GM}{r^2}$

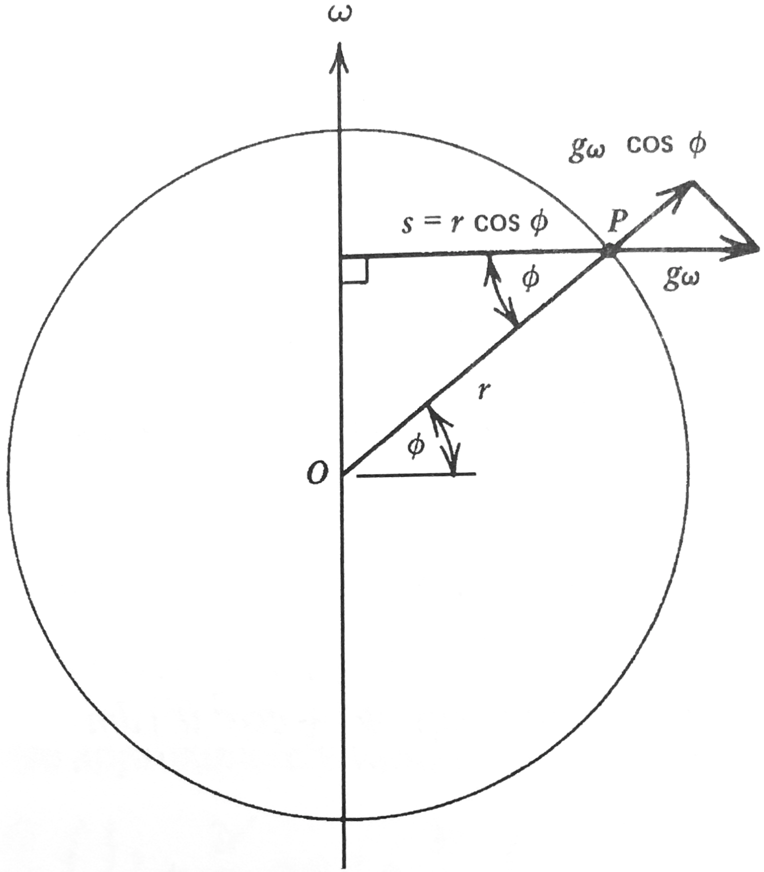
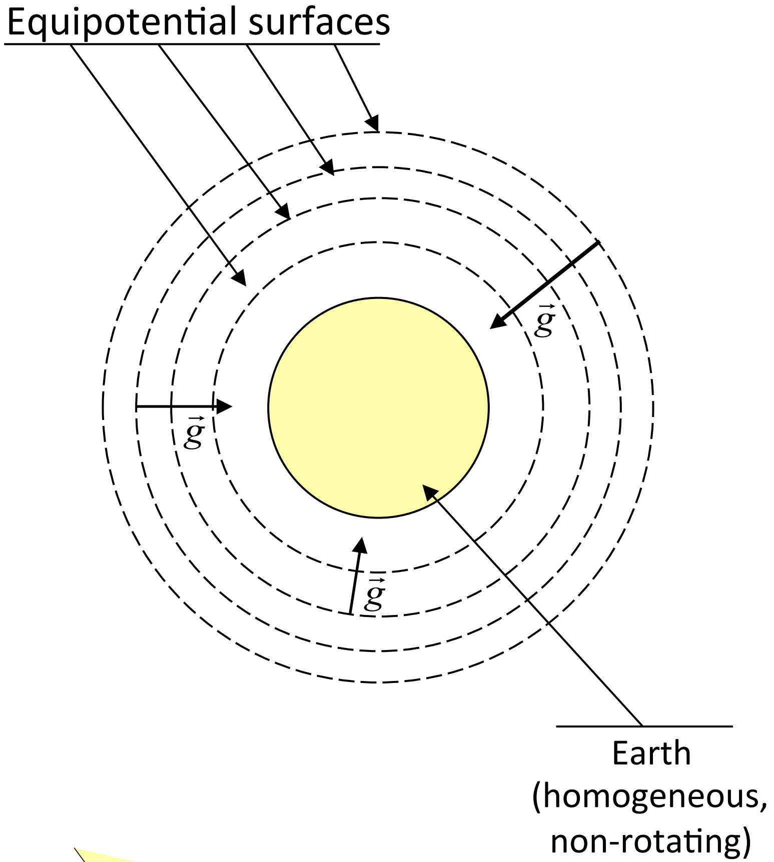
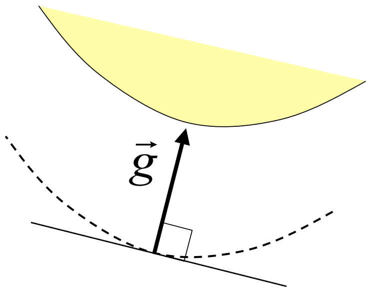


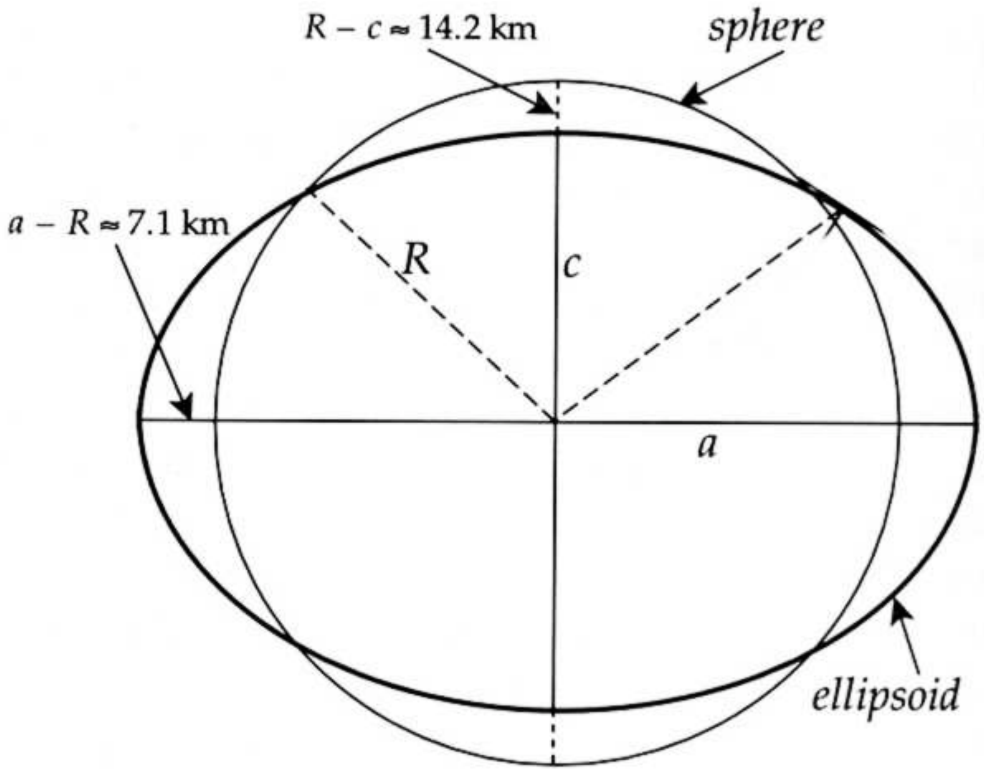
Figure 5.4 Centrifugal acceleration at a point on the Earth's surface.



Equipotential surfaces

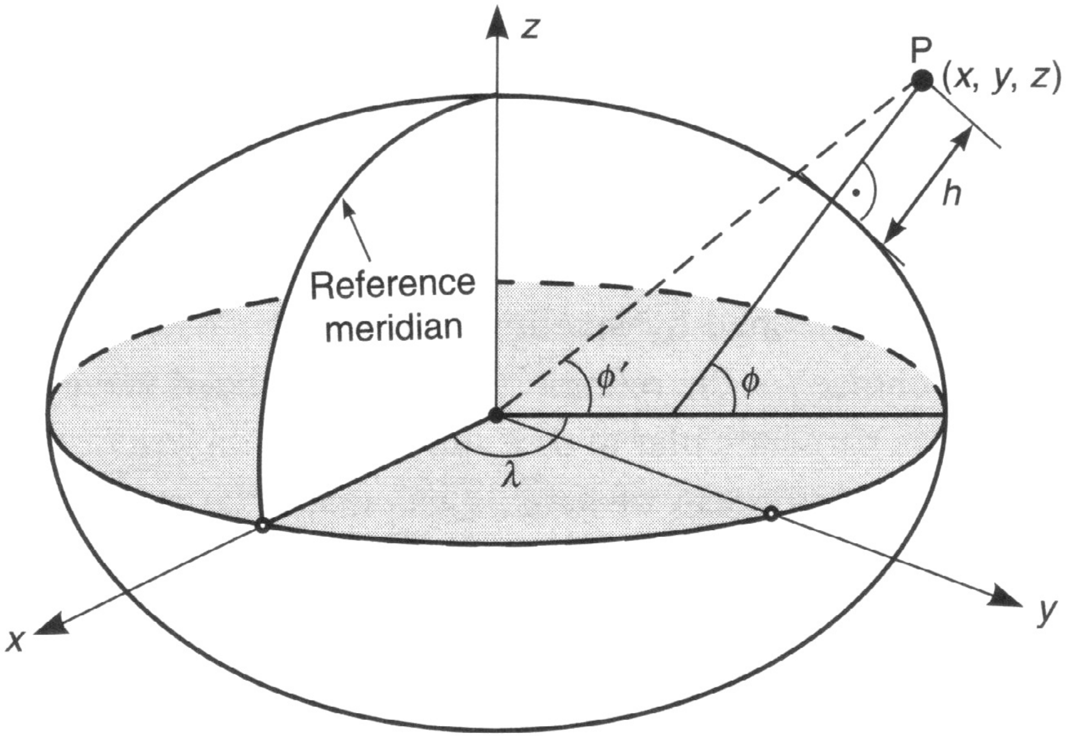
Earth
(homogeneous,
non-rotating)





a = 6378.136 km
c = 6356.751 km
R = 6371.000 km

Comparison between the WGS-84 ellipsoid and a sphere of identical volume



Conversion between ECEF and ellipsoidal coordinates can be made using the following formulas:

$$N = \frac{a}{\left(1 - e^2 \sin^2 \phi\right)^{1/2}}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} (N+h)\cos\phi\cos\lambda \\ (N+h)\cos\phi\sin\lambda \\ \left(N\left(1-e^2\right)+h\right)\sin\phi \end{bmatrix}$$

$$\tan\lambda = y/x$$

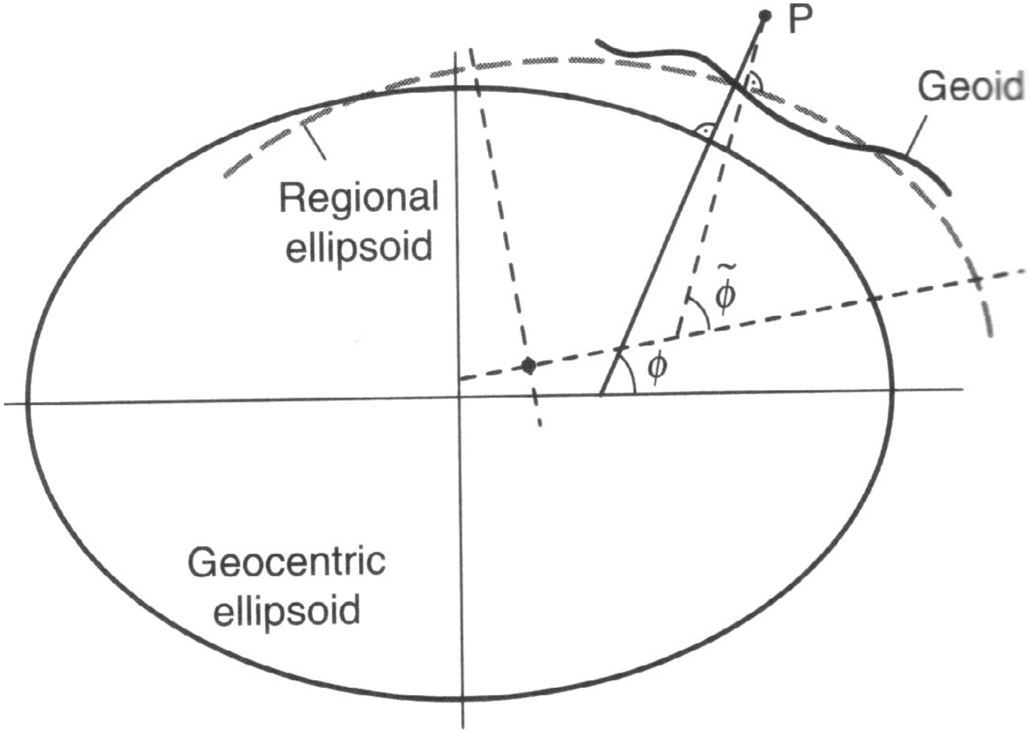
$$p = \sqrt{x^2 + y^2}$$

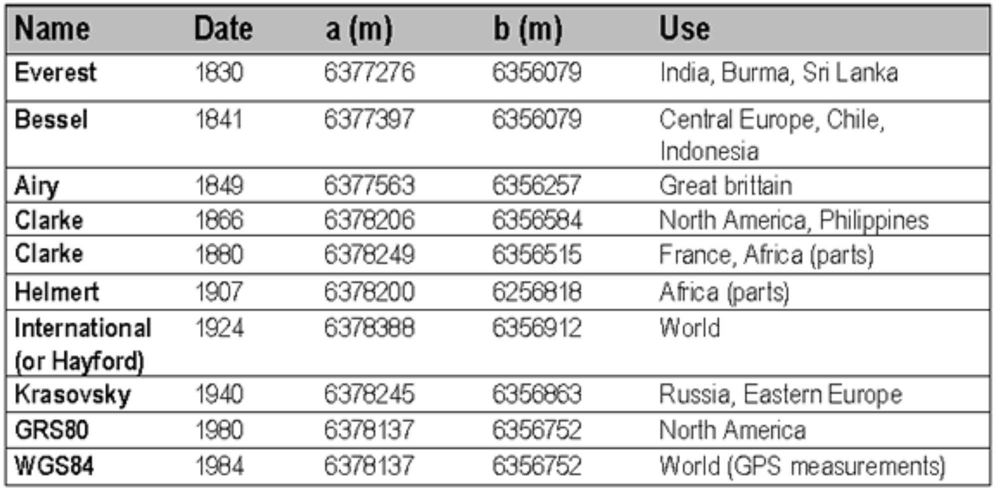
$$N = \frac{a}{\left(1 - e^2 \sin^2 \phi\right)^{1/2}}$$

$$h = \frac{p}{\cos\phi} - N$$

$$\tan\phi = \frac{z}{p}\left(1 - e^2 \frac{N}{N+h}\right)$$

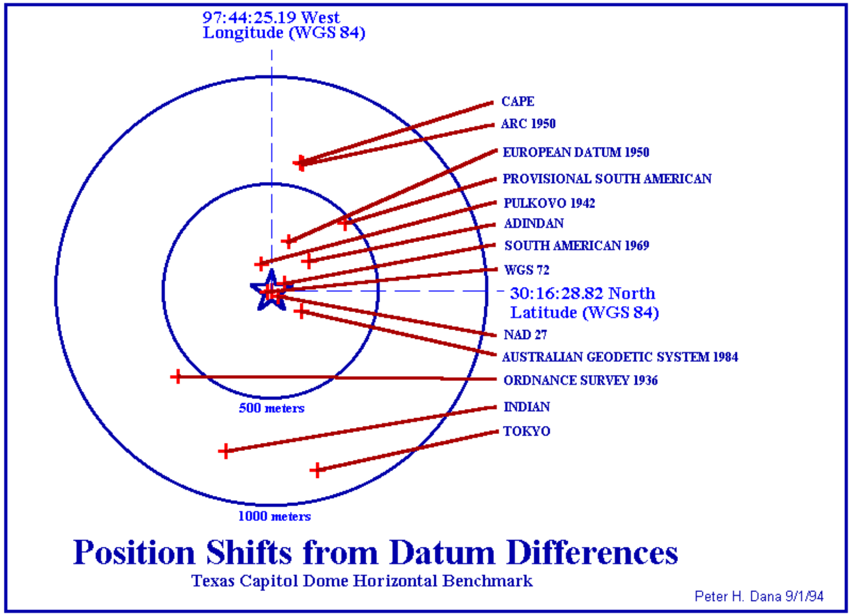
How could this happen? (well, it did not…)





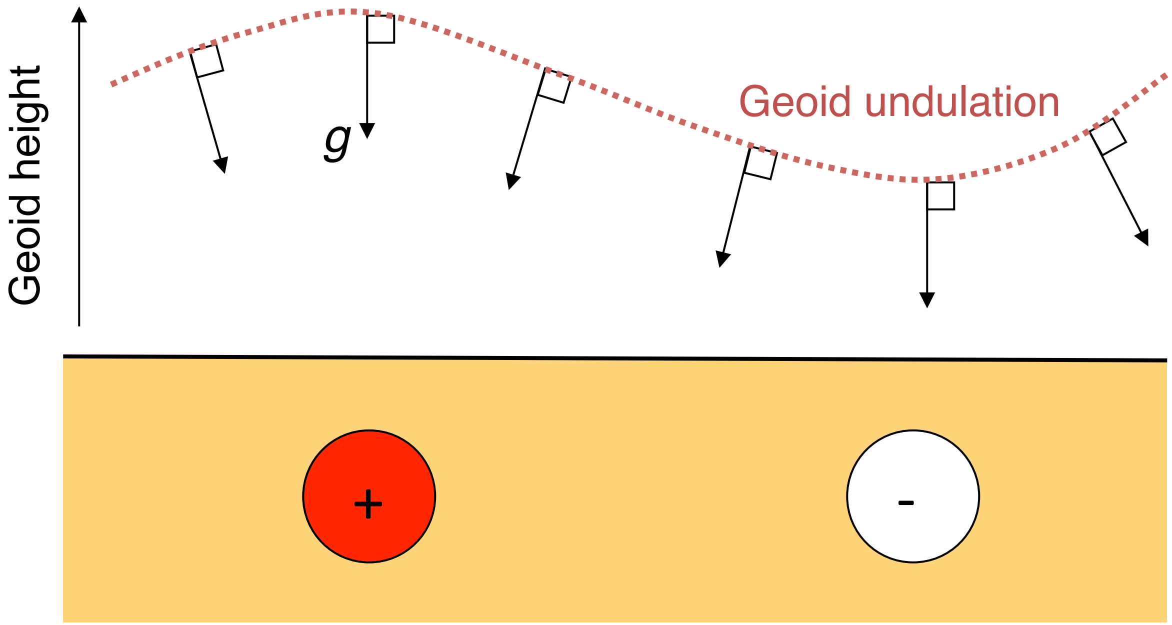
Name	Date	a (m)	b (m)	Use
Everest	1830	6377276	6356079	India, Burma, Sri Lanka
Bessel	1841	6377397	6356079	Central Europe, Chile, Indonesia
Airy	1849	6377563	6356257	Great brittain
Clarke	1866	6378206	6356584	North America, Philippines
Clarke	1880	6378249	6356515	France, Africa (parts)
Helmert	1907	6378200	6256818	Africa (parts)
International (or Hayford)	1924	6378388	6356912	World
Krasovsky	1940	6378245	6356863	Russia, Eastern Europe
GRS80	1980	6378137	6356752	North America
WGS84	1984	6378137	6356752	World (GPS measurements)

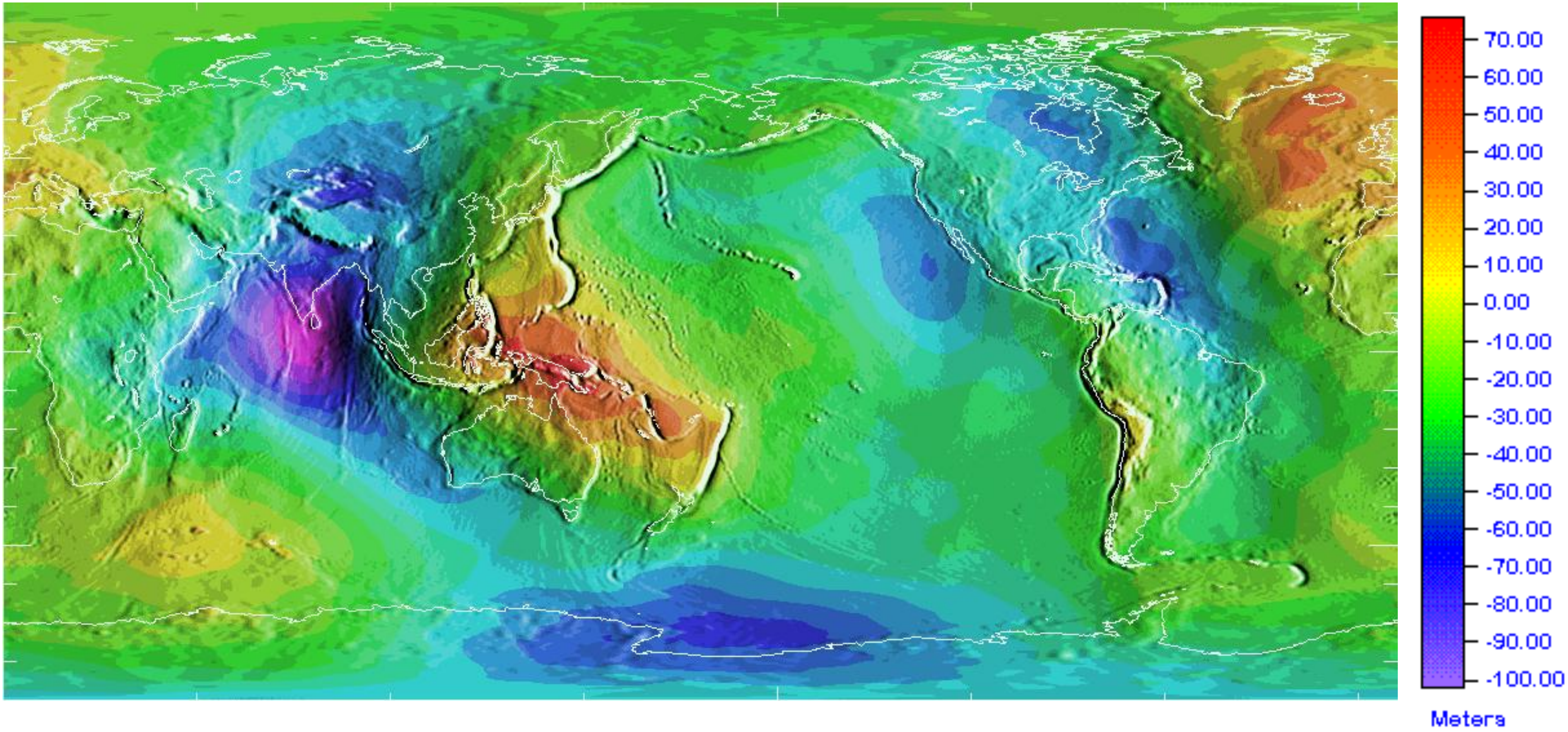
http://www.kartografie.nl

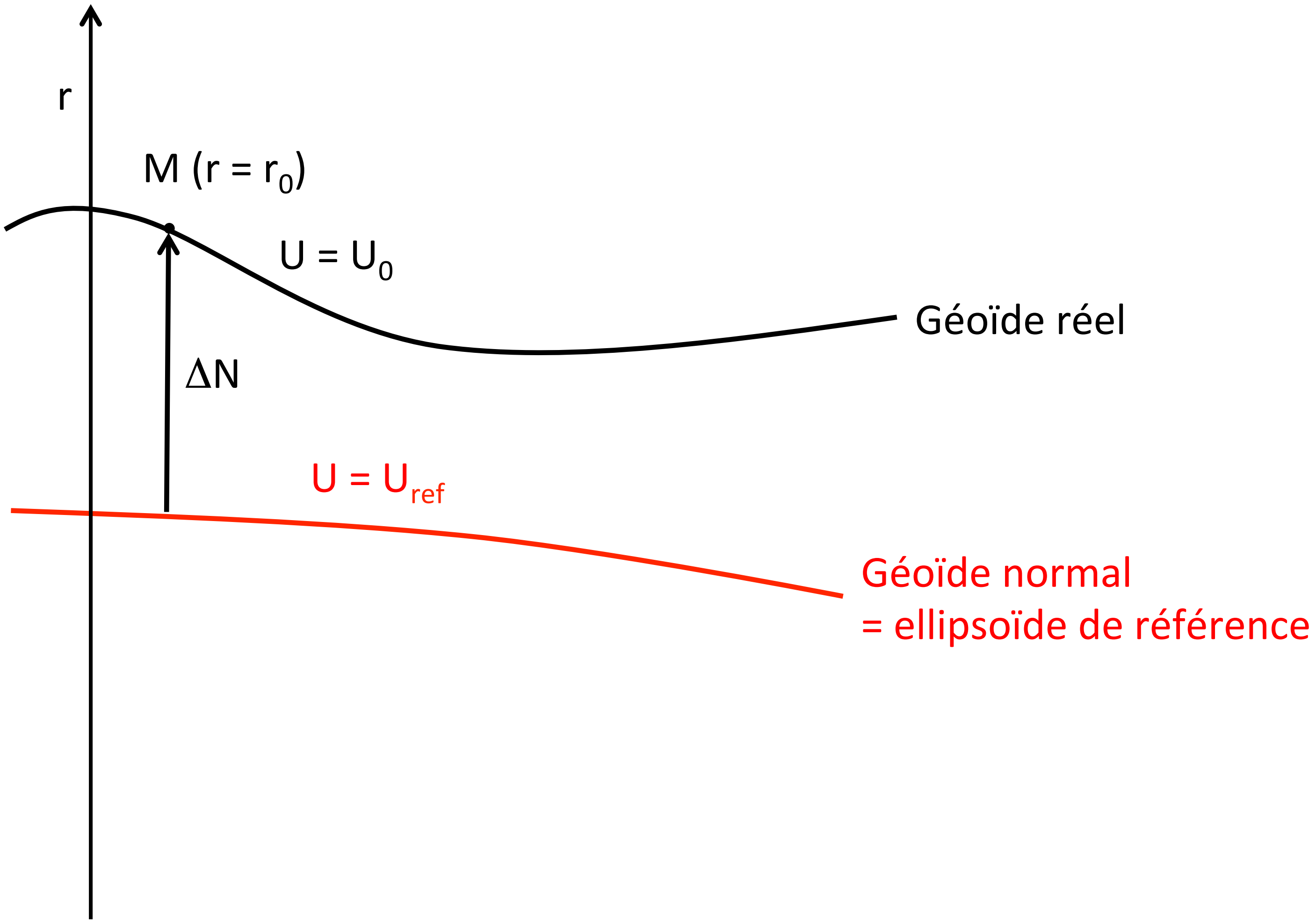


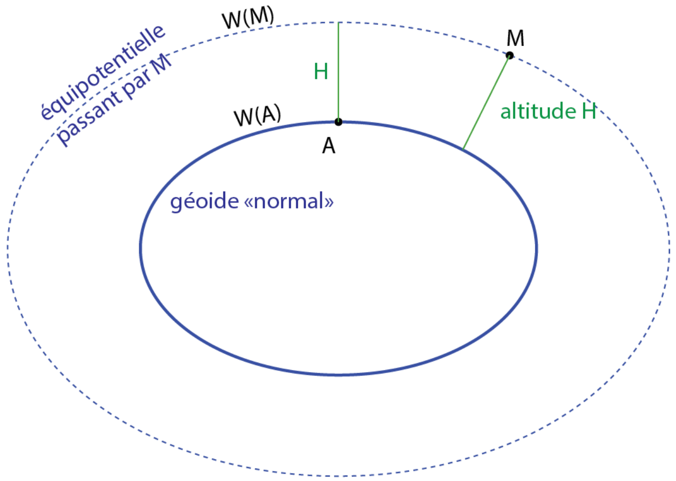
Position Shifts from Datum Differences

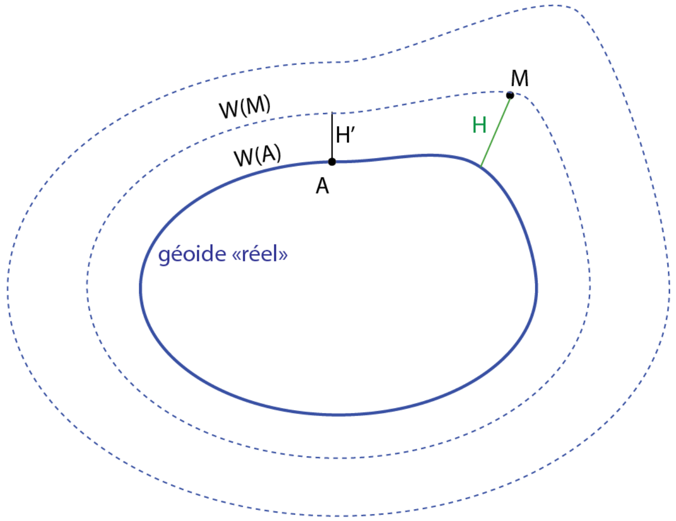
Texas Capitol Dome Horizontal Benchmark

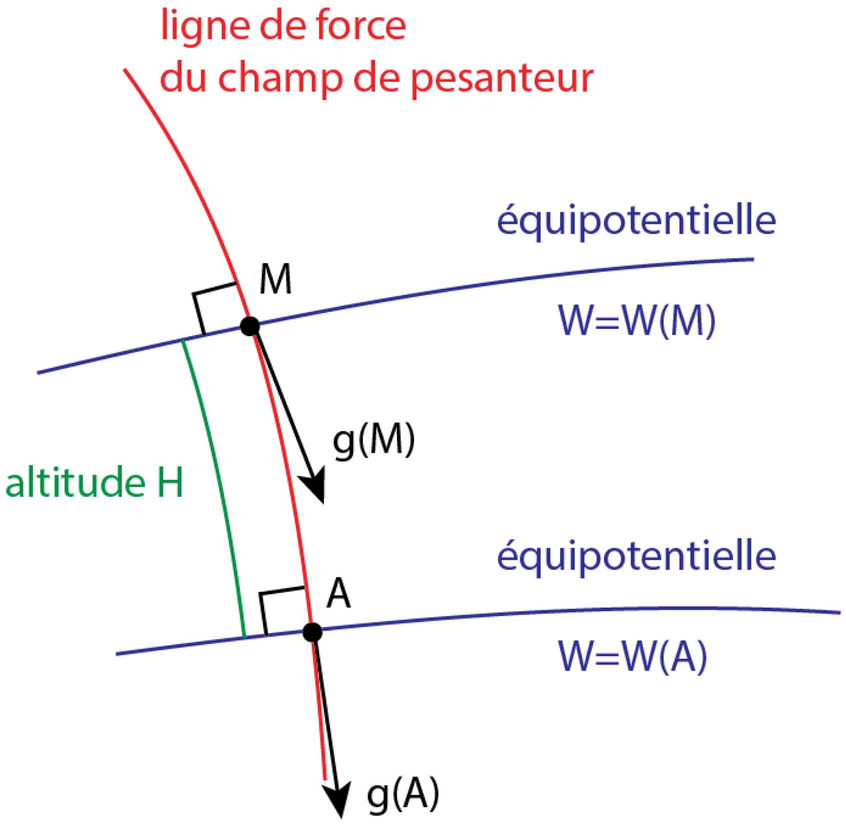

Geoid height
Geoid undulation
g
+
-

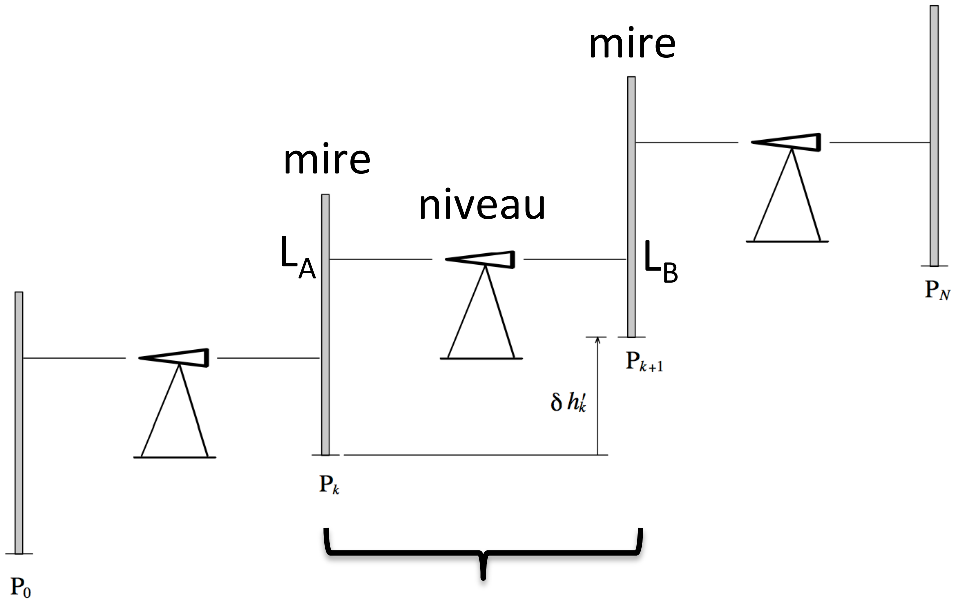

70.00
60.00
50.00
40.00
30.00
20.00
10.00
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
-60.00
-70.00
-80.00
-90.00
-100.00
Meters


r
M (r = r0)
U = U0
Géoïde réel
ΔN
U = Uref
Géoïde normal
= ellipsoïde de référence


W(M)
équipotentielle
passant par M
M
H
altitude H
W(A)
A
géoide «normal»


M
W(M)
H'
H
W(A)
A
géoide «réel»

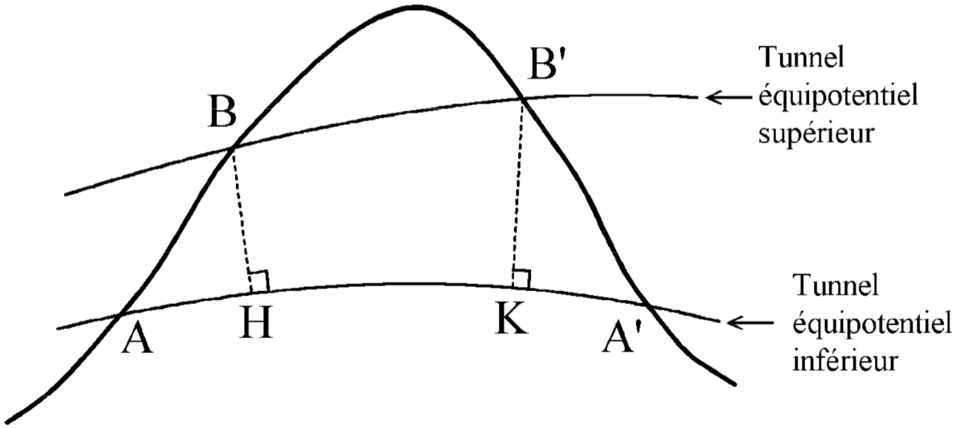

ligne de force
du champ de pesanteur
équipotentielle
M
W=W(M)
g(M)
altitude H
équipotentielle
A
W=W(A)
g(A)



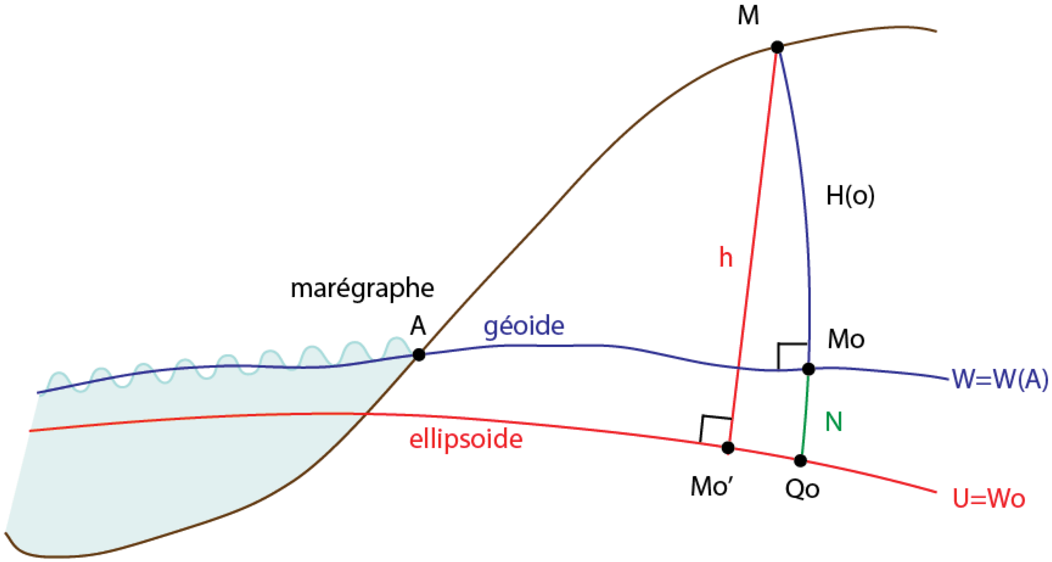
Une nivelée, L_A = mesure avant, L_B = mesure arrière

$\delta h_k' = L_A - L_B$

Un cheminement

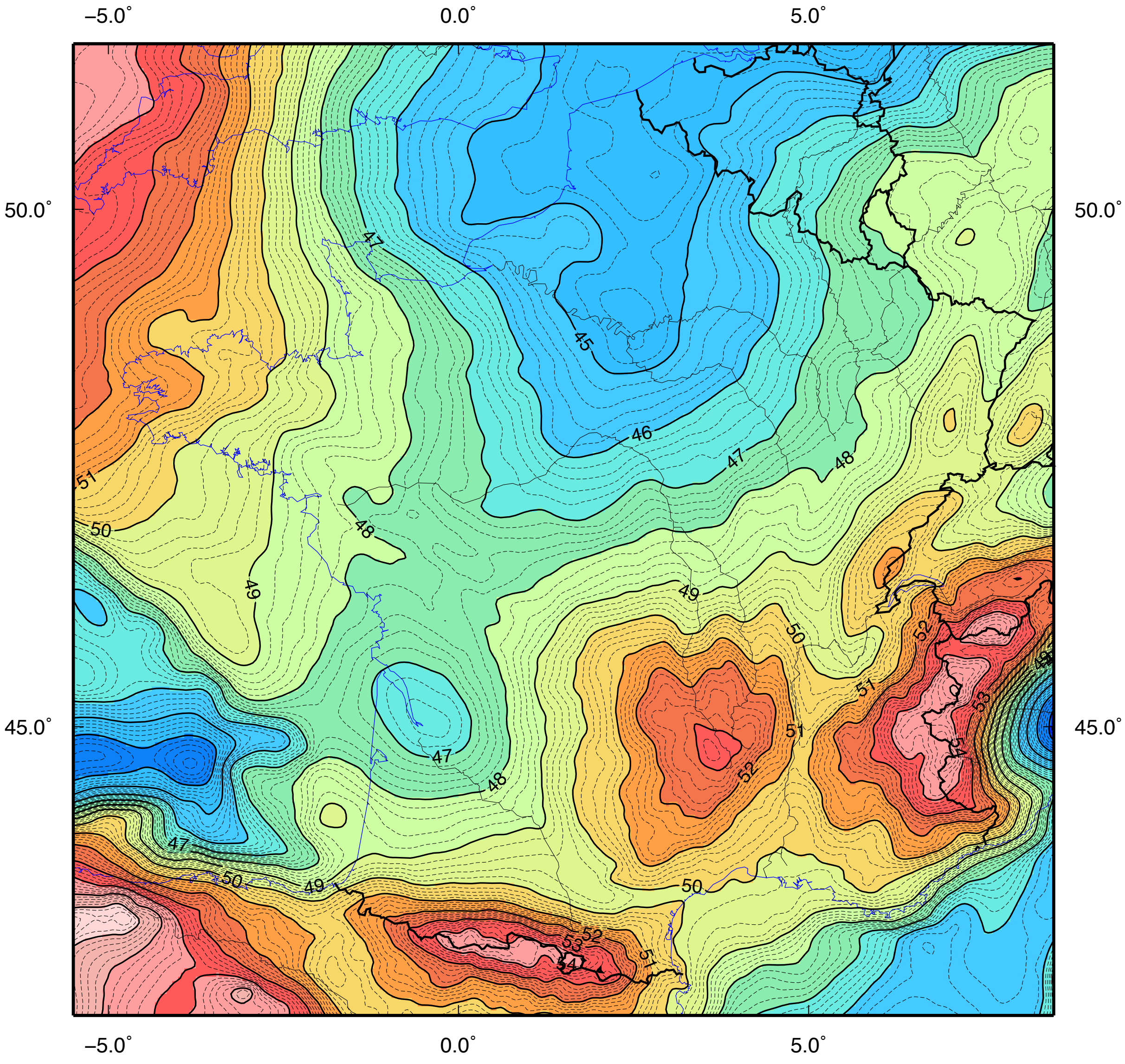


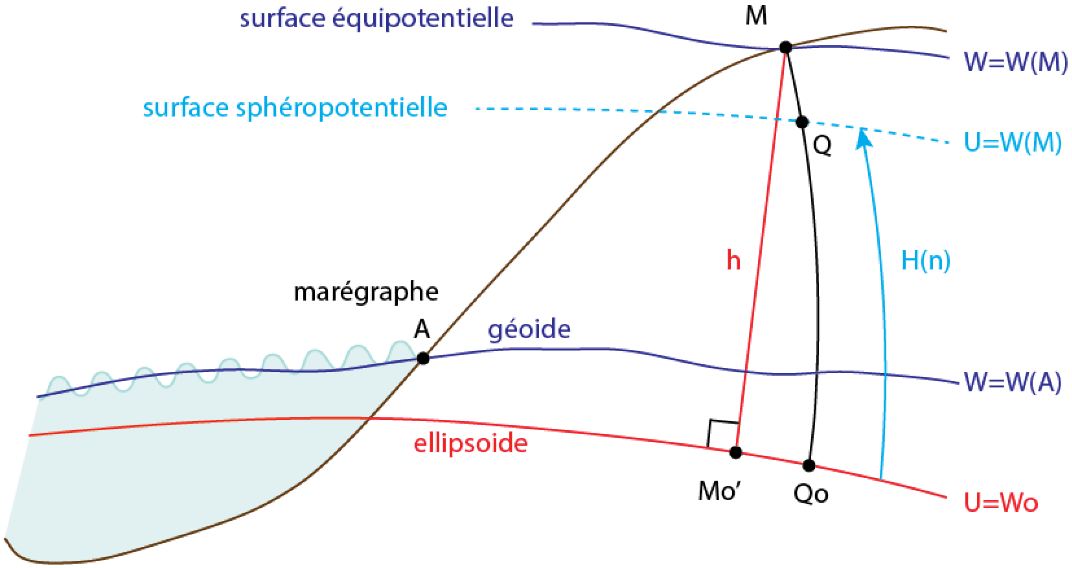
Le paradoxe des tunnels
équipotentiels (Duquenne, 2008)

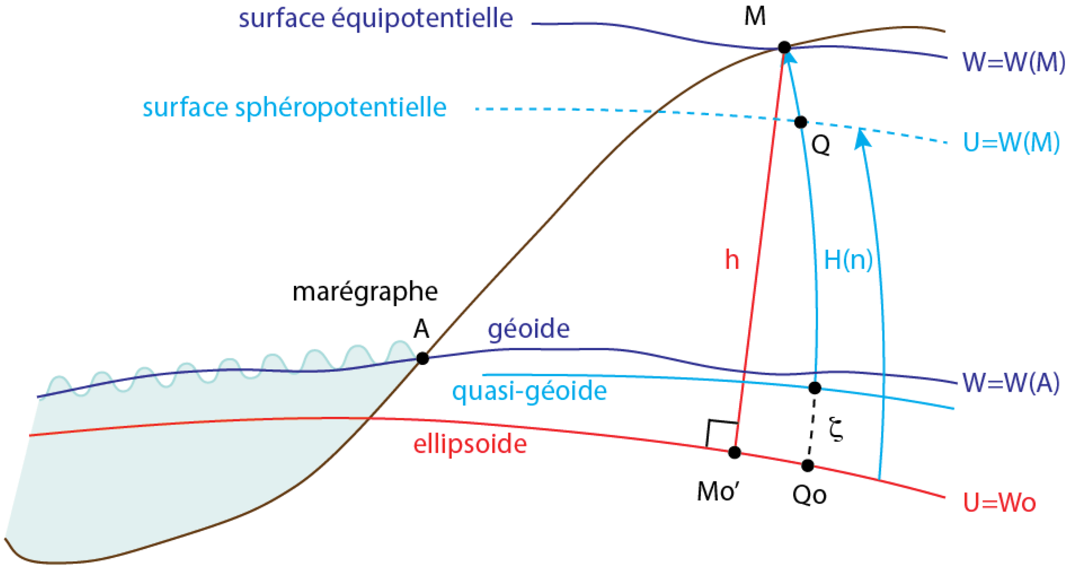

M
H(o)
h
marégraphe
A
géoide
Mo
W=W(A)
N
ellipsoide
Mo'
Qo
U=Wo

Un modèle de géoïde, EGM2008

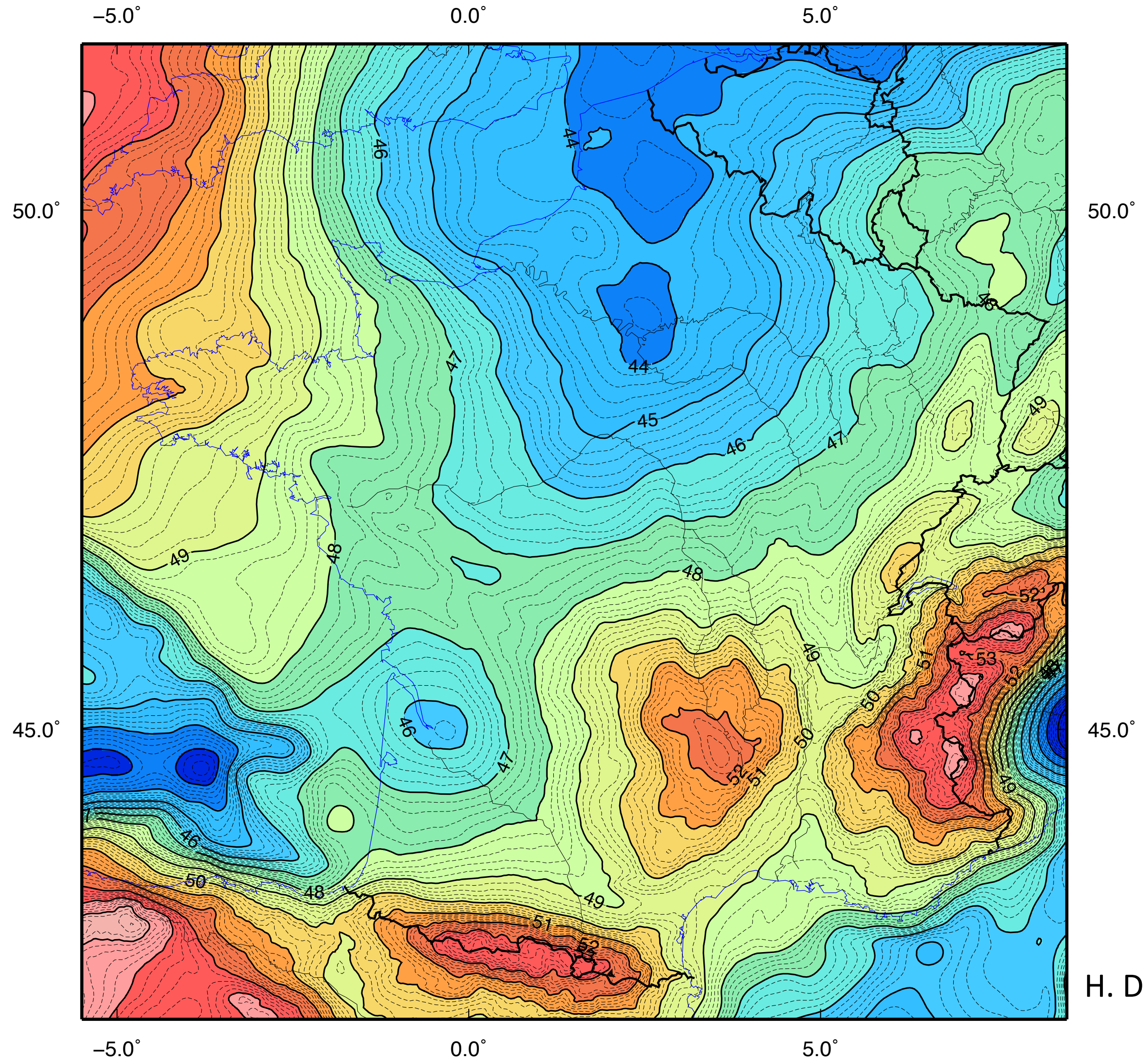
EGM2008: obtenu à partir de données gravimétriques terrestres et maritimes, de données d'altimétrie satellitaire, et utilise des Modèles Numériques de Terrain (MNT) pour évaluer la contribution du relief au géoïde.




surface équipotentielle
M
W=W(M)
surface sphéropotentielle
U=W(M)
Q
h
H(n)
marégraphe
A
géoide
W=W(A)
ellipsoide
Mo'
Qo
U=Wo


surface équipotentielle
M
W=W(M)
surface sphéropotentielle
Q
U=W(M)
h
H(n)
marégraphe
A
géoide
W=W(A)
quasi-géoide
ζ
ellipsoide
Mo′
Qo
U=Wo

Le quasi-géoïde français QGF98



H. Duquenne, IGN

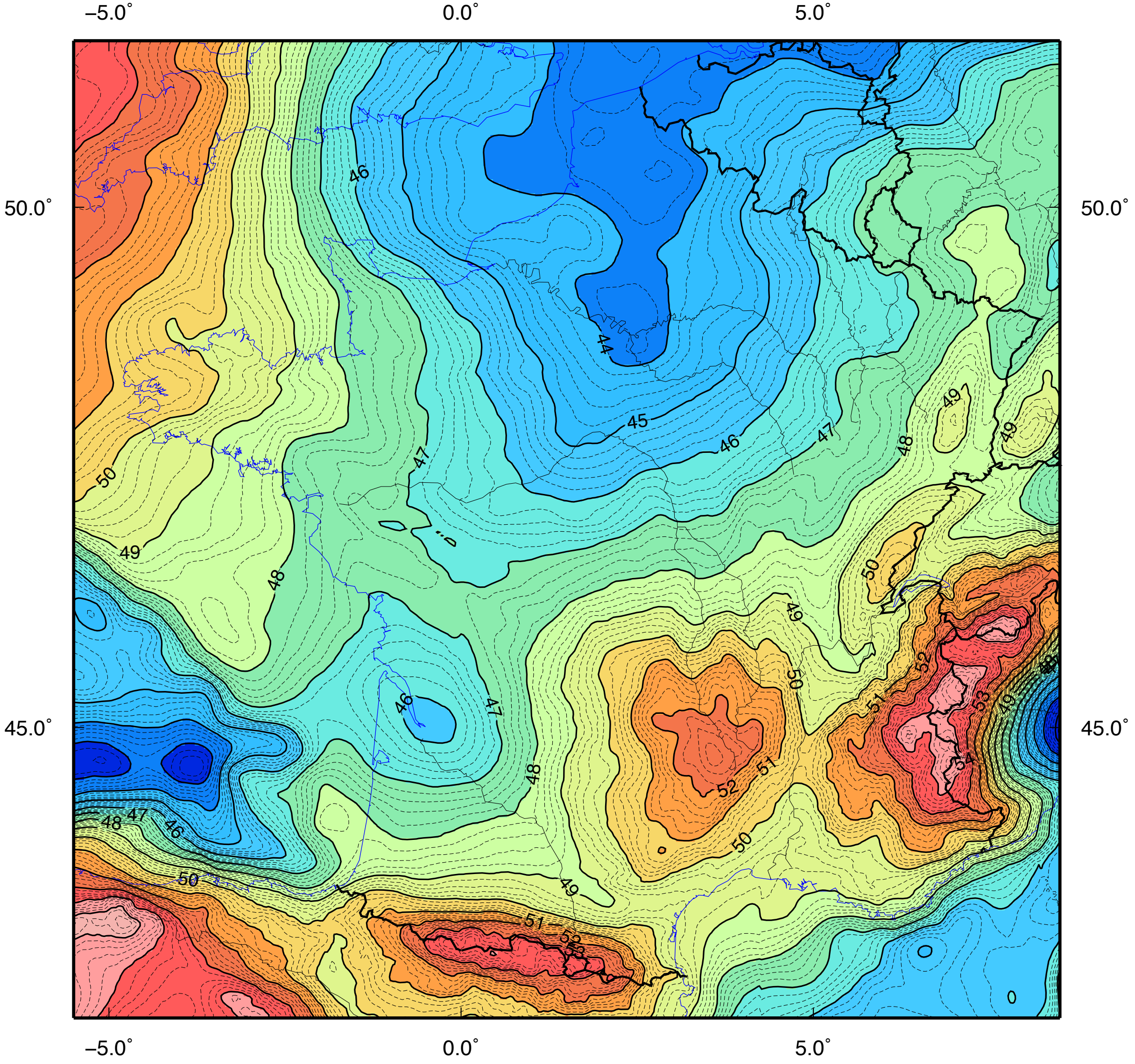
Altitudes normales et orthométriques

- Une grille de corrections entre les altitudes orthométriques (NGF-Lallemand) et normales (NGF-IGN69) est fournie par l'IGN.

- Une valeur moyenne C par carte au 50000ème telle que:

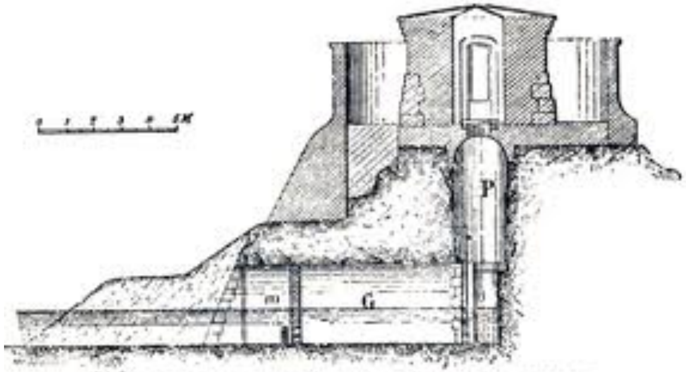
$$H^{(o)} + C = H^{(n)}$$

Grille de conversion des altitudes




Le point fondamental


Le marégraphe de Marseille



L'OBSERVATOIRE MARÉGRAPHIQUE DE MARSEILLE.
Fig. 1. — Coupe de l'Observatoire.

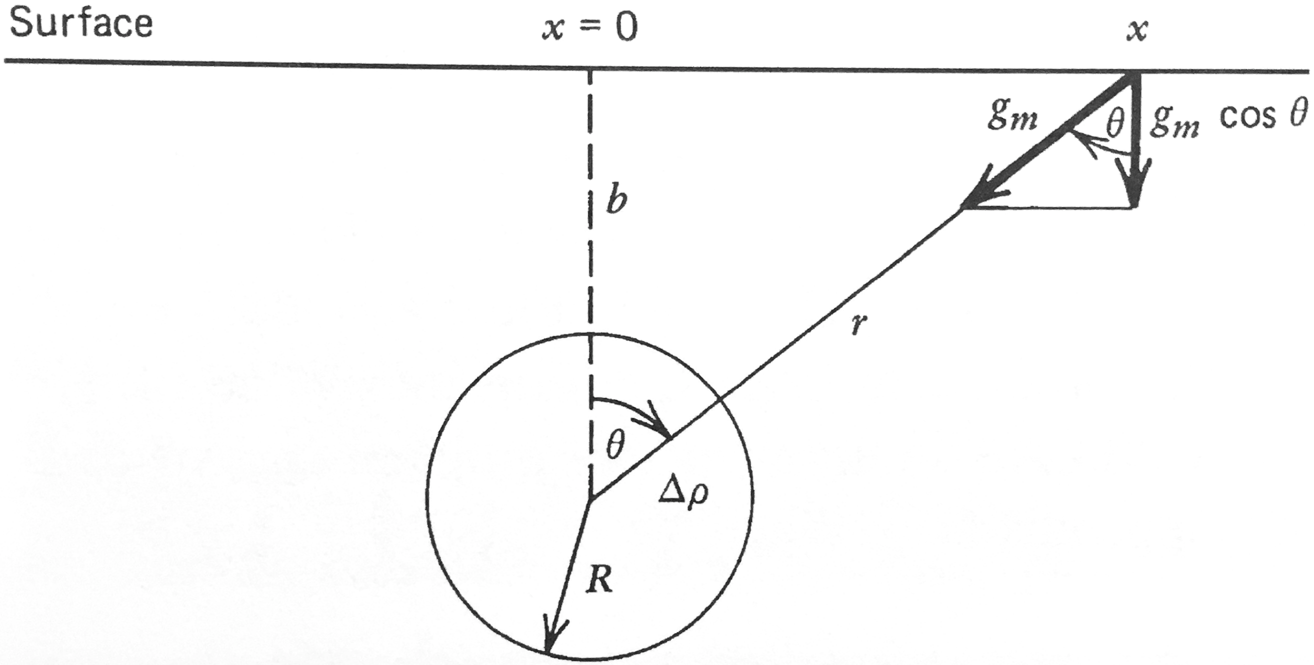
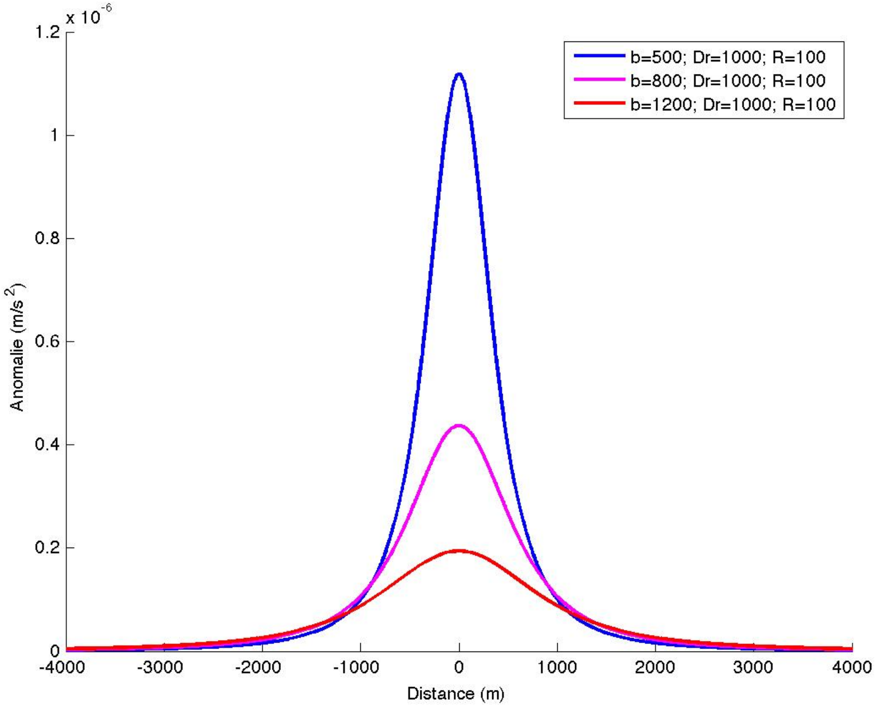
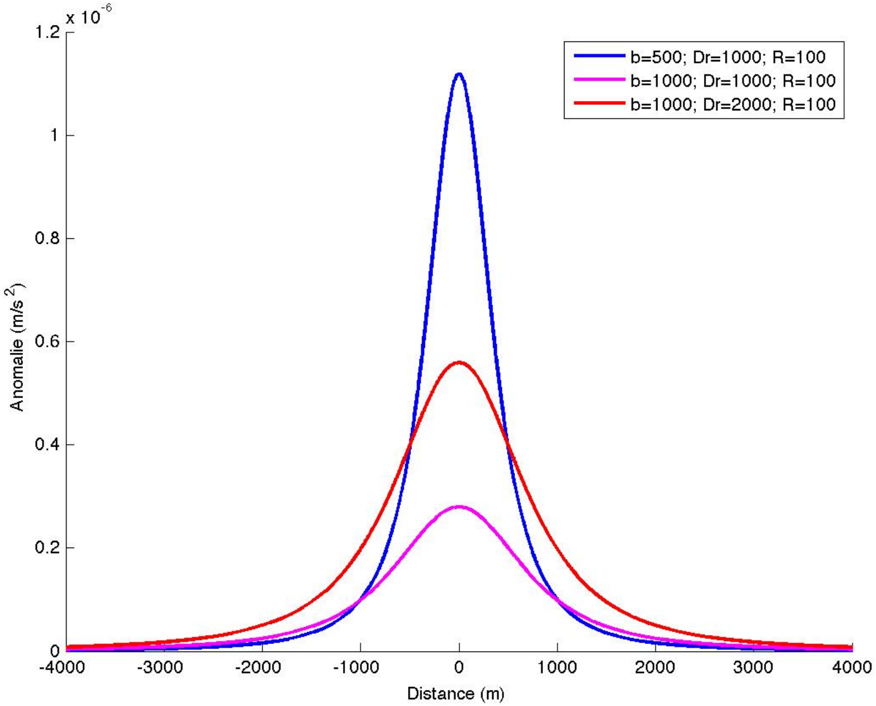


Figure 5.7 The gravitational attraction due to a sphere of anomalous density $\Delta\rho$ and radius R buried at a depth b beneath the surface.


x 10^-6
Anomalie (m/s^2)
1.2
1
0.8
0.6
0.4
0.2
0
-4000
-3000
-2000
-1000
0
1000
2000
3000
4000
Distance (m)
b=500; Dr=1000; R=100
b=800; Dr=1000; R=100
b=1200; Dr=1000; R=100


x 10^-6
Anomalie (m/s^2)
1.2
1
0.8
0.6
0.4
0.2
0
-4000
-3000
-2000
-1000
0
1000
2000
3000
4000
Distance (m)
b=500; Dr=1000; R=100
b=1000; Dr=1000; R=100
b=1000; Dr=2000; R=100

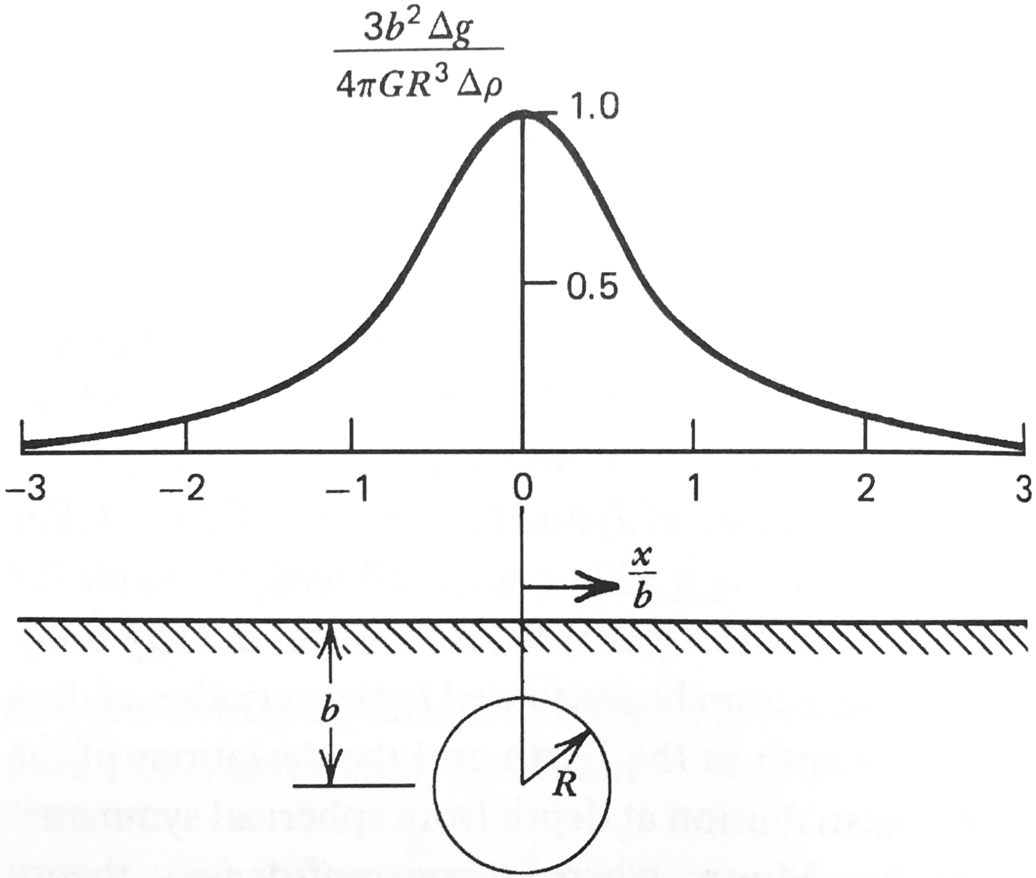


Figure 5.8 The surface gravity anomaly resulting from a spherical body of radius *R* whose center is at a depth *b*, as in

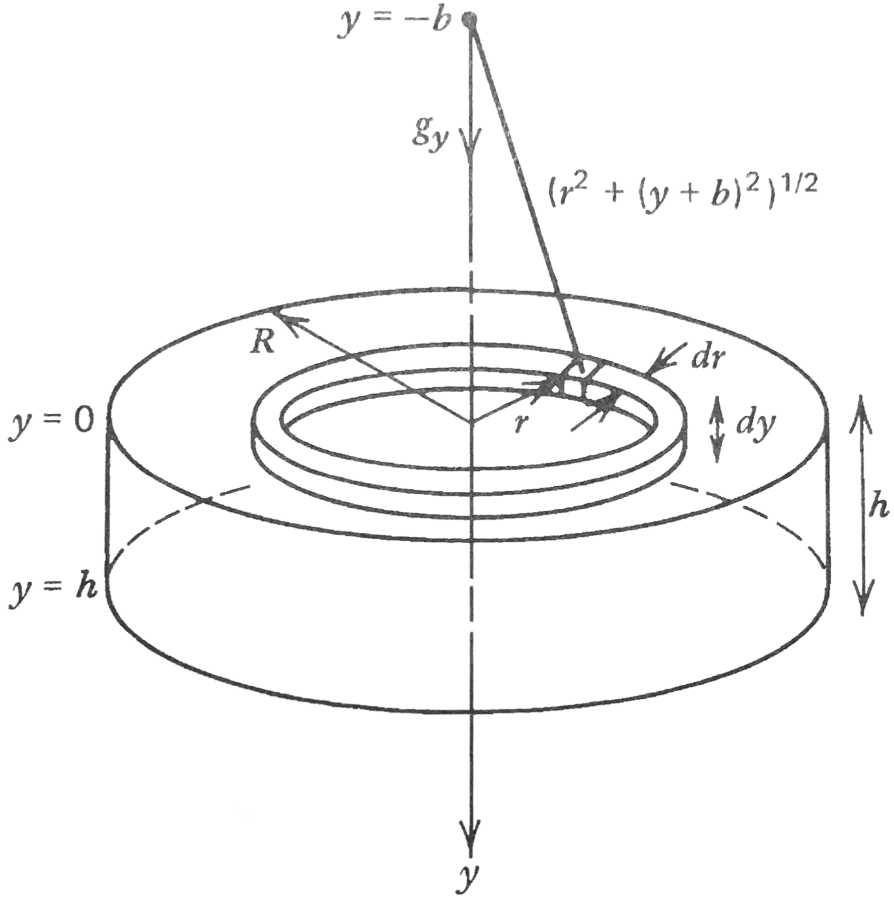


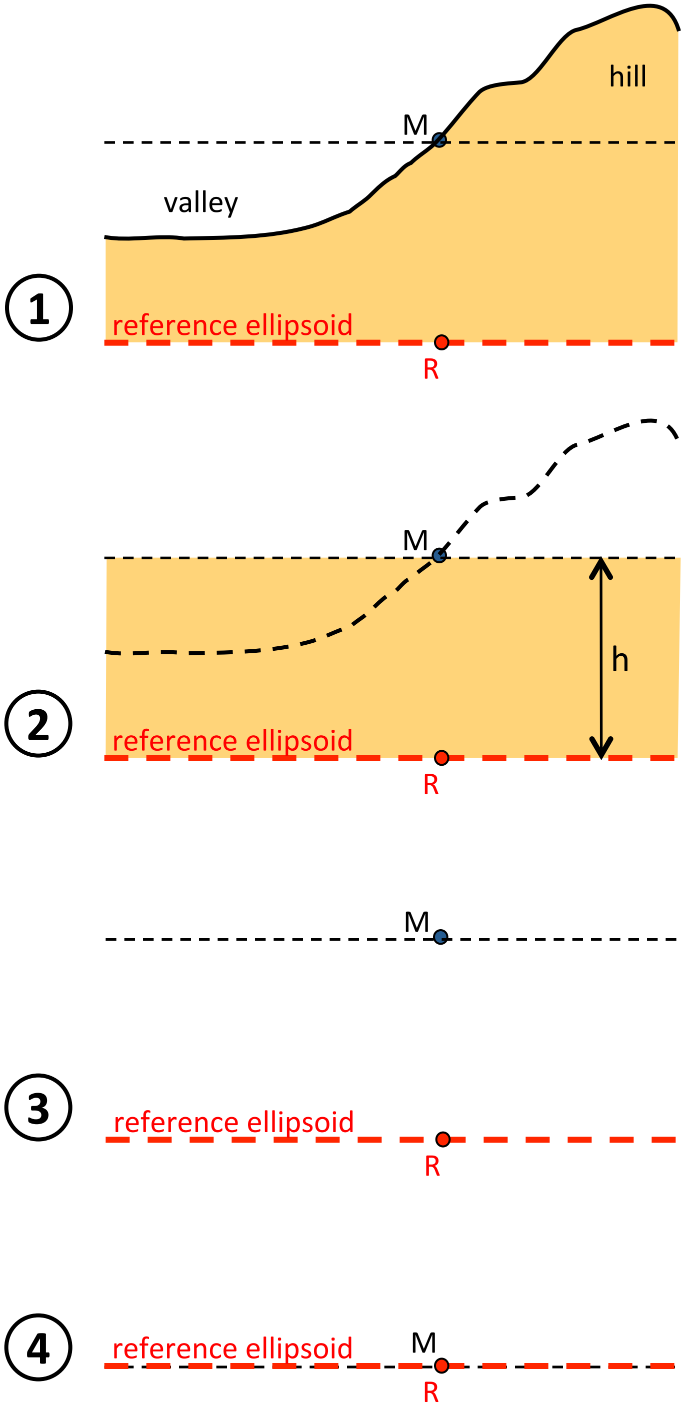
Figure 5.12 Coordinate system used to determine the gravitational attraction of a circular disk at a point along its axis.

Reduction of gravity measurements

- Recall that, if the Earth was an homogeneous ellipsoid:

$$g = g_o(1 + k_1 \sin^2 \Phi - k_2 \sin^2 2\Phi)$$

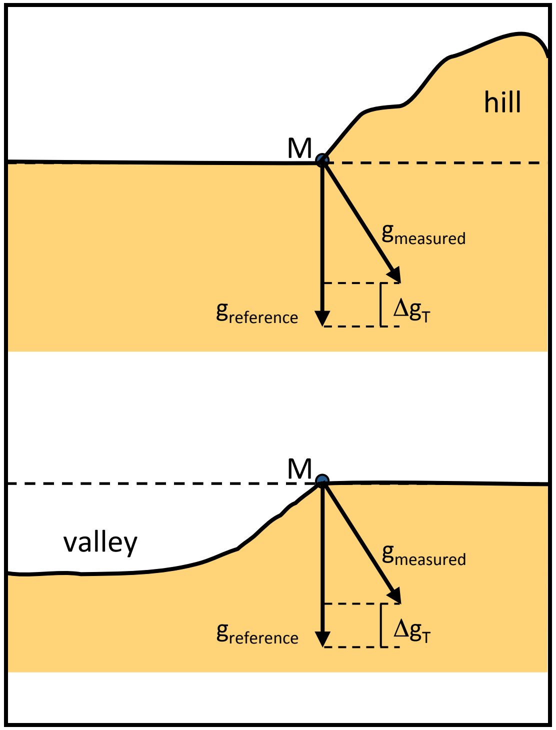
- Gravity measurements:
  - Objective: look for deviations from this reference value
  - Problem: measurements are (usually) not made on the reference ellipsoid…
  - Solution: “reduce” the measurements to “bring” then on the ellipsoid
- Reduction = “correct” the measurements from the effect of:
  - Attraction of terrain around the measurement site: 1 → 2
  - Attraction of rock mass between *M* and *R*: 2 → 3
  - Elevation of *M* w.r.t. reference ellipsoid: 3 → 4
- What do we learn if:
  - $g_{reduced} = g_{reference}$?
  - $g_{reduced} \neq g_{reference}$?



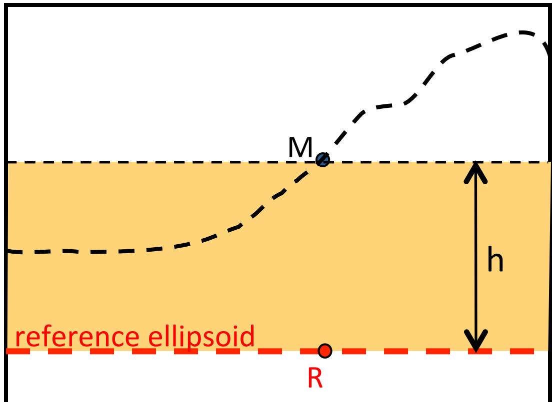
Gravity corrections

Terrain correction: Compensates for the reduction of *g* due to terrain around the measurement site

- Always added to $g_{measured}$
- Complex calculation: discretize topographic map or use DEM



Bouguer plate correction: Compensate for the gravitational attraction of a plate of constant thickness h

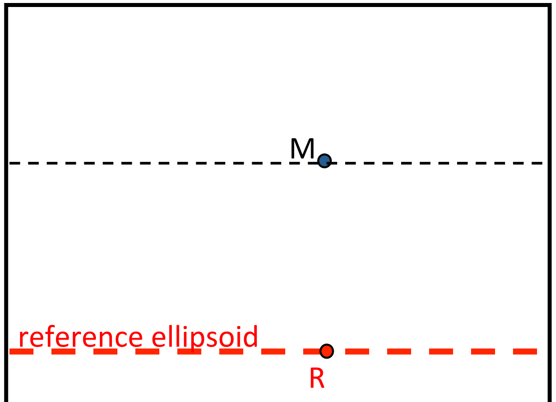


$$\Delta g_{BP} = 2\pi G\rho h$$

$$\Rightarrow \Delta g_{BP} = 0.0419\rho h$$

ρ = plate density (e.g. 2.67)
h = plate thickness

Free-air correction: Compensates for the elevation of the measurement site w.r.t. the ellipsoid

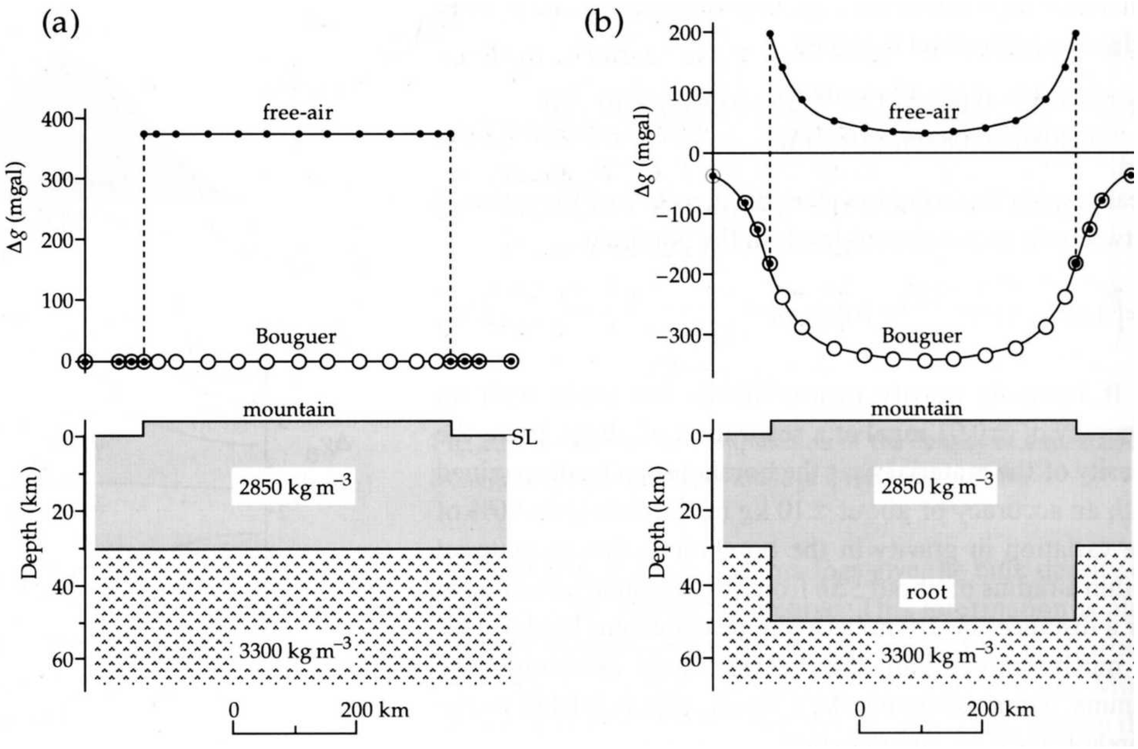


$$\frac{dg}{dr} = -2\frac{g}{R}$$

$$\Delta g_{FA} = 0.3086\, mGal/m$$

(with g = 981,000 mGal
and R = 6371 km)

Bouguer and free-air gravity anomalies



(a) Mountain is supported by the strength of the crust
(b) Mountain is supported by a crustal root that projects into the denser mantle

Bouguer anomalies

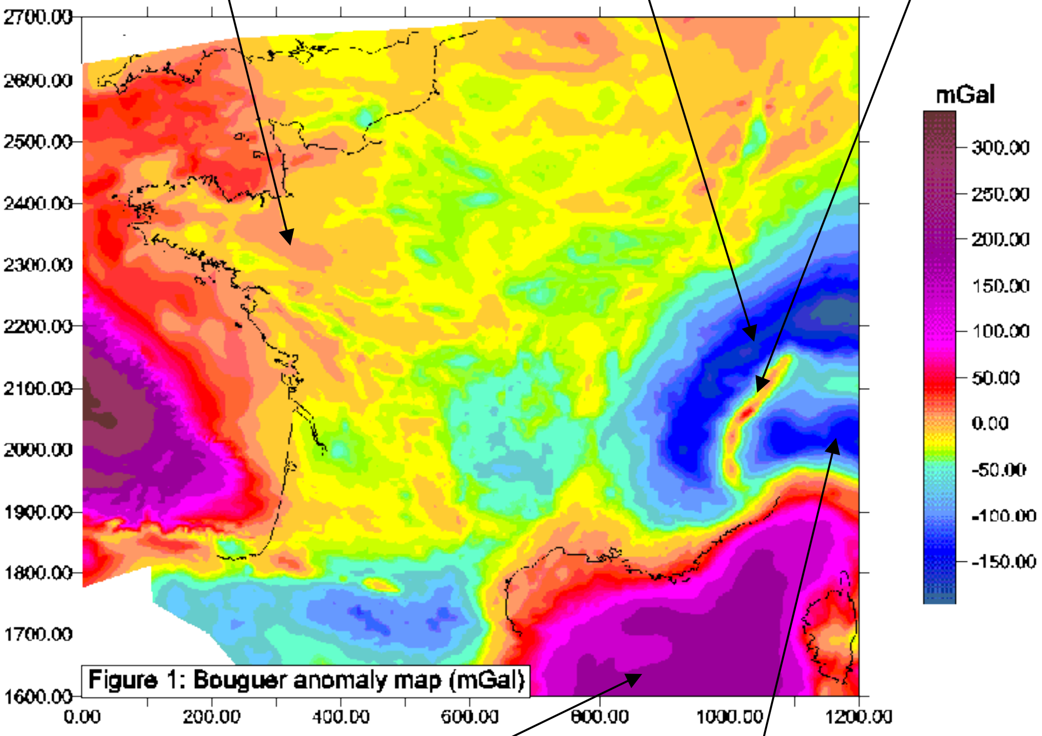


Figure 1: Bouguer anomaly map (mGal)

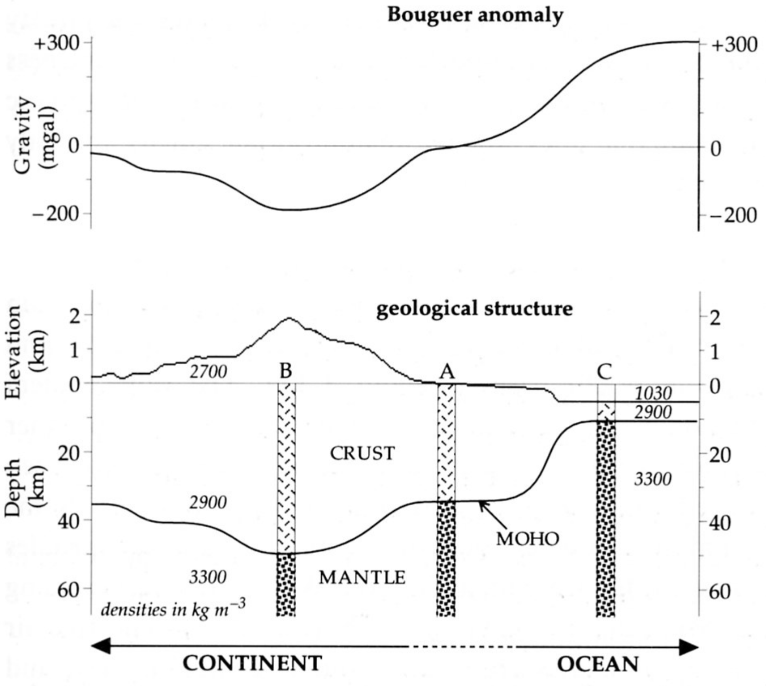
Alps: miocene mountain range

Brittany: hercinyan peneplain

Ivrea body: mantle rocks

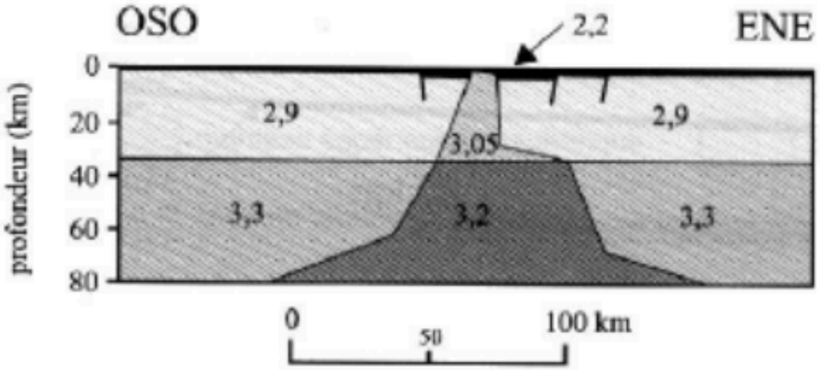
Ligurian basin: oceanic crust

Po valley: thick sedimentary basin

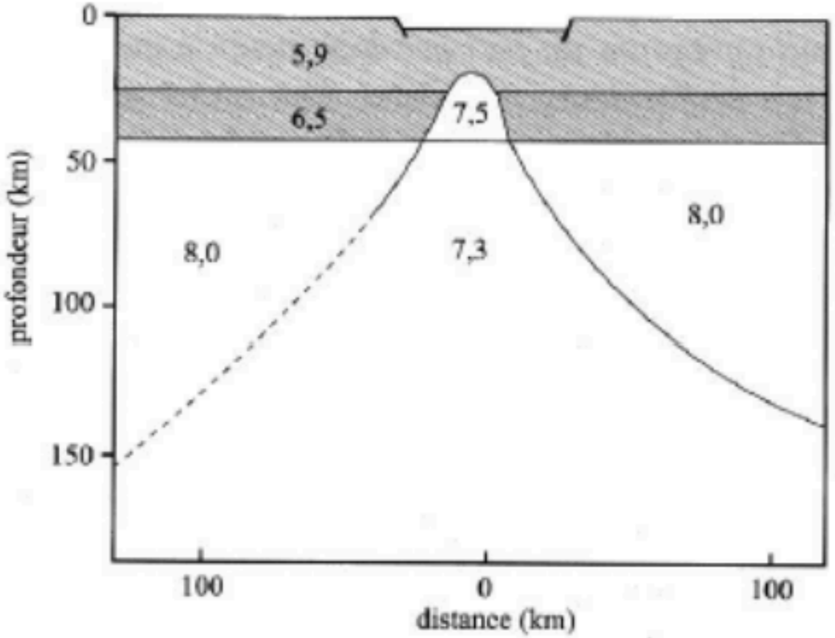


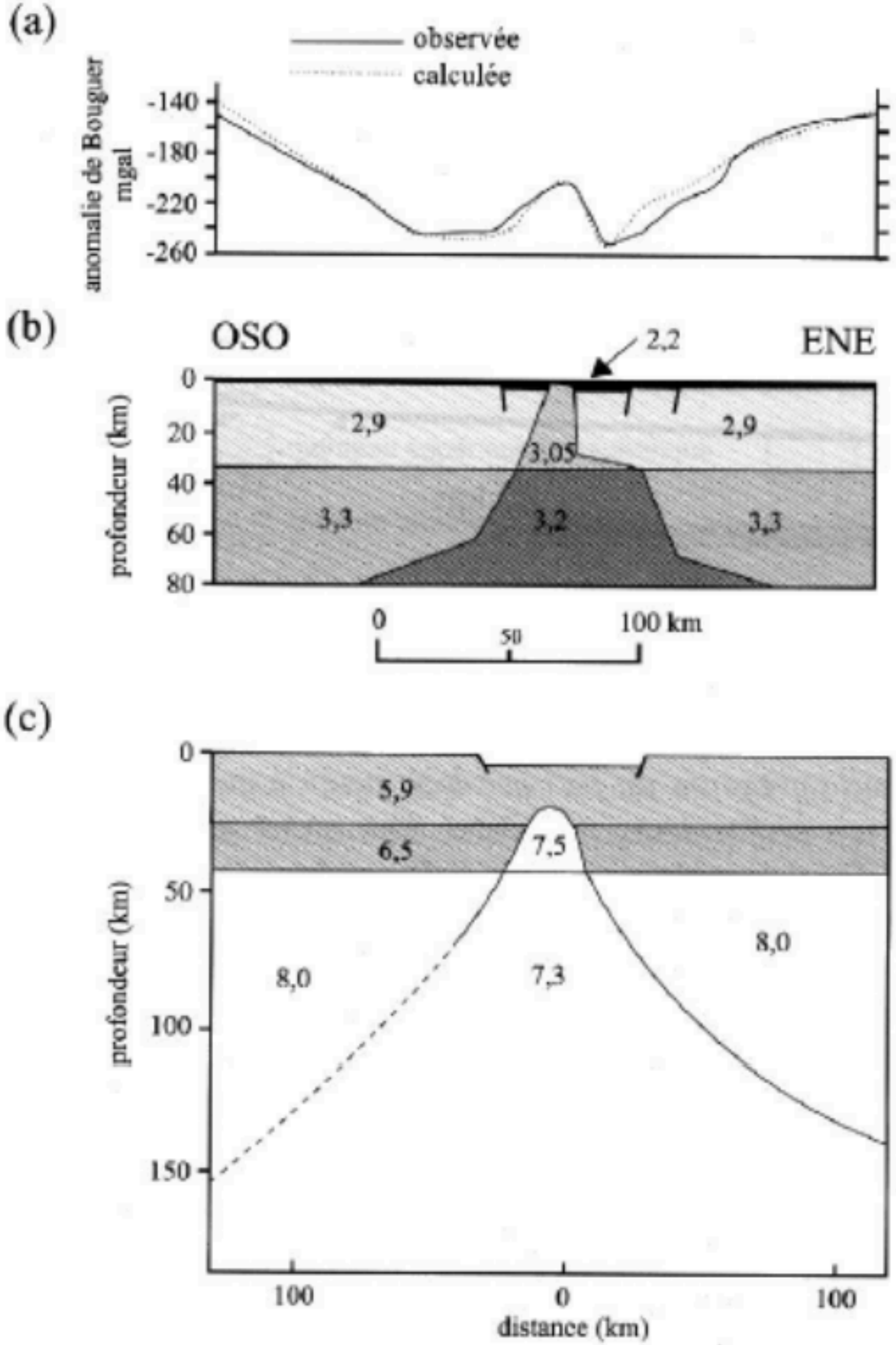
Hypothetical Bouguer anomaly over continental and oceanic areas.

(b)


OSO
ENE
2,2
0
20
40
60
80
profondeur (km)
2,9
2,9
3,05
3,3
3,2
3,3
0
50
100 km

(c)


0
50
100
150
profondeur (km)
5,9
6,5
7,5
8,0
7,3
8,0
100
0
100
distance (km)


(a)
observée
calculée
anomalie de Bouguer mgal
-140
-180
-220
-260
(b)
OSO
ENE
2,2
0
20
40
60
80
profondeur (km)
2,9
3,05
2,9
3,3
3,2
3,3
0
50
100 km
(c)
0
50
100
150
profondeur (km)
5,9
6,5
7,5
8,0
7,3
8,0
100
0
100
distance (km)

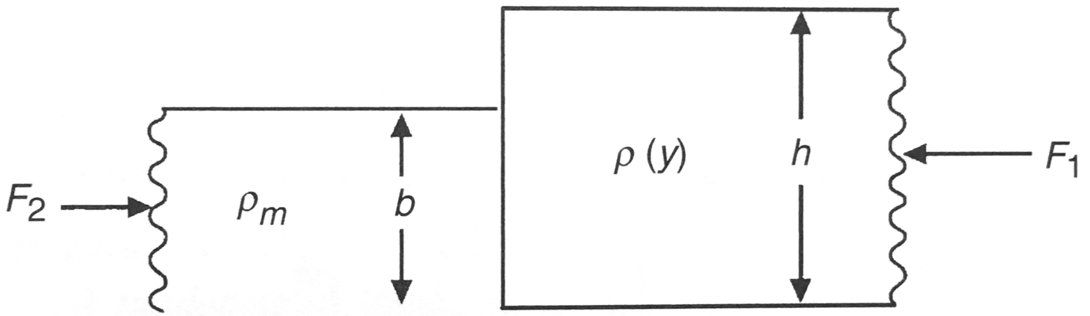


Figure 5.23 Force balance on a section of continental crust and lithosphere.

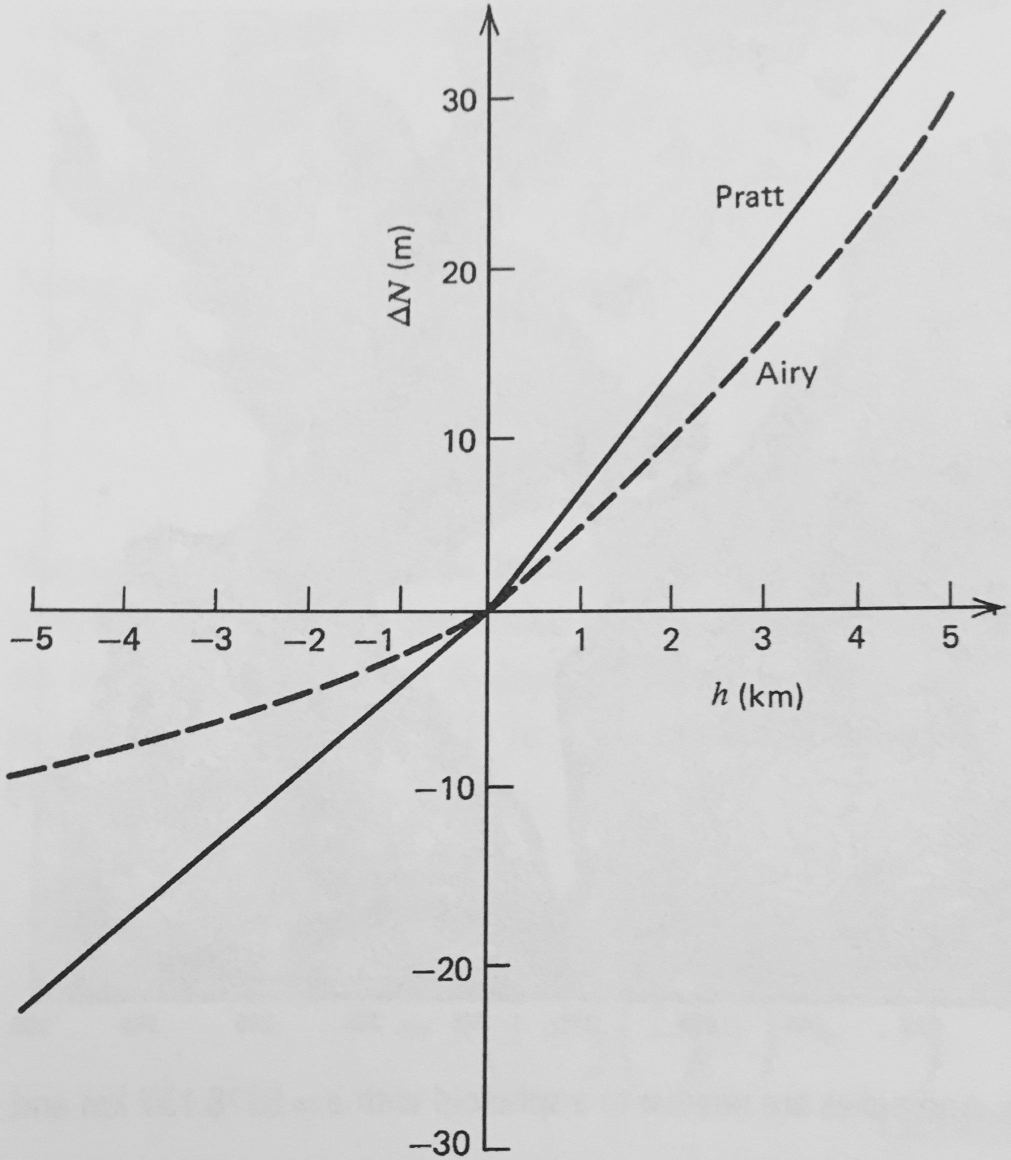


Figure 5.19 Geoid anomaly as a function of topographic elevation above and below sea level. For Pratt compensation $\rho_0 = 3100$ kg m^{-3} and $W = 100$ km. For Airy compensation $\rho_m = 3300$ kg m^{-3}, $\rho_c = 2800$ kg m^{-3}, and $H = 30$ km.

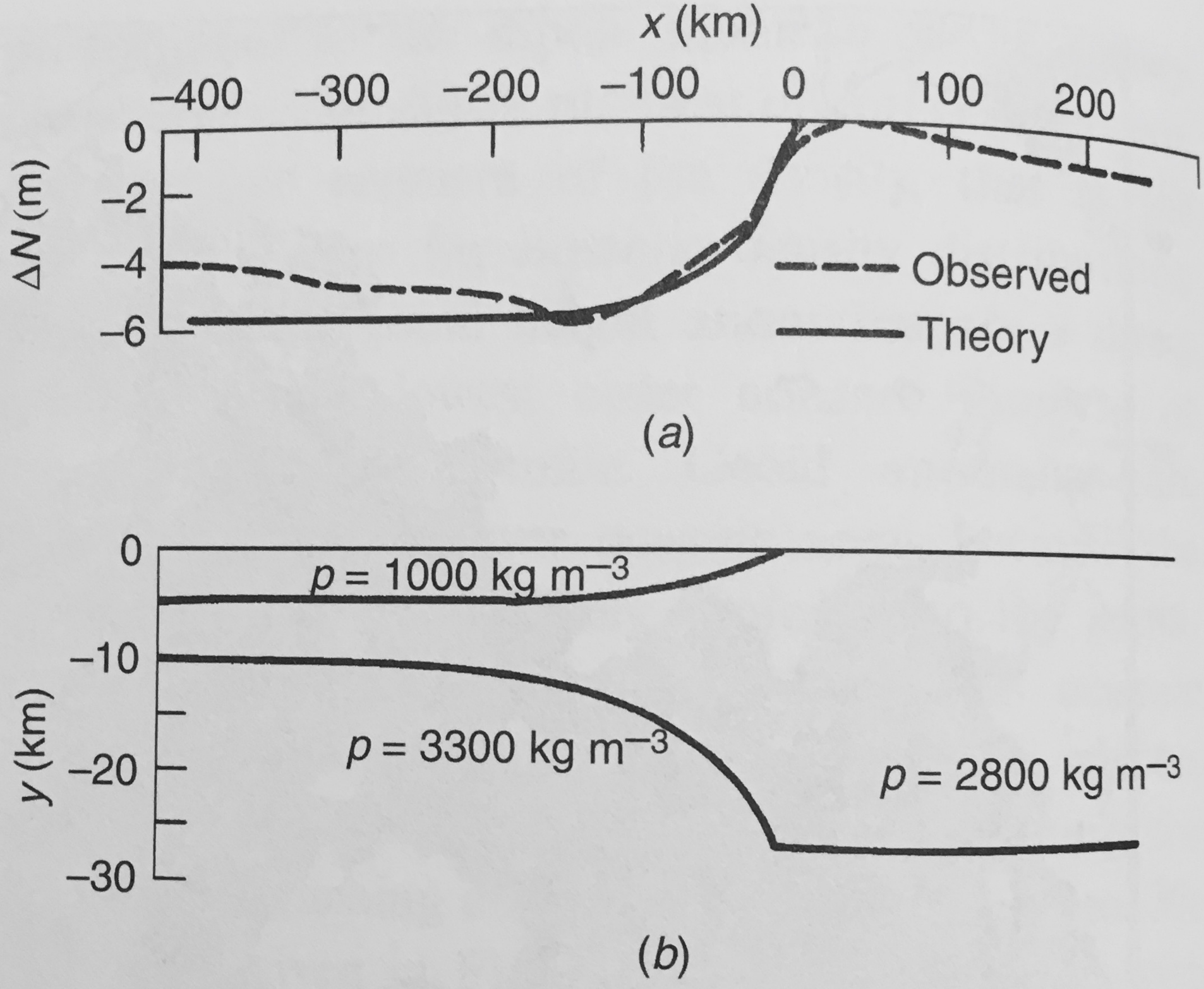


Figure 5.20 (*a*) Observed geoid anomaly across the Atlantic continental margin of North America at 40.5°N compared with the predicted anomaly from Equation (5.149) (*b*) The distribution of density used in the calculation.