Elements of Geodesy
Part 2

Earth Rotation
Time systems
Conventional Systems and Franes
Tides, loading

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GPS Geodesy - Spring 2008
The Earth in Space

• Why should we care?
  – Stars have long been essential to positioning and navigation
  – Basic physics
• Newton’s law are valid in inertial frame = no force due to the motion of the frame itself
• Counter-example: the fictitious (i.e., non-inertial) centrifugal force -- see to the right.
• In practice for us: orbit calculations MUCH easier in frame tied to space…

Newton’s 2nd law:

\[ \vec{F}_i = m \vec{a}_i \]  
(inertial frame…)

Transformation from inertial to rotating frame at uniform speed \( \omega \):

\[ \left( \frac{d}{dt} \right)_i = \left( \frac{d}{dt} \right)_r + \vec{\omega} \times \ldots \]  
(cf. classical mechanics)

Positions become:

\[ \left( \frac{d\vec{r}}{dt} \right)_i = \left( \frac{d\vec{r}}{dt} \right)_r + \vec{\omega} \times \vec{r} \Rightarrow \vec{v}_i = \vec{v}_r + \vec{\omega} \times \vec{r} \]

Velocities become:

\[ \left( \frac{d\vec{v}_i}{dt} \right)_i = \left( \frac{d[\vec{v}_r + \vec{\omega} \times \vec{r}]}{dt} \right)_r + \vec{\omega} \times [\vec{v}_r + \vec{\omega} \times \vec{r}] \]

\[ \Rightarrow \vec{a}_i = \vec{a}_r + 2\vec{\omega} \times \vec{v}_r + \vec{\omega} \times (\vec{\omega} \times \vec{r}) \]

Newton’s 2nd law in rotating frame:

\[ \vec{F}_i - 2m\vec{\omega} \times \vec{v}_r - m\vec{\omega} \times (\vec{\omega} \times \vec{r}) = m\vec{a}_r \]

Force applied in inertial frame  Coriolis Force  Centrifugal Force
Space-fixed versus Earth-fixed Reference Systems

GPS orbit, inertial frame

GPS orbit, Earth-fixed frame

Figures by T. Herring (MIT)
Space-fixed coordinate system

• The coordinate system:
  – Origin = Earth’s center of mass
  – Pole = Celestial Ephemeris Pole, CEP
  – Earth’s rotation axis coincides with CEP
    (actually angular momentum axis)
  – 2 fundamental planes:
    • Celestial equator, perpendicular to CEP
    • Ecliptic equator = Earth’s orbital plane (its pole = North Ecliptic Pole, NEP)
  – Angle between these planes = obliquity \( \varepsilon \approx 23.5^\circ \)
  – Intersection of these planes: vernal equinox
• Coordinates of an object (e.g. star) uniquely defined by:
  – Right ascension \( \alpha \) = angle in equatorial plane
    measured CW from the vernal equinox
  – Declination \( \delta \) = angle above or below the
    equatorial plane
• Or by:
  – Ecliptic longitude \( \lambda \)
  – Ecliptic latitude \( \beta \)
• \((\alpha, \delta)\) and \((\lambda, \beta)\) related through rotation of \( \varepsilon \)
  about direction of vernal equinox.
Oscillation of axes

- Earth rotation vector $\omega_E$: $\vec{\omega}_E = \vec{\omega}\parallel \vec{\omega}_e$

- $\omega_E$ oscillates because of:
  - Gravitational torque exerted by the Moon, Sun and planets
  - Displacements of matter in different parts of the planet
    (including fluid envelopes) and other excitation mechanisms

- Oscillations of the Earth rotational vector:
  - Oscillations of unit vector $\omega$:
    - With respect to inertial space (= stars), because of luni-solar tides: **precession** and **nutation**
    - With respect to Earth’s crust: **polar motion**
  - Oscillations of norm = variations of speed of rotation = variations of time
Precession and Nutation

- Because of luni-solar tides, the earth rotation axis oscillates with respect to inertial space (= stars)
- This oscillation is partitioned into:
  - Secular **precession** (~25,730 yrs): Sun (or Moon) attraction on Earth equatorial buldge, which is not in the ecliptic plane => torque that tends to bring the equator in the ecliptic plane (opposed by the centrifugal force due to Earth’s rotation)
  - Periodic **nutation** (main period 18.6 yrs):
    - e.g. Sun or Moon pass equatorial plane => tidal torque = 0 => creates semi-annual and semi-monthly periods
    - There are semi-annual and semi-monthly irregularities as well
    - Results in small oscillations (also called forced nutations) superimposed on the secular precession

Now pointing to Polaris in Ursa Minor

Pointed to Alpha Draconis 3000 B.C.
Precession and Nutation

- For an accurate treatment of precession and nutations, one must account for precise lunar and solar orbital parameters: use of astronomical tables (`luntab, soltab` in GAMIT)

- In addition, any process that modifies the Earth’s moment of inertia will induce variations of its rotation vector.

- Recall that angular momentum $L=I\omega$ and must be conserved. For instance:
  - Solid Earth tides
  - Oceanic effects: tidal + non-tidal (currents, winds, pressure)
  - Atmospheric effects: pressure distribution
  - Free core nutations: rotation axis of core and mantle not aligned => 432 day period nutation

Offsets in longitude $d\Psi$ and in obliquity $d\varepsilon$ of the celestial pole with respect to (old) precession-nutation model (IAU 1980).

1 arcsec ~30 m at the surface of the Earth => 1 mas ~ 3 cm
Chandler Wobble

• If rotation axis of rotating solid does not coincide with polar major axis of inertia: for the angular momentum to remain constant (assuming no external forcing) the angular velocity vector has to change.
  – True for any irregularly shaped rotating solid body.
  – Happens in the absence of external forcing => “free wobble” (or “free nutation”)

• On Earth:
  – Free wobble is called “Chandler wobble” in honor of S.C. Chandler who first observed it in 1891
  – With respect to Earth surface: position of instantaneous rotation pole moves around polar major axis of inertia
  – Theory gives $\omega_c = 305$ days, observations = 430 days…

![Diagram of Chandler Wobble]
• Observed Chandler wobble: period = 433 days, amplitude ~0.7 arcsec (~15 m on Earth surface)

• Difference between observed and theoretical value?
  – A mystery for a while as astronomers were search for a 305 days period without finding it…
  – Due to non-perfect elasticity of Earth: some energy is transferred into anelastic deformation => damping of the Chandler wobble.

• Calculations using the anelasticity of Earth ($Q$) show that Chandler wobble should rapidly damp to zero (in less than 100 years) => there must be an excitation mechanism that keeps it going.

• Excitation mechanism not understood until recently: two-thirds of the Chandler wobble is caused by ocean-bottom pressure changes, the remaining one-third by fluctuations in atmospheric pressure (Gross, GRL, 2000).

• Polar motion actually has three major components:
  – Chandler wobble.
  – Annual oscillation forced by seasonal displacement of air and water masses, 0.15” ~ 2 m
  – Diurnal and semi-diurnal variations forced by oceanic tides ~ 0.5 m amplitude

• Non-oscillatory motion with respect to Earth’s crust: polar wander, 3.7 mas/yr towards Groenland

Polar motion for the period 1996-2000.5 (dotted line) and polar wander since 1890 (doc. IERS). Axes in arcsec: 1 arcsec ~30m at the surface of the Earth.
Time

- Atomic time scale: based on frequency standards (mostly Cesium)
  - Highly stable (e.g., $10^{-13}$ at $10^4$ sec (~3 hours) ~3 ns)
  - Access to atomic time is direct.

- Astronomic time scales: based on Earth rotation
  - Sidereal time = directly related to Earth’s rotation, not practical to measure for most Earth’s applications
  - Universal time = more practical, related to apparent diurnal motion of Sun about Earth.
  - Not stable: Earth’s rotation varies with time.
  - Instability of TAI is about 6 orders of magnitude smaller than that of UT1
  - Access to astronomical times is not direct, requires observation of stars or sun.

Stability of atomic frequency standards.

Allan variance = $1/2$ the time average of squares of differences between successive readings over the sampling period = measurement of stability of clocks.
International Atomic Time = TAI

- Defined by its unit, the atomic second (on the geoid) = 9,192,631,770 periods of the radiation corresponding to the transition between two hyperfine levels of Ce133
- Origin so that its first epoch coincides with Universal Time UT1 (see later)
- TAI day = 86,400 seconds
- Realized by >200 atomic clocks worldwide, weighted mean calculated by BIPM (www.bipm.org).

- Heated cesium to boil of atoms, different energy state
- Send them through a tube and use magnetic field to select atoms with right energy state
- Apply intense microwave field, sweeping microwave frequency about 9,192,631,770 Hz => atoms change energy state
- Collect atoms that have changed frequency state at end of tube and count them
- Adjust sweeping frequency to maximize # of atoms received at end of tube => then lock frequency
- Divide frequency by 9,192,631,770 Hz to get one pulse-per-second output
Sidereal Time - ST

- Directly related to Earth’s rotation in celestial frame
- Definition = Earth rotates 360 degrees (in celestial frame) in 1 sidereal day
- If measured at observer’s location w.r.t. true vernal equinox = Local Apparent ST
- LAST measured by observations to distant stars and extragalactic radio sources
- If corrected for nutation and precession (w.r.t. mean vernal equinox) = Local Mean ST
- If measured at Greenwich = GAST or GMST
Universal Time - UT

- Related to apparent motion of the sun around the Earth
- When referred to the Greenwich meridian = Universal Time (UT)
- UT is measured directly using ground stations (VLBI)
- UT not uniform because of polar motion => UT corrected for polar motion = UT1
  - UT1 has short term instabilities at the level of $10^{-8}$
  - Duration of the day slowly decaying (~0.002 s/century) because of secular deceleration of Earth rotation.
TAI - UT relation?

- Because of the secular deceleration of the Earth's rotation, TAI increases continuously with respect to UT1.

- If legal time was based on TAI, coincidence with solar day could not be maintained (in a couple of years TAI - UT1 can increase by a few seconds).

- Compromise: use highly stable atomic time, but adjust time to match irregular Earth rotation => UTC
  - Unit = atomic second
  - By definition, |UT1 – UTC| < 0.9 s
  - UTC changed in steps of 1 full second (leap second) if |UT1 – UTC| > 0.9 s, responsibility of the IERS (June 30 or Dec. 31)
  - UTC = broadcast time used for most civilian applications (your watch!)

- Currently: TAI-UTC = 33 s
UT - ST relation?

- It takes 365 solar days for Earth to be back at the same place w.r.t. sun.
- During 1 sidereal day, observer A has rotated 360 degrees.
- Need to rotate by additional $\alpha$ to complete 1 solar day:
  - Solar day is longer than sidereal.
  - Observer accumulates some "lead time" w.r.t. solar day.
  - At a rate of 1/365 of a solar day = 3 min 44.90 sec per solar day.
- Therefore:
  - Sidereal day = solar day - 3 min 44.90 sec = 86,164.10 s.
  - Earth angular rotation:
    $\omega = \frac{2\pi}{86,164.10} = 7,292,115 \times 10^{-5}$ rad s$^{-1}$ (see GRS definition).
GPS time (GPST)

- Atomic scale, unit = SI second
- GPST = TAI – 19 s
- Coincident with UTC on January 6th, 1980, 00:00 UTC
- Not incremented by leap seconds
- Currently: TAI - UTC = 33 s => GPST = UTC + 14 s

http://www.leapsecond.com/java/gpsclock.htm
Length-of-day = LOD

- Length of Day (LOD) = difference between 86,400 sec SI and length of astronomical day:
  - Long term variations:
    - Dynamics of liquid core
    - Climate
  - Short term variations:
    - Zonal tides
    - Seasonal = climate
    - Residual: Cf. 1983 El Nino event
Time systems/Calendar

• **Julian Date** (JD): number of mean solar days elapsed since January 1\textsuperscript{st}, 4713 B.C., 12:00

• **Modified Julian Date** (MJD) = JD – 2,400,00.5
  – Ex.: GPS standard epoch, JD = 2,444,244.5 (January 6\textsuperscript{th}, 1980, 00:00 UTC)
  – Ex.: Standard epoch J2000.0, JD = 2,451,545.0 (January 1\textsuperscript{st}, 2000, 12:00 UTC)

• **Day Of Year** (DOY): day since January 1\textsuperscript{st} of the current year

• GPS calendar:
  – GPS week: Week since GPS standard epoch
  – GPS day of week: Sunday = 0 to Saturday = 6
  – GPS second: Second since GPS standard epoch
Reference Systems

• Introduced to help model geodetic observations as a function of unknown parameters of interest
  – Positions:
    • Space-fixed = celestial, tied to extra-galactic objects
    • Terrestrial = tied to solid Earth
  – Time systems: based on quantum physics or Earth rotation

• Systems vs. Frames:
  – Reference System = set of prescriptions, conventions, and models required to define at any time a triad of axes.
  – Reference Frame = practical means to “access” or “realize” a system (e.g., existing stations of known coordinates)
Reference Systems

• Systems and frames:
  – Defined by international body: the international Earth Rotation and Reference Systems Service = IERS (http://www.iers.org)
  – Mission of the IERS: “To provide to the worldwide scientific and technical community reference values for Earth orientation parameters and reference realizations of internationally accepted celestial and terrestrial reference systems”
  – Updated on a regular basis as models or measurements improve.

• Basic units and fundamental constants:
  – Meter = length unit (length of path traveled by light in vacuum in 1/299,792,458 of a second - CGPM 1983)
  – Kilogram = mass unit (mass of international prototype - CGPM 1901)
  – Second = time unit (9192631770 periods of the radiation corresponding to the transition between two hyperfine levels of Ce133 - CGPM 1967)
Conventional Systems

- **Conventional Inertial System:**
  - Orthogonal system, center = Earth center of mass, defined at standard epoch J2000 (January 1st, 2000, 12:00 UT)
  - \( Z \) = position of the Earth’s angular momentum axis at standard epoch J2000
  - \( X \) = points to the vernal equinox
  - It is materialized by precise equatorial coordinates of extragalactic radio sources observed in Very Long Baseline Interferometry (VLBI) = Inertial Reference Frame.

- **Conventional Terrestrial System:**
  - Orthogonal system center = Earth center of mass
  - \( Z \) = position of the Earth’s angular momentum axis at standard epoch J2000
  - \( X \) = Greenwich meridian
  - It is materialized by a set of ground control stations of precisely know positions and velocities = Terrestrial Reference Frame
International Celestial Reference Frame

- Directions of the ICRS pole and right ascensions origin maintained fixed relative to the quasars within +/-20 microarcseconds.
- The ICRS is accessible by means of coordinates of reference extragalactic radio sources.
- It is realized by VLBI estimates of equatorial coordinates of a set of extragalactic compact radio sources, the International Celestial Reference frame.
- ICRS can be connected to the International Terrestrial Reference System through Earth Orientation Parameters (EOP -- more later).

An extragalactic source “seen” by a radio telescope at 8 GHz

212 high-astrometric-quality objects define the ICRF axes
International Terrestrial Reference Frame

- Realized by a set of ground control stations, in the framework the IERS
- International Terrestrial Reference Frame (http://itrf.ensg.ign.fr/):
  - First version in 1989 (ITRF-89), current version ITRF-2005
  - Set of station positions, velocities, and epochs in an Earth centered-Earth fixed (=ECEF) Terrestrial System
  - And associated variance-covariance matrix
  - Since ITRF derives from measurements, it changes (improves) with its successive realizations
- More on ITRF and reference frames later.
Earth Orientation Parameters (EOP)

- The Earth's orientation is defined as the **rotation between**:
  - A **rotating geocentric set of axes linked to the Earth** (the terrestrial system materialized by the coordinates of observing stations) = Terrestrial System
  - A **non-rotating geocentric set of axes linked to inertial space** (the celestial system materialized by coordinates of stars, quasars, objects of the solar system) = Inertial System

- Inertial coordinates are transformed to terrestrial by combining rotations:

\[ r_t = R_2(-x_p)R_1(-y_p)R_3(GAST)N(t)P(t)r_i \]

Terrestrial  Polar motion  UT1  Nutation  Precession  Inertial
Earth Orientation Parameters (EOP)

• Nutation and Precession:
  – Rotation matrices derived from geophysical model + corrections provided by IERS.
  – \( P(t) = \) rotate from reference epoch (J2000) to observation epoch --> obtain mean equator and equinox.
  – \( N(t) = \) rotate from mean to instantaneous true equator and vernal equinox.
  – At this stage Earth rotation axis (Z-axis) coincides with CEP.

• \( R_3 = \) rotate about Z axis (=CEP) by GAST
  => X-axis coincides with vernal equinox.

• Polar motion:
  – \( R_1 = \) rotate about X axis by \(-y_p\) (small angle)
  – \( R_2 = \) rotate about Y axis by \(-x_p\) (small angle)

\[
R_3(GAST) = \begin{bmatrix}
\cos(GAST) & \sin(GAST) & 0 \\
-\sin(GAST) & \cos(GAST) & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
R_1(-y_p) = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & -y_p \\
0 & y_p & 1
\end{bmatrix}
\]

\[
R_2(-x_p) = \begin{bmatrix}
1 & 0 & x_p \\
0 & 1 & 0 \\
-x_p & 0 & 1
\end{bmatrix}
\]
Earth Orientation Parameters (EOP)

• Rotation parameters are given in astronomical tables provided by the IERS:
  – For the most precise positioning, they are used as a priori values (sometimes even adjusted) in the data inversion.
  – For non-precise applications, only $R_3(GAST)$ matters.
• In practice, the IERS provides five Earth Orientation Parameters (EOP):
  – Celestial pole offsets ($d\Psi, d\Ep$): corrections to a given precession and nutation model
  – Universal time (UT1) = UT corrected for polar motion, provided as UT1-TAI
  – 2 coordinates of the pole ($x_p, -y_p$) of the CEP (corrected for nutation and precession) with respect to terrestrial Z-axis axis = polar motion
Tides

- Earth and Moon are coupled by gravitational attraction: each one rotates around the center of mass of the pair.
- Moon (or Sun) gravitational force = sum of:
  - Part constant over the Earth --> orbital motions (think of Earth as a point mass) = orbital force
  - Remainder, varies over the Earth --> tides (Earth is not a point mass…) = tidal force
- Tidal force:
  - Causes no net force on Earth => does not contribute to orbital motion
  - Fixed w.r.t. Moon
- In addition: the Earth rotates => tidal force varies with time at given location.
Tidal frequencies

• One can show that the Earth’s tidal potential can be written as:

\[ V_I = \frac{3}{4} GM \frac{R^2}{r^3} \left[ \left( \frac{1}{3} - \sin^2 \varphi_P \right) (1 - 3 \sin^2 \delta) + \sin 2\varphi_P \sin 2\delta \cos(h) \right. \]
\[ + \left. \cos^2 \varphi_P \cos^2 \delta \cos(2h) \right] \]

→ Long-period

→ Diurnal

→ Semi-diurnal

with \( \varphi_P, \lambda_P \) = latitude, longitude of observer, \( r \) = Earth’s radius, \( R \) = earth-moon distance, \( \delta, \alpha \) = declination, right-ascension of celestial body, \( h \) = hour angle given by:

\[ h = \lambda_P + GAST - \alpha \]

• \( R, \delta, h \): periodic variations with time ⇒ 3 terms have periodic variations:
  – Moon: 14 days, 24 hours, and 12 hours.
  – Sun: ~180 days, 24 hours, and 12 hours.
  – 3 distinct groups of tidal frequencies: twice-daily, daily, and long period

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<th>Name</th>
<th>Period</th>
<th>Acceleration (nm s-2)</th>
<th>Amplitude, ocean (m)</th>
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<td>Semi-annual</td>
<td>Ssa</td>
<td>182.62 d</td>
<td>14.8</td>
</tr>
</tbody>
</table>
Earth Tides

• Love showed that displacement on spherical Earth is proportional to tidal potential and Love numbers (for harmonic degree n):

\[ d_r = \frac{h_n}{g} V_T \tilde{e}_r \]
\[ d_i = \frac{l_n}{g} \nabla V_T \tilde{e}_i = \frac{l_n}{g} \frac{\partial V_T}{\partial \theta} + \frac{l_n}{g \sin \theta} \frac{\partial V_T}{\partial \lambda} \]

- Love numbers for displacement = \( h_n \) and \( l_n \) (+ \( k_n \) for potential)
- Depend on elastic properties of the Earth (from seismic velocities)
- Solid Earth: largest contribution from harmonic degree 2: \( h_2 = 0.609; l_2 = 0.085; k_2 = 0.3 \)

• Displacements due to Earth tides:
  - Ex. in Greenland (figures)
  - Large compared to the precision of GPS => must be corrected (using model)
Ocean tides

- Tidal force results in displacements of Earth’s constituents:
  - Oceans
  - Solid Earth
  - Atmosphere

- Ocean tides:
  - Measured with tide gauges
  - Complex pattern, local amplifications.

- Once tides are removed, what do tide gauges measure?

The ocean tides for harmonic M2 (period of 12 hours and 25 minutes). The color represents the amplitude and the contour lines indicate the phase lag of the tides with a spacing of 60 degrees. (Doc. H.G. Scherneck)
Loading effects

• Earth ~elastic solid => deforms under load, as a function of:
  – Load (=source) characteristics (spatial and temporal evolution of mass)
  – Elastic properties: quantified by loading Love numbers: $h_n'$, $l_n'$, $k_n'$

• Radial deformation of elastic Earth:
  
  \[ u_r(\lambda, \varphi) = \frac{3}{\rho_e} \sum_{n=0}^{\infty} \frac{h'_n}{2n+1} q_n(\lambda, \varphi) \]

  $\rho_e$ = mean Earth density
  $h'_n$ = load Love number (function of assumed rheology)
  N = degree of spherical harmonic series
  $q_n(\lambda, \varphi)$ = spherical harmonic expansion of surface load

• Ocean tides are one of the major sources of load:
  – Result is centimeters of radial motion
  – Higher power at diurnal and semi-diurnal periods (K1, M2)
  – Loading effect larger in coastal regions, decreases land-ward

Fig. T. Herring
Loading effects

- Additional loading sources include
  - Atmospheric pressure:
    - Centimeters: ~0.5 mm/mbar
    - Amplitude related to weather patterns => mostly short period
  - Ground water:
    - Rivers + soil + snow
    - Centimeters: ~0.5 mm/cm of water
    - Higher power at seasonal period
- Ultimate GPS precision requires correcting for loading effects:
  - Ocean loading:
    - Routine
    - Ocean tidal models accurate (except some local areas)
  - Atm and hydro:
    - Still experimental
    - Must rely on pressure and continental water measurements (and models…)

Fig. T. vanDam
Fig. T. Herring

VanDam et al., 2001

mm
Tidal friction and LOD

- If the Earth was purely elastic => tidal bulge aligned with the Moon (a)
- But Earth tidal response is not instantaneous because of Earth anelasticity
  - => slight delay between high tide and Moon alignment (~12 min)
  - => creates torque that tends to bring the tidal bulge axis back into the Earth-Moon direction, in a direction opposite to the Earth’s rotation
  - Deceleration of the Earth’s rotation => increase of length-of-day: ~2 s/100,000 yr
  - Analysis of growth rings of fossil corals, 350 Ma old => 1 day = 22 hr, 1 year = 400 days

- The Earth tidal bulge creates a similar torque on the Moon, in opposite direction (conservation of Earth-Moon angular momentum) => deceleration of the Moon revolution
- Because of Kepler’s third Law (period² / a³=const), the Moon-Earth distance increases (~3.8 cm/yr)
Geophysical Fluids and Earth Rotation

- Geophysical fluids act on Earth rotation $\omega$ in 2 ways:
  - By the torques ($\tau$) they exert on the planet
  - By the modifying the Earth moment of inertia ($I$)
- Since angular momentum of the whole Earth system has to be conserved (recall that $L=I\omega$ and $\tau=dL/dt$): torques and changes in moment of inertia result in changes in $\omega$.
- Any geophysical process involving mass transport will trigger Earth rotation variations with spatial and temporal characteristics function of the triggering process:
  - Atmosphere: pressure systems = air mass moving around the planet
  - Oceans: water displacements due to tides, wind, thermohaline fluxes
  - Whole earth: body tide, mantle flow (e.g. post-glacial rebound), tectonic plate motion, earthquakes
  - Liquid core: the “geodynamo”
  - Continents: snow and groundwater

Chao, 2004: LOD/UT1 variations with respect to various geophysical excitation sources ranging from core flow, to El Ninos, to ocean tides
Geophysical Fluids and J₂dot

- Recall that the dynamic oblateness of the Earth \( J_2 \) relates to the Earth moment of inertia via:

\[
J_2 = \frac{C - A}{Ma^2}
\]

- Similar to tidal torque exerted by moon and sun on Earth that cause precession, Earth exerts a torque on artificial satellites \( \Rightarrow \) precession of the satellite orbit = “regression of the nodes”

- In the simplified case of a circular orbit, one can show that the angular change in the position of the node is given by:

\[
\frac{\Delta \Omega}{2\pi} = -\frac{3}{2} a^2 r^2 J_2 \cos i
\]

with \( i \) = inclination of satellite’s orbit plane w.r.t. equatorial plane, \( r \) = orbital radius

- Since one can track the position of satellites from Earth-based stations, \( \Delta \Omega \) can be measured \( \Rightarrow J_2 \) can be determined using geodetic observations.

- Actually, higher-order coefficients as well \( \Rightarrow \) determination of the “satellite geoid”.

- In practice, this is more involved since orbits are elliptical.

⇒ Until 1996: \( J_2 \) decreases, a result of postglacial rebound (1,000 years scale)

⇒ 1997-1998: \( J_2 \) increases because of surge in ice melt and mass shift in oceans (also strong El Nino event, 1998 warmest global mean temperature on record).