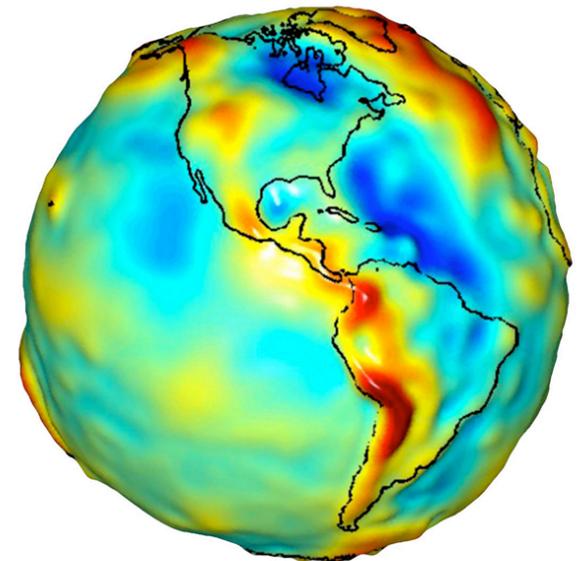


Elements of Geodesy

Part 2

Earth Rotation
Time systems
Conventional Systems and Frames
Tides, loading

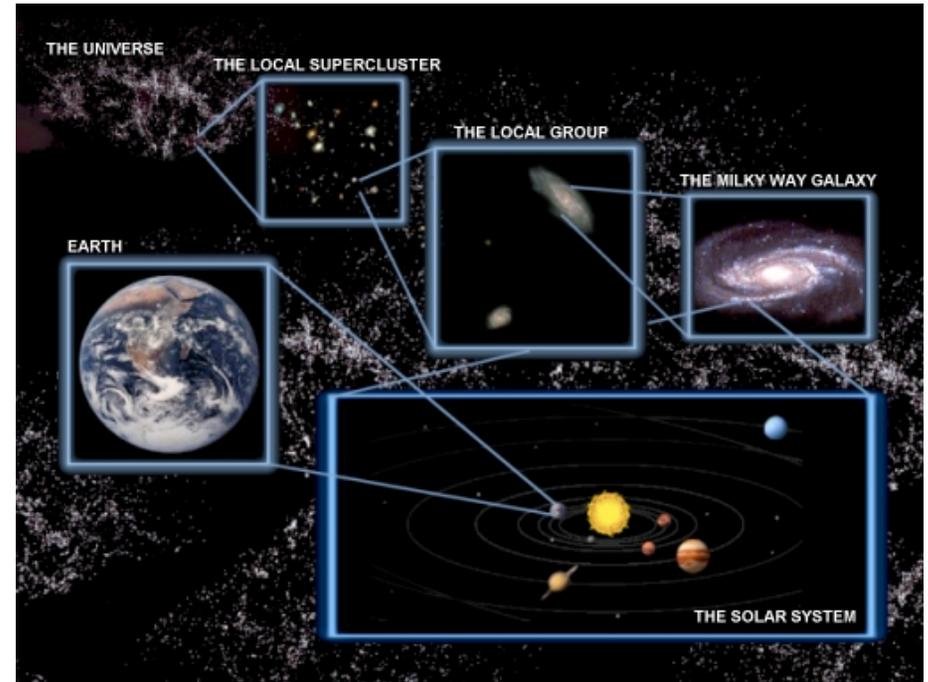
Prof. E. Calais
Purdue University - EAS Department
CIVL 3273 – ecalais@purdue.edu



Geoid of Western Hemisphere. Image from
University of Texas Center for Space Research and
NASA.

The Earth in Space

- Why should we care?
 - Stars have long been essential to positioning and navigation
 - Basic physics
- Newton's law are valid in inertial frame = no force due to the motion of the frame itself
- Counter-example: the fictitious (*i.e.*, non-inertial) centrifugal force -- see to the right.
- In practice for us: orbit calculations MUCH easier in frame tied to space...



Newton's 2nd law: $\vec{F}_i = m\vec{a}_i$ (inertial frame...)

Transformation from inertial to rotating frame at uniform speed ω : $\left(\frac{d}{dt}\right)_i = \left(\frac{d}{dt}\right)_r + \vec{\omega} \times \dots$ (cf. classical mechanics)

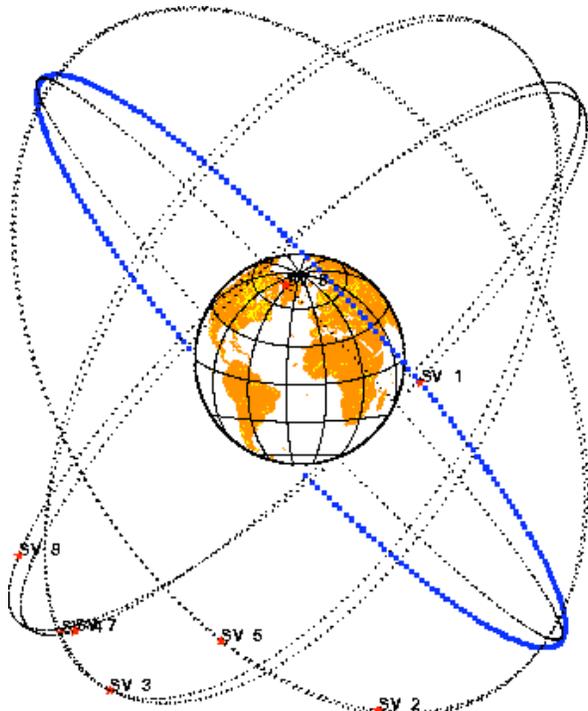
Positions become: $\left(\frac{d\vec{r}}{dt}\right)_i = \left(\frac{d\vec{r}}{dt}\right)_r + \vec{\omega} \times \vec{r} \Rightarrow \vec{v}_i = \vec{v}_r + \vec{\omega} \times \vec{r}$

Velocities become: $\left(\frac{d\vec{v}_i}{dt}\right)_i = \left(\frac{d[\vec{v}_r + \vec{\omega} \times \vec{r}]}{dt}\right)_r + \vec{\omega} \times [\vec{v}_r + \vec{\omega} \times \vec{r}]$
 $\Rightarrow \vec{a}_i = \vec{a}_r + 2\vec{\omega} \times \vec{v}_r + \vec{\omega} \times (\vec{\omega} \times \vec{r})$

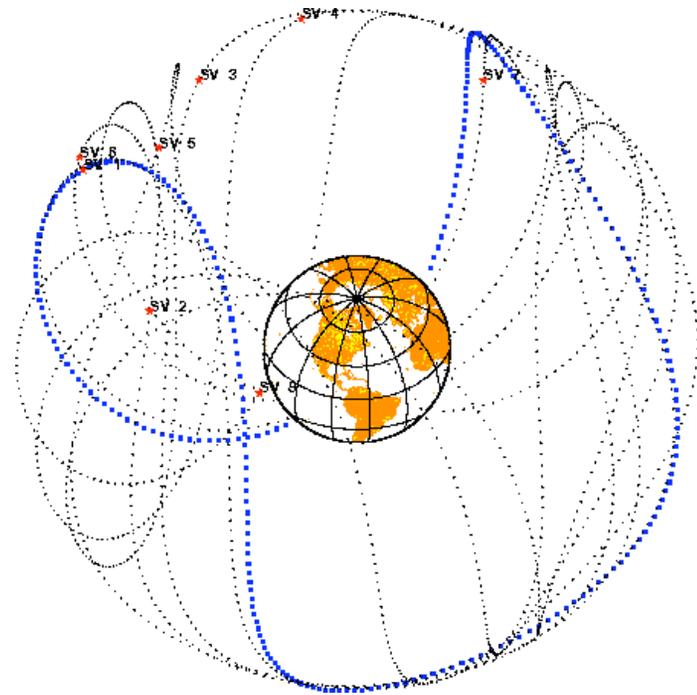
Newton's 2nd law in rotating frame: $\boxed{\vec{F}_i} - \boxed{2m\vec{\omega} \times \vec{v}_r} - \boxed{m\vec{\omega} \times (\vec{\omega} \times \vec{r})} = m\vec{a}_r$

Force applied in inertial frame
Coriolis Force
Centrifugal Force

Space-fixed versus Earth-fixed Reference Systems



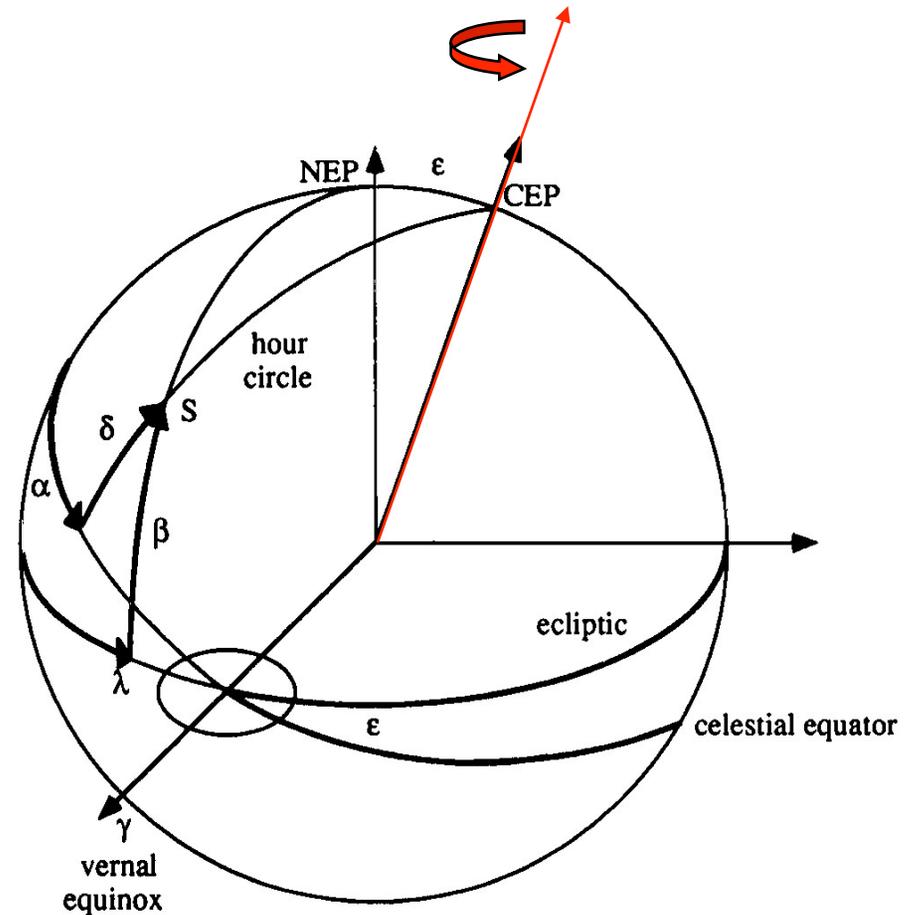
GPS orbit, inertial frame



GPS orbit, Earth-fixed frame

Space-fixed coordinate system

- The coordinate system:
 - Origin = Earth's center of mass
 - Pole = Celestial Ephemeris Pole, CEP
 - Earth's rotation axis coincides with CEP (actually angular momentum axis)
 - 2 fundamental planes:
 - Celestial equator, perpendicular to CEP
 - Ecliptic equator = Earth's orbital plane (its pole = North Ecliptic Pole, NEP)
 - Angle between these planes = obliquity $\epsilon \sim 23.5^\circ$
 - Intersection of these planes: vernal equinox
- Coordinates of an object (e.g. star) uniquely defined by:
 - Right ascension α = angle in equatorial plane measured CW from the vernal equinox
 - Declination δ = angle above or below the equatorial plane
- Or by:
 - Ecliptic longitude λ
 - Ecliptic latitude β
- (α, δ) and (λ, β) related through rotation of ϵ about direction of vernal equinox.

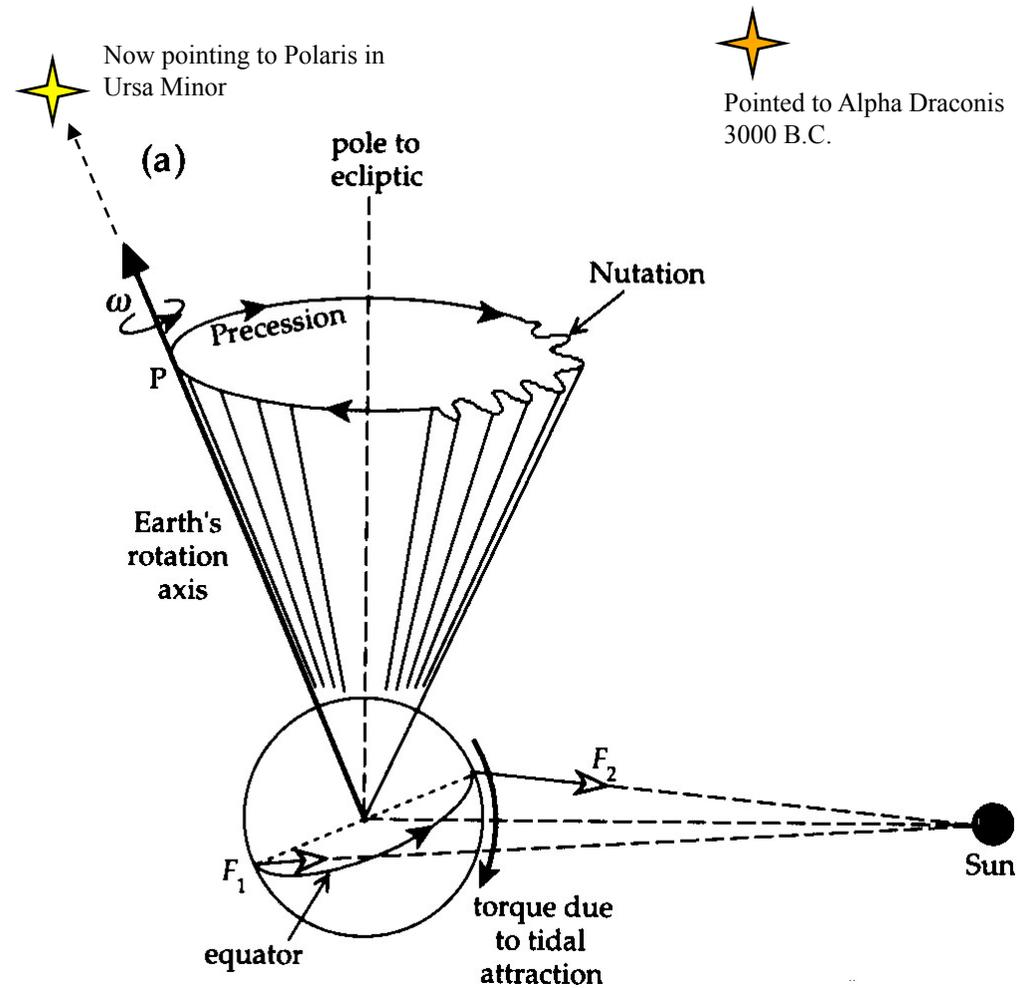


Oscillation of axes

- Earth rotation vector ω_E : $\vec{\omega}_E = \vec{\omega} \|\vec{\omega}_e\|$
- ω_E oscillates because of:
 - Gravitational torque exerted by the Moon, Sun and planets
 - Displacements of matter in different parts of the planet (including fluid envelopes) and other excitation mechanisms
- Oscillations of the Earth rotational vector:
 - Oscillations of unit vector ω :
 - With respect to inertial space (= stars), because of luni-solar tides: **precession** and **nutation**
 - With respect to Earth's crust: **polar motion**
 - Oscillations of norm = variations of speed of rotation = variations of time

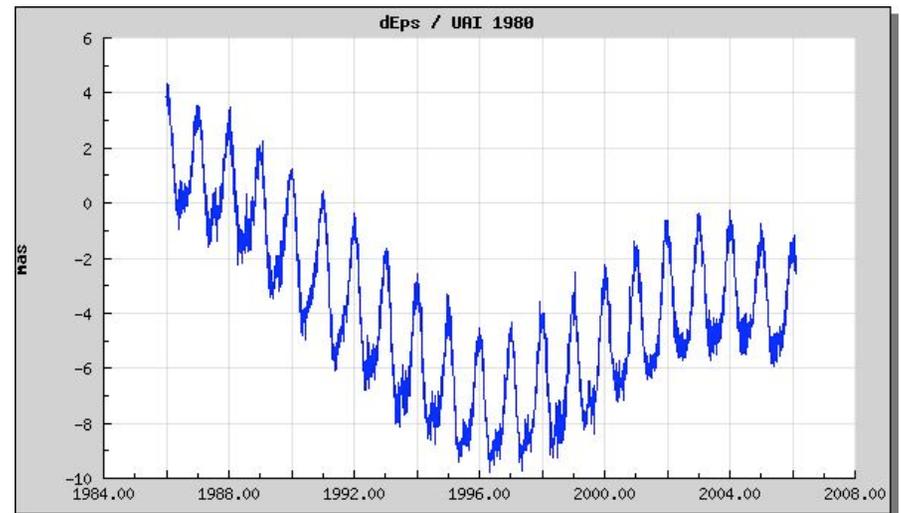
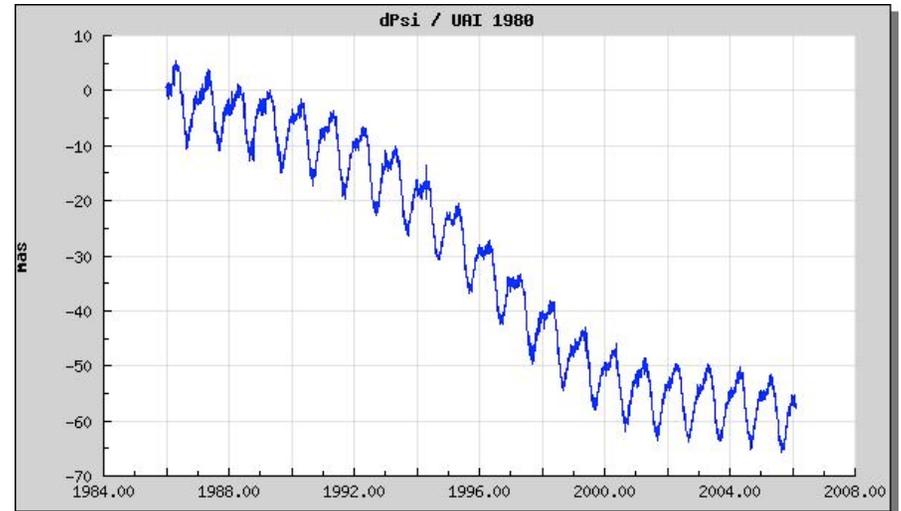
Precession and Nutation

- Because of luni-solar tides, the earth rotation axis oscillates with respect to inertial space (= stars)
- This oscillation is partitioned into:
 - Secular **precession** (~25 730 yrs): Sun (or Moon) attraction on Earth equatorial buldge, which is not in the ecliptic plane => torque that tends to bring the equator in the ecliptic plane (opposed by the centrifugal force due to Earth's rotation)
 - Periodic **nutation** (main period 18.6 yrs):
 - e.g. Sun or Moon pass equatorial plane => tidal torque = 0 => creates semi-annual and semi-monthly periods
 - There are semi-annual and semi-monthly irregularities as well
 - Results in small oscillations (also called forced nutations) superimposed on the secular precession



Precession and Nutation

- For an accurate treatment of precession and nutations, one must account for precise lunar and solar orbital parameters: use of astronomical tables (luntab, soltab in GAMIT)
- In addition, any process that modifies the Earth's moment of inertia will induce variations of its rotation vector.
- Recall that angular momentum $L=I\omega$ and must be conserved). For instance:
 - Solid Earth tides
 - Oceanic effects: tidal + non-tidal (currents, winds, pressure)
 - Atmospheric effects: pressure distribution
 - Free core nutations: rotation axis of core and mantle not aligned => 432 day period nutation

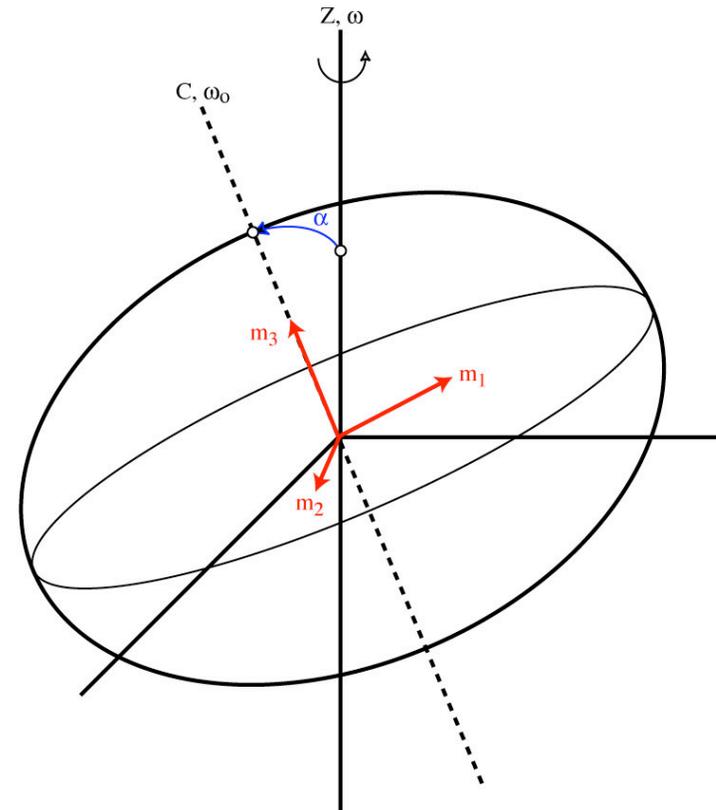


Offsets in longitude $d\Psi$ and in obliquity $dEps$ of the celestial pole with respect to (old) precession-nutation model (IAU 1980).

1 arcsec \sim 30 m at the surface of the Earth => 1 mas \sim 3 cm

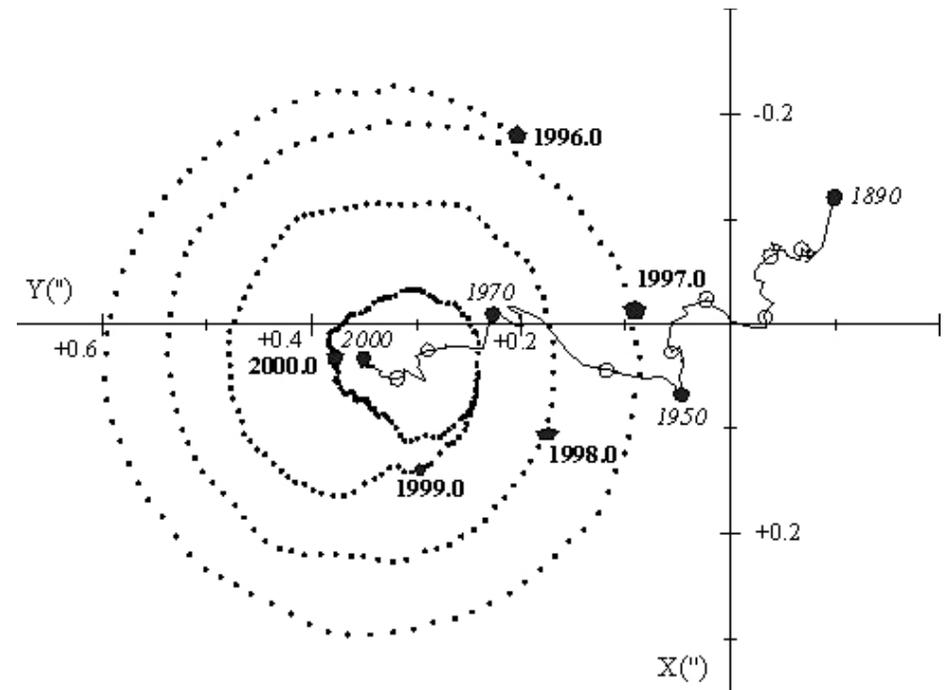
Chandler Wobble

- If rotation axis of rotating solid does not coincide with polar major axis of inertia: for the angular momentum to remain constant (assuming no external forcing) the angular velocity vector has to change.
 - True for any irregularly shaped rotating solid body.
 - Happens in the absence of external forcing => “free wobble” (or “free nutation”)
- On Earth:
 - Free wobble is called “Chandler wobble” in honor of S.C. Chandler who first observed it in 1891
 - With respect to Earth surface: position of instantaneous rotation pole moves around polar major axis of inertia
 - Theory gives $\omega_c = 305$ days, observations = 430 days...



Polar Motion

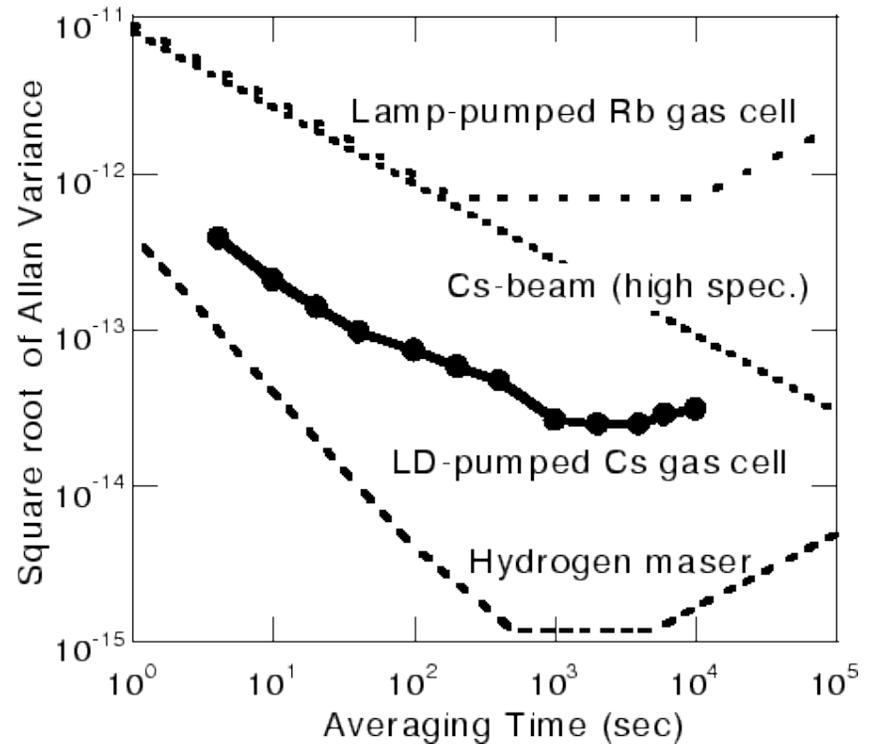
- Observed Chandler wobble: period = 433 days, amplitude ~ 0.7 arcsec (~ 15 m on Earth surface)
- Difference between observed and theoretical value?
 - A mystery for a while as astronomers were search for a 305 days period without finding it...
 - Due to non-perfect elasticity of Earth: some energy is transferred into anelastic deformation => damping of the Chandler wobble.
- Calculations using the anelasticity of Earth (Q) show that Chandler wobble should rapidly damp to zero (in less than 100 years) => there must be an excitation mechanism that keeps it going.
- Excitation mechanism not understood until recently: two-thirds of the Chandler wobble is caused by ocean-bottom pressure changes, the remaining one-third by fluctuations in atmospheric pressure (Gross, GRL, 2000).
- Polar motion actually has three major components:
 - Chandler wobble.
 - Annual oscillation forced by seasonal displacement of air and water masses, $0.15'' \sim 2$ m
 - Diurnal and semi-diurnal variations forced by oceanic tides ~ 0.5 m amplitude
- Non-oscillatory motion with respect to Earth's crust: polar wander, 3.7 mas/yr towards Groenland



Polar motion for the period 1996-2000.5 (dotted line) and polar wander since 1890 (doc. IERS). Axes in arcsec: 1 arcsec ~ 30 m at the surface of the Earth.

Time

- **Atomic time scale: based on frequency standards (mostly Cesium)**
 - Highly stable (e.g., 10^{-13} at 10^4 sec (~3 hours) ~3 ns)
 - Access to atomic time is direct.
- **Astronomic time scales: based on Earth rotation**
 - Sidereal time = directly related to Earth's rotation, not practical to measure for most Earth's applications
 - Universal time = more practical, related to apparent diurnal motion of Sun about Earth.
 - Not stable: Earth's rotation varies with time.
 - Instability of TAI is about 6 orders of magnitude smaller than that of UT1
 - Access to astronomical times is not direct, requires observation of stars or sun.

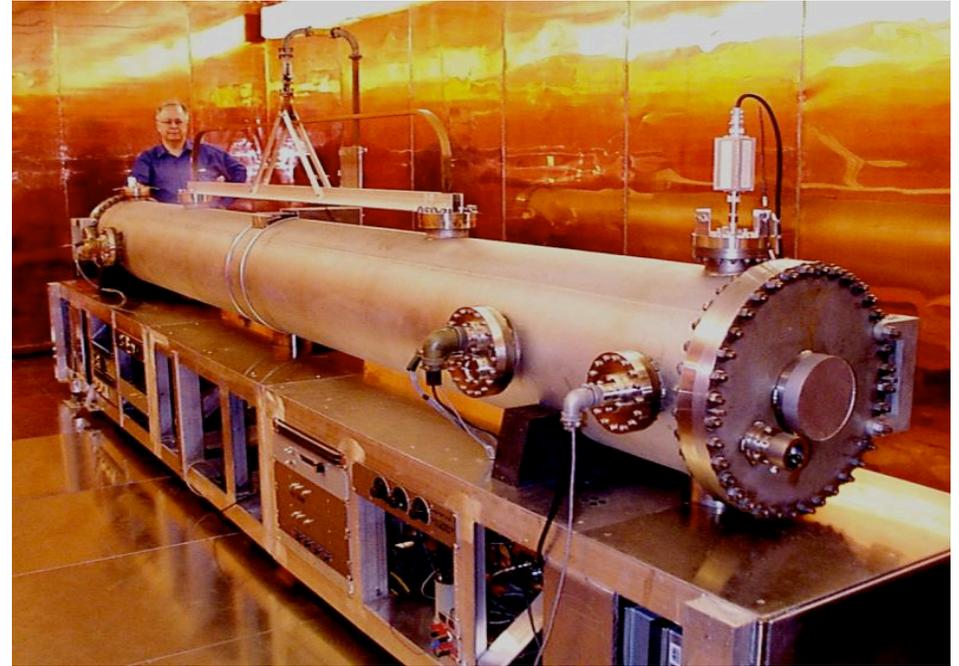


Stability of atomic frequency standards.

Allan variance = $1/2$ the time average of squares of differences between successive readings over the sampling period = measurement of stability of clocks .

International Atomic Time = TAI

- Defined by its unit, the atomic second (on the geoid) = 9,192,631,770 periods of the radiation corresponding to the transition between two hyperfine levels of Ce^{133}
- Origin so that its first epoch coincides with Universal Time UT1 (see later)
- TAI day = 86,400 seconds
- Realized by >200 atomic clocks worldwide, weighted mean calculated by BIPM (www.bipm.org).

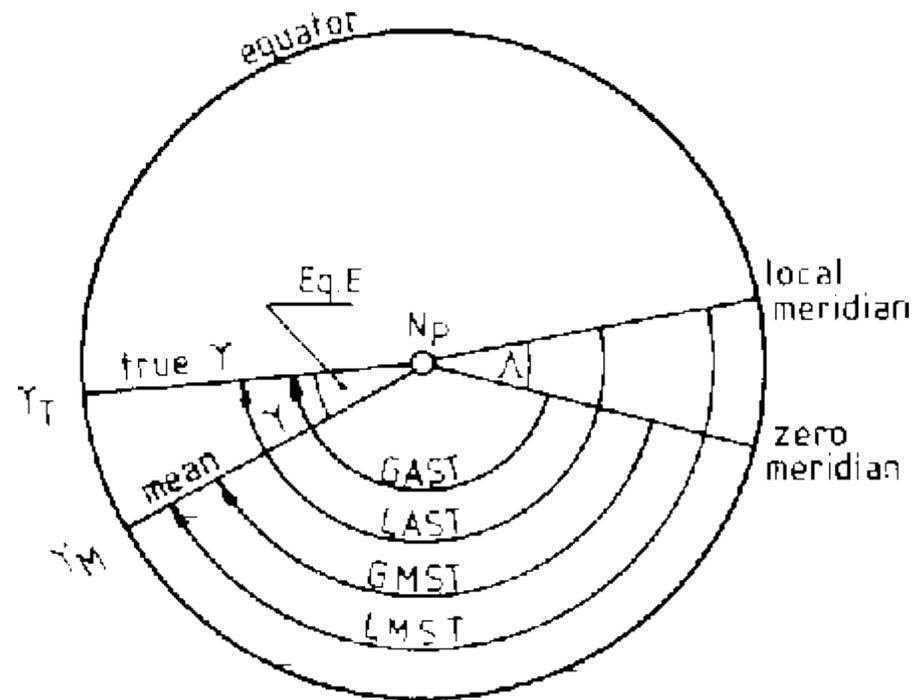


Traditional Cesium beam frequency standard

- Heated cesium to boil off atoms, different energy state
- Send them through a tube and use magnetic field to select atoms with right energy state
- Apply intense microwave field, sweeping microwave frequency about 9,192,631,770 Hz => atoms change energy state
- Collect atoms that have changed frequency state at end of tube and count them
- Adjust sweeping frequency to maximize # of atoms received at end of tube => then lock frequency
- Divide frequency by 9,192,631,770 Hz to get one pulse-per-second output

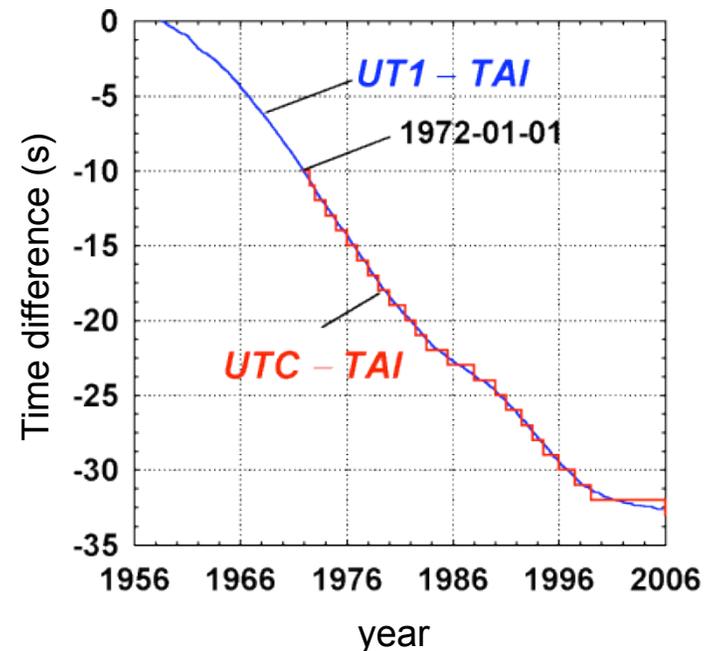
Sidereal Time - ST

- Directly related to Earth's rotation in celestial frame
- Definition = Earth rotates 360 degrees (in celestial frame) in 1 sidereal day
- If measured at observer's location w.r.t. true vernal equinox = Local Apparent ST
- LAST measured by observations to distant stars and extragalactic radio sources
- If corrected for nutation and precession (w.r.t. mean vernal equinox) = Local Mean ST
- If measured at Greenwich = GAST or GMST



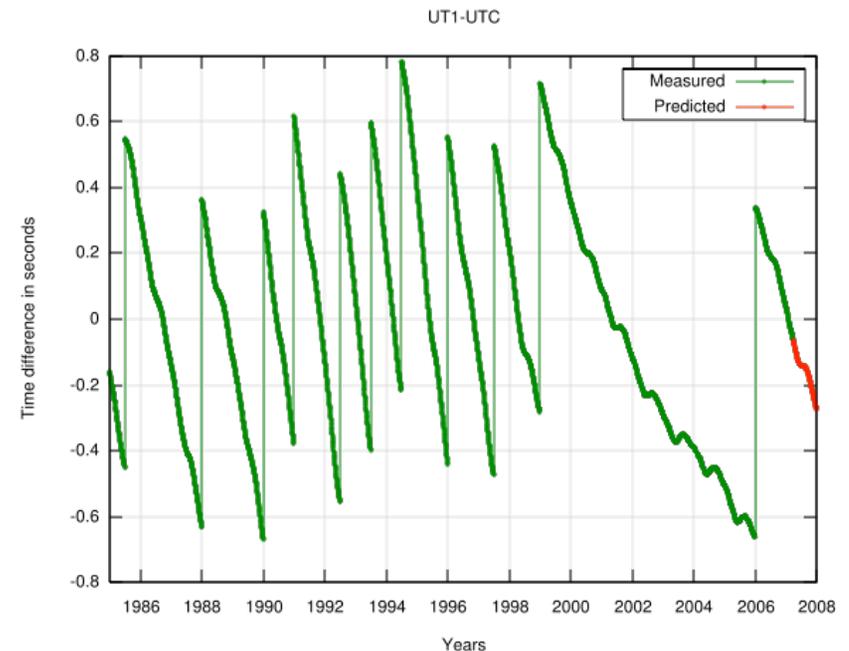
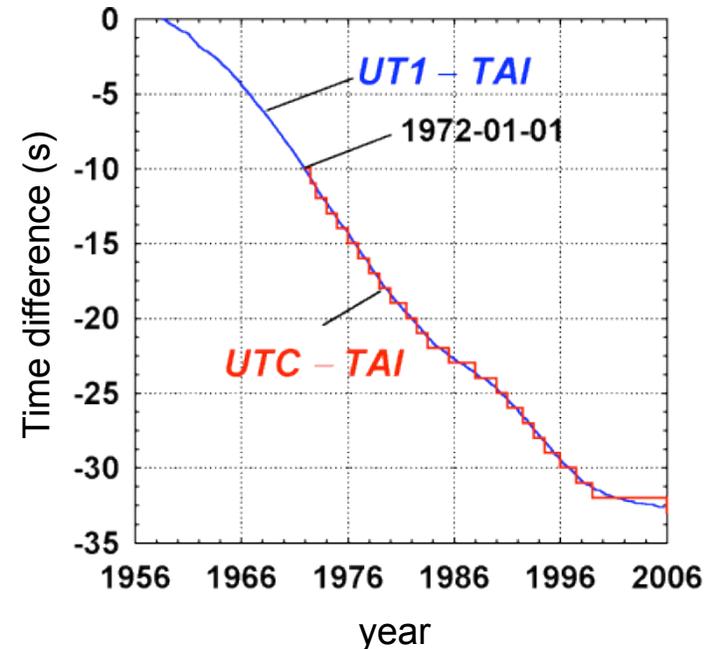
Universal Time - UT

- Related to apparent motion of the sun around the Earth
- When referred to the Greenwich meridian = Universal Time (UT)
- UT is measured directly using ground stations (VLBI)
- UT not uniform because of polar motion => UT corrected for polar motion = UT1
 - UT1 has short term instabilities at the level of 10^{-8}
 - Duration of the day slowly decaying (~ 0.002 s/century) because of secular deceleration of Earth rotation.



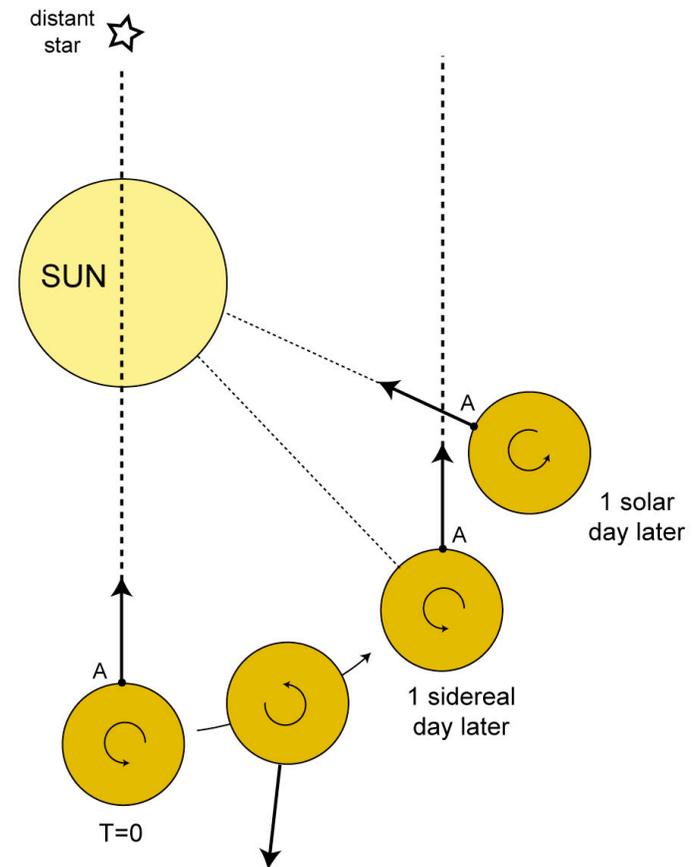
TAI - UT relation?

- Because of the secular deceleration of the Earth's rotation, TAI increases continuously with respect to UT1.
- If legal time was based on TAI, coincidence with solar day could not be maintained (in a couple of year TAI - UT1 can increase by a few seconds).
- Compromise: use highly stable atomic time, but adjust time to match irregular Earth rotation => UTC
 - Unit = atomic second
 - By definition, $|\text{UT1} - \text{UTC}| < 0.9 \text{ s}$
 - UTC changed in steps of 1 full second (leap second) if $|\text{UT1} - \text{UTC}| > 0.9 \text{ s}$, responsibility of the IERS (June 30 or Dec. 31)
 - UTC = broadcast time used for most civilian applications (your watch!)
- Currently: $\text{TAI} - \text{UTC} = 33 \text{ s}$



UT - ST relation?

- It takes 365 solar days for Earth to be back at same place w.r.t. sun
- During 1 sidereal day, observer A has rotated 360 degrees
- Need to rotate by additional α to complete 1 solar day:
 - Solar day is longer than sidereal
 - Observer accumulates some "lead time" w.r.t. solar day every solar day
 - At a rate of $1/365$ of a solar day = 3 min 44.90 sec per solar day
- Therefore:
 - Sidereal day = solar day - 3 min 44.90 sec = 86,164.10 s
 - Earth angular rotation:
 $\omega = 2\pi/86,164.10 = 7,292,115 \times 10^{-5} \text{ rad s}^{-1}$ (see GRS definition)



GPS time (GPST)

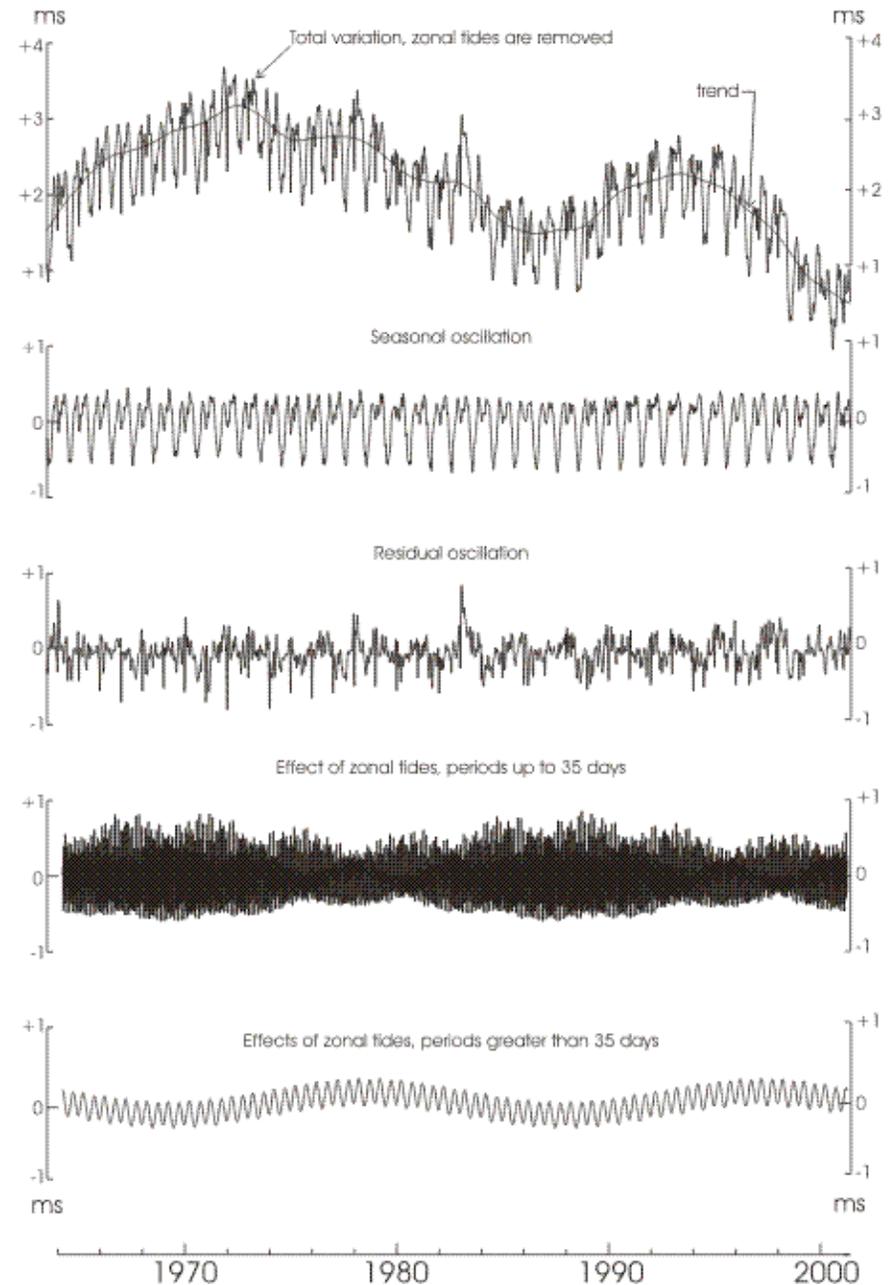
- Atomic scale, unit = SI second
- $GPST = TAI - 19 \text{ s}$
- Coincident with UTC on January 6th, 1980, 00:00 UTC
- Not incremented by leap seconds
- Currently: $TAI - UTC = 33 \text{ s} \Rightarrow GPST = UTC + 14 \text{ s}$

local	2008-01-08 22:19:42	Tuesday	day 008	timezone UTC-5
UTC	2008-01-09 03:19:42	Wednesday	day 009	MJD 54474.13868
GPS	2008-01-09 03:19:56	week 1461	271196 s	cycle 1 week 0437 day 3
Loran	2008-01-09 03:20:05	GRI 9940	84 s until	next TOC 03:21:06 UTC
TAI	2008-01-09 03:20:15	Wednesday	day 009	33 leap seconds

<http://www.leapsecond.com/java/gpsclock.htm>

Length-of-day = LOD

- Length of Day (LOD) = difference between 86 400 sec SI and length of astronomical day:
 - Long term variations:
 - Dynamics of liquid core
 - Climate
 - Short term variations:
 - Zonal tides
 - Seasonal = climate
 - Residual: Cf. 1983 El Nino event



Time systems/Calendar

- **Julian Date (JD)**: number of mean solar days elapsed since January 1st, 4713 B.C., 12:00
- **Modified Julian Date (MJD)** = $JD - 2,400,00.5$
 - Ex.: GPS standard epoch, JD = 2,444,244.5 (January 6th, 1980, 00:00 UTC)
 - Ex.: Standard epoch J2000.0, JD = 2,451,545.0 (January 1st, 2000, 12:00 UTC)
- **Day Of Year (DOY)**: day since January 1st of the current year
- GPS calendar:
 - GPS week: Week since GPS standard epoch
 - GPS day of week: Sunday = 0 to Saturday = 6
 - GPS second: Second since GPS standard epoch

Reference Systems

- Introduced to help model geodetic observations as a function of unknown parameters of interest
 - Positions:
 - Space-fixed = celestial, tied to extra-galactic objects
 - Terrestrial = tied to solid Earth
 - Time systems: based on quantum physics or Earth rotation
- Systems vs. Frames:
 - Reference System = set of prescriptions, conventions, and models required to define at any time a triad of axes.
 - Reference Frame = practical means to “access” or “realize” a system (e.g., existing stations of known coordinates)

Reference Systems



- Systems and frames:
 - Defined by international body: the international Earth Rotation and Reference Systems Service = IERS (<http://www.iers.org>)
 - Mission of the IERS: *“To provide to the worldwide scientific and technical community reference values for Earth orientation parameters and reference realizations of internationally accepted celestial and terrestrial reference systems”*
 - Updated on a regular basis as models or measurements improve.
- Basic units and fundamental constants:
 - Meter = length unit (length of path traveled by light in vacuum in 1/299,792,458 of a second - CGPM 1983)
 - Kilogram = mass unit (mass of international prototype - CGPM 1901)
 - Second = time unit (9192631770 periods of the radiation corresponding to the transition between two hyperfine levels of Ce133 - CGPM 1967)

Conventional Systems



- **Conventional Inertial System:**

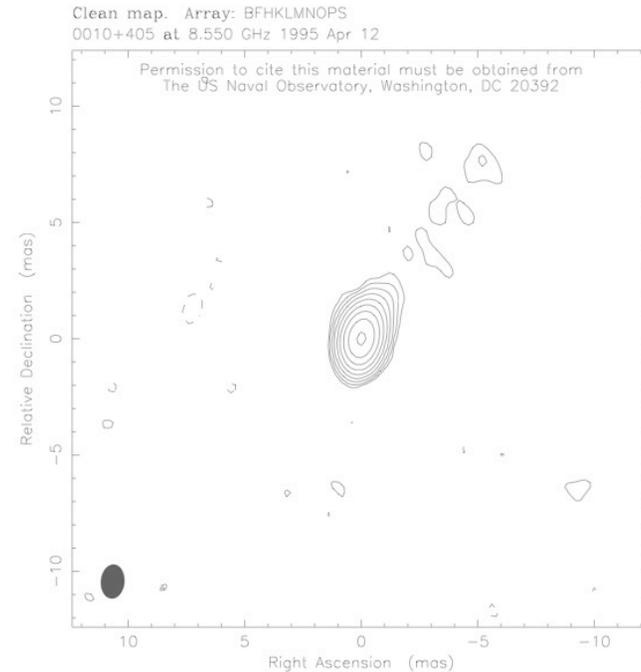
- Orthogonal system, center = Earth center of mass, defined at standard epoch J2000 (January 1st, 2000, 12:00 UT)
- Z = position of the Earth's angular momentum axis at standard epoch J2000
- X = points to the vernal equinox
- It is materialized by precise equatorial coordinates of extragalactic radio sources observed in Very Long Baseline Interferometry (VLBI) = Inertial Reference **Frame**.
- First realization of the International Celestial Reference Frame (ICRF) in 1995.

- **Conventional Terrestrial System:**

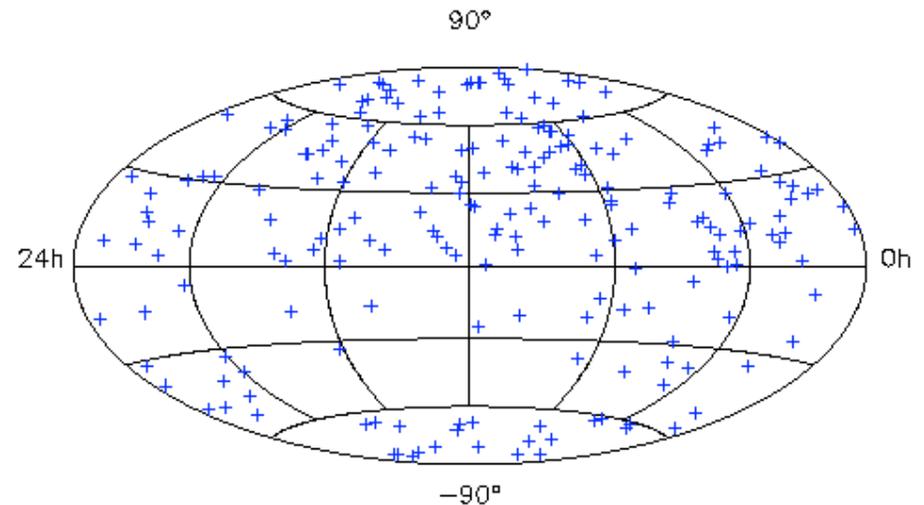
- Orthogonal system center = Earth center of mass
- Z = position of the Earth's angular momentum axis at standard epoch J2000
- X = Greenwich meridian
- It is materialized by a set of ground control stations of precisely known positions and velocities = Terrestrial Reference **Frame**

International Celestial Reference Frame

- Directions of the ICRS pole and right ascensions origin maintained fixed relative to the quasars within ± 20 microarcseconds.
- The ICRS is accessible by means of coordinates of reference extragalactic radio sources.
- It is realized by VLBI estimates of equatorial coordinates of a set of extragalactic compact radio sources, the International Celestial Reference frame.
- ICRS can be connected to the International Terrestrial Reference System through Earth Orientation Parameters (EOP -- more later).



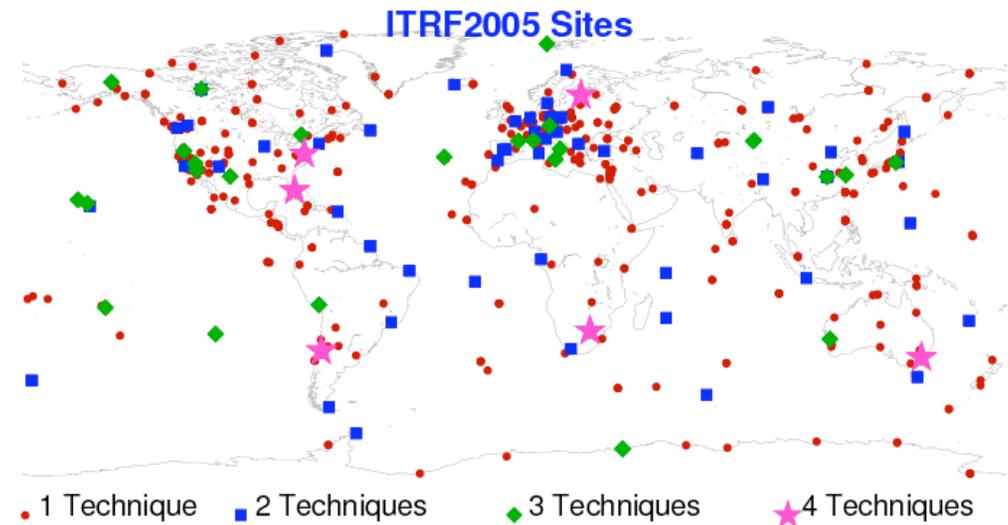
An extragalactic source "seen" by a radio telescope at 8 GHz



212 high-astrometric-quality objects define the ICRF axes

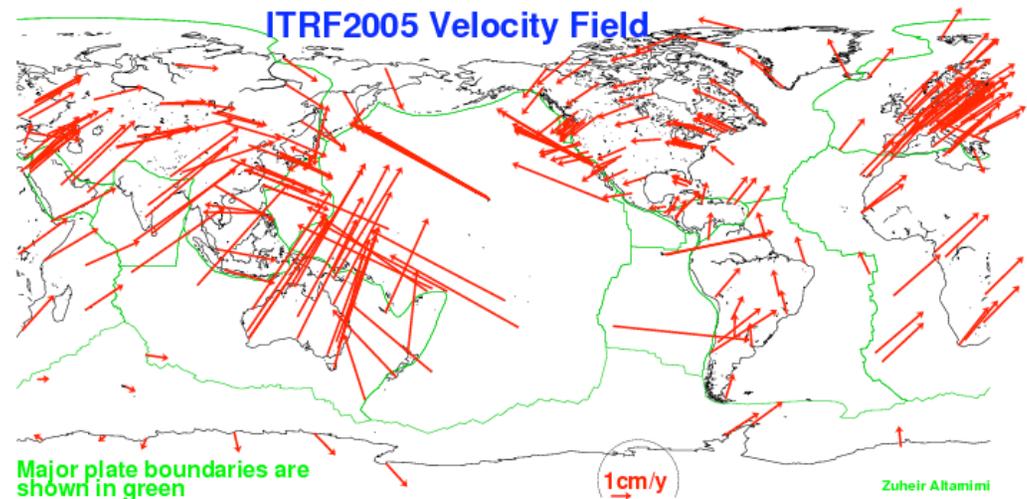
International Terrestrial Reference Frame

- Realized by a set of ground control stations, in the framework the IERS
- International Terrestrial Reference Frame (<http://itrf.ensg.ign.fr/>):
 - First version in 1989 (ITRF-89), current version ITRF-2005
 - Set of station positions, velocities, and epochs in an Earth centered-Earth fixed (=ECEF) Terrestrial System
 - And associated variance-covariance matrix
 - Since ITRF derives from measurements, it changes (improves) with its successive realizations
- More on ITRF and reference frames later.



Map of ground geodetic stations used in the definition of the ITRF-2005.
Note that some stations benefit from several observation techniques in colocation

Below: surface velocities associated with ITRF2005



Earth Orientation Parameters (EOP)

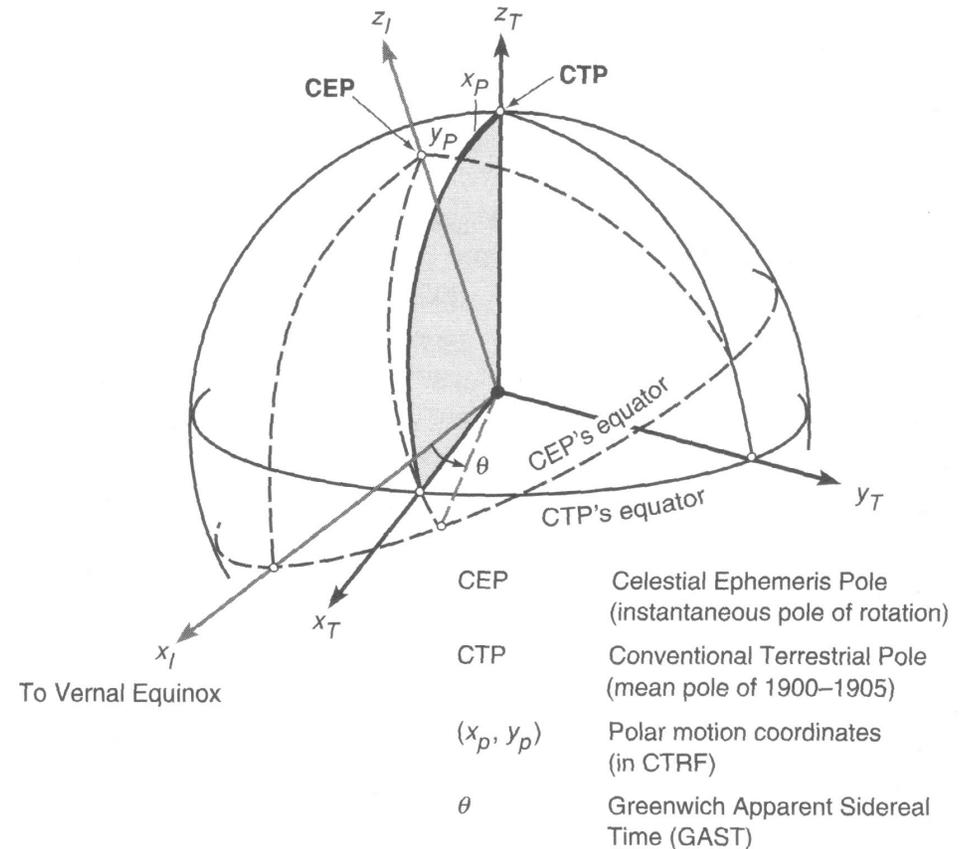
- The Earth's orientation is defined as the **rotation between**:
 - A **rotating geocentric set of axes linked to the Earth** (the terrestrial system materialized by the coordinates of observing stations) = Terrestrial System
 - A **non-rotating geocentric set of axes linked to inertial space** (the celestial system materialized by coordinates of stars, quasars, objects of the solar system) = Inertial System
- Inertial coordinates are transformed to terrestrial by combining rotations:

$$r_t = R_2(-x_p)R_1(-y_p)R_3(GAST)N(t)P(t)r_i$$

Terrestrial Polar motion UT1 Nutation Precession Inertial

Earth Orientation Parameters (EOP)

- Nutation and Precession:
 - Rotation matrices derived from geophysical model + corrections provided by IERS.
 - $P(t)$ = rotate from reference epoch (J2000) to observation epoch --> obtain mean equator and equinox.
 - $N(t)$ = rotate from mean to instantaneous true equator and vernal equinox.
 - At this stage Earth rotation axis (Z-axis) coincides with CEP.
- R_3 = rotate about Z axis (=CEP) by GAST
=> X-axis coincides with vernal equinox.
- Polar motion:
 - R_1 = rotate about X axis by $-y_p$ (small angle)
 - R_2 = rotate about Y axis by $-x_p$ (small angle)



$$R_3(GAST) = \begin{bmatrix} \cos(GAST) & \sin(GAST) & 0 \\ -\sin(GAST) & \cos(GAST) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1(-y_p) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -y_p \\ 0 & y_p & 1 \end{bmatrix}$$

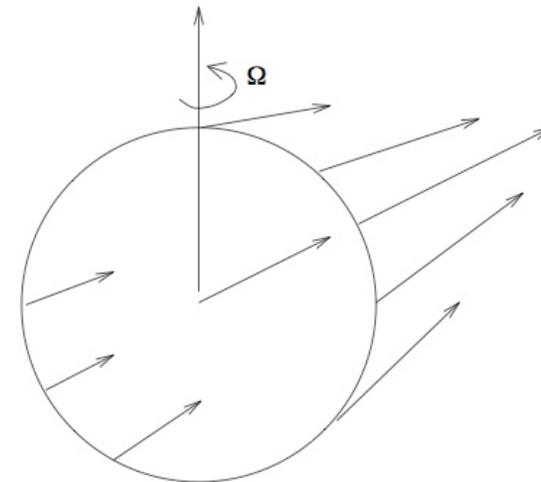
$$R_2(-x_p) = \begin{bmatrix} 1 & 0 & x_p \\ 0 & 1 & 0 \\ -x_p & 0 & 1 \end{bmatrix}$$

Earth Orientation Parameters (EOP)

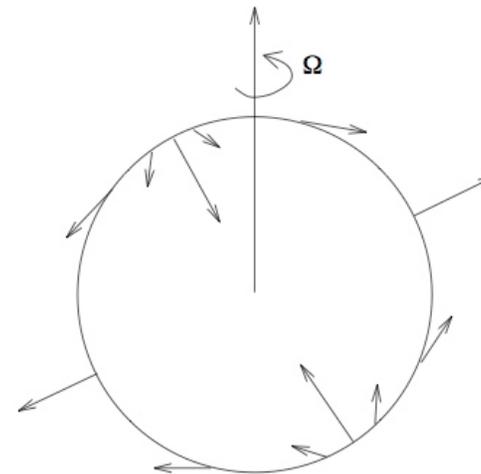
- Rotation parameters are given in astronomical tables provided by the IERS:
 - For the most precise positioning, they are used as a priori values (sometimes even adjusted) in the data inversion.
 - For non-precise applications, only $R_3(\text{GAST})$ matters.
- In practice, the IERS provides **five Earth Orientation Parameters (EOP)** :
 - Celestial pole offsets ($d\text{Psi}$, $d\text{Eps}$): corrections to a given precession and nutation model
 - Universal time (UT1) = UT corrected for polar motion, provided as UT1-TAI
 - 2 coordinates of the pole ($x_p, -y_p$) of the CEP (corrected for nutation and precession) with respect to terrestrial Z-axis axis = polar motion

Tides

- Earth and Moon are coupled by gravitational attraction: each one rotates around the center of mass of the pair.
- Moon (or Sun) gravitational force = sum of:
 - Part constant over the Earth --> orbital motions (think of Earth as a point mass) = orbital force
 - Remainder, varies over the Earth --> tides (Earth is not a point mass...) = tidal force
- Tidal force:
 - Causes no net force on Earth => does not contribute to orbital motion
 - Fixed w.r.t. Moon
- In addition: the Earth rotates => tidal force varies with time at given location.



Total gravitational force: larger at closer distance to the Moon. Orbital force = average of all arrows.



Tidal force: total force minus orbital force.

Moon
●

Moon
●

Tidal frequencies

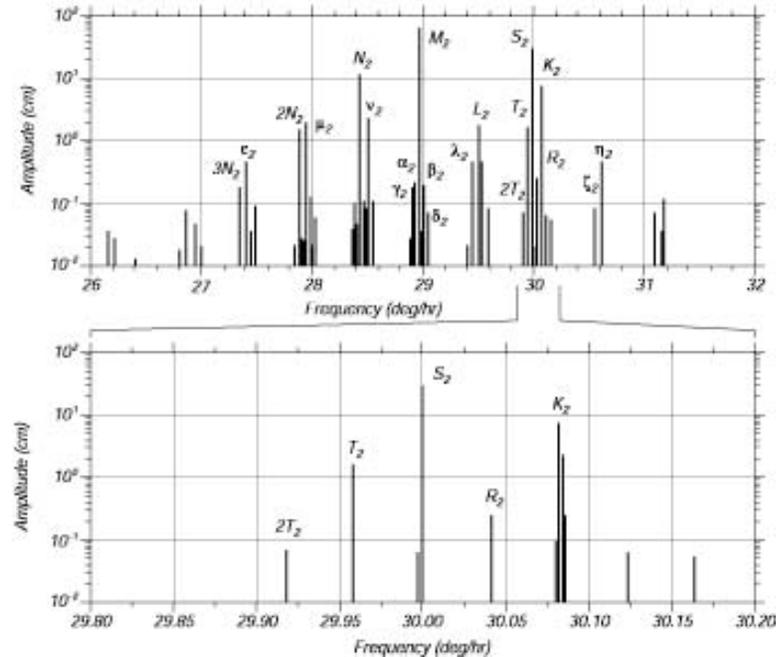
- One can show that the Earth's tidal potential can be written as:

$$V_T = \frac{3}{4} GM \frac{r^2}{R^3} \left[\begin{array}{l} \left(\frac{1}{3} - \sin^2 \varphi_p \right) (1 - 3 \sin^2 \delta) \\ + \sin 2\varphi_p \sin 2\delta \cos(h) \\ + \cos^2 \varphi_p \cos^2 \delta \cos(2h) \end{array} \right] \begin{array}{l} \rightarrow \text{Long-period} \\ \rightarrow \text{Diurnal} \\ \rightarrow \text{Semi-diurnal} \end{array}$$

with φ_p, λ_p = latitude, longitude of observer, r = Earth's radius, R = earth-moon distance, δ, α = declination, right-ascension of celestial body, h = hour angle given by:

$$h = \lambda_p + GAST - \alpha$$

- R, δ, h : periodic variations with time => 3 terms have periodic variations:
 - Moon: 14 days, 24 hours, and 12 hours.
 - Sun: ~180 days, 24 hours, and 12 hours.
 - 3 distinct groups of tidal frequencies: twice-daily, daily, and long period



Constituent	Name	Period	Acceleration (nm s ⁻²)	Amplitude, ocean (m)
Semi-diurnal				
Principal lunar	M2	12.4206 h	375.6	0.242334
Principal solar	S2	12.0000 h	174.8	0.112841
Lunar elliptic	N2	12.6584 h	71.9	0.046398
Lunisolar	K2	11.9673 h	47.5	0.030704
Diurnal				
Lunisolar	K1	23.9344 h	436.9	0.141565
Principal lunar	O1	25.8194 h	310.6	0.100514
Principal solar	P1	24.0659 h	144.6	0.046843
Elliptic lunar	Q1	26.8684 h	59.5	0.019256
Long Period				
Fortnightly	Mf	13.66 d	31.9	0.041742
Monthly	Mm	27.55 d	16.8	0.022026
Semi-annual	Ssa	182.62 d	14.8	0.019446

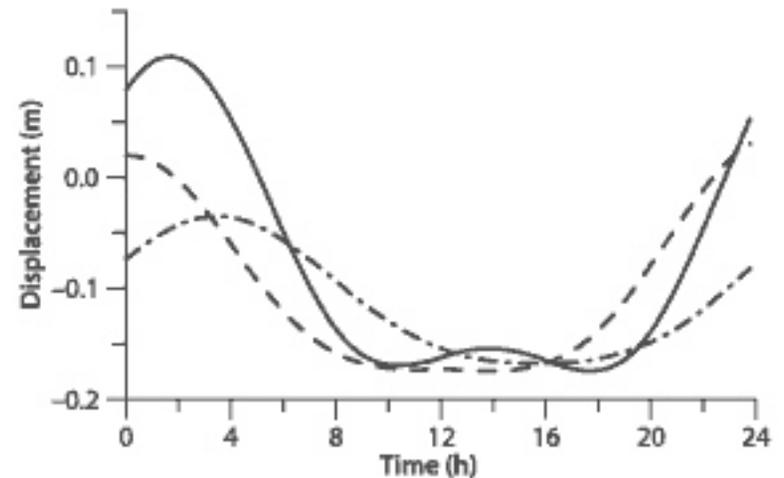
Earth Tides

- Love showed that displacement on spherical Earth is proportional to tidal potential and Love numbers (for harmonic degree n):

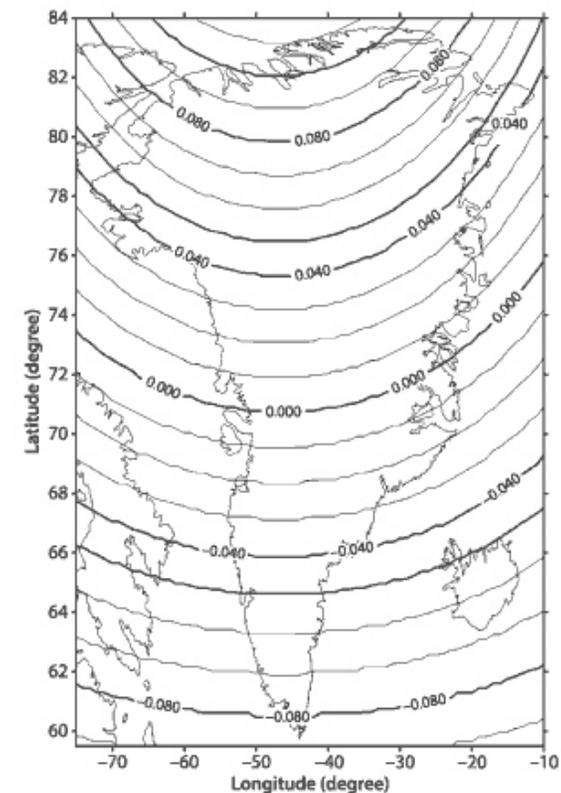
$$d_r = \frac{h_n}{g} V_T \vec{e}_r$$

$$d_t = \frac{l_n}{g} \nabla V_T \vec{e}_t = \frac{l_n}{g} \frac{\partial V_T}{\partial \theta} \vec{\theta} + \frac{l_n}{g \sin \theta} \frac{\partial V_T}{\partial \lambda} \vec{\lambda}$$

- Love numbers for displacement = h_n and l_n (+ k_n for potential)
- Depend on elastic properties of the Earth (from seismic velocities)
- Solid Earth: largest contribution from harmonic degree 2: $h_2=0.609$; $l_2=0.085$; $k_2=0.3$
- Displacements due to Earth tides:
 - Ex. in Greenland (figures)
 - Large compared to the precision of GPS => must be corrected (using model)

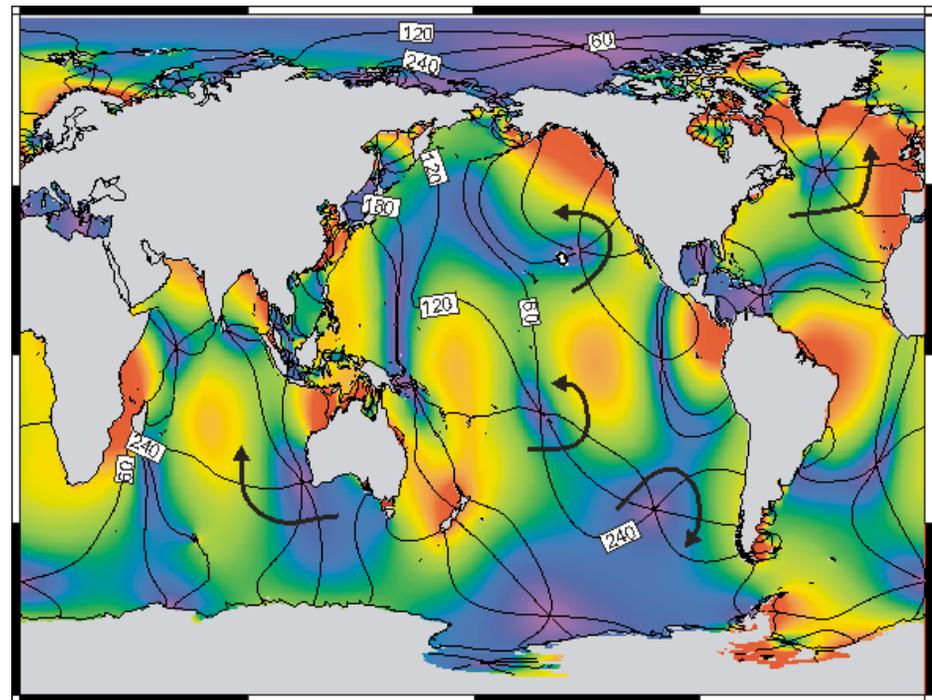
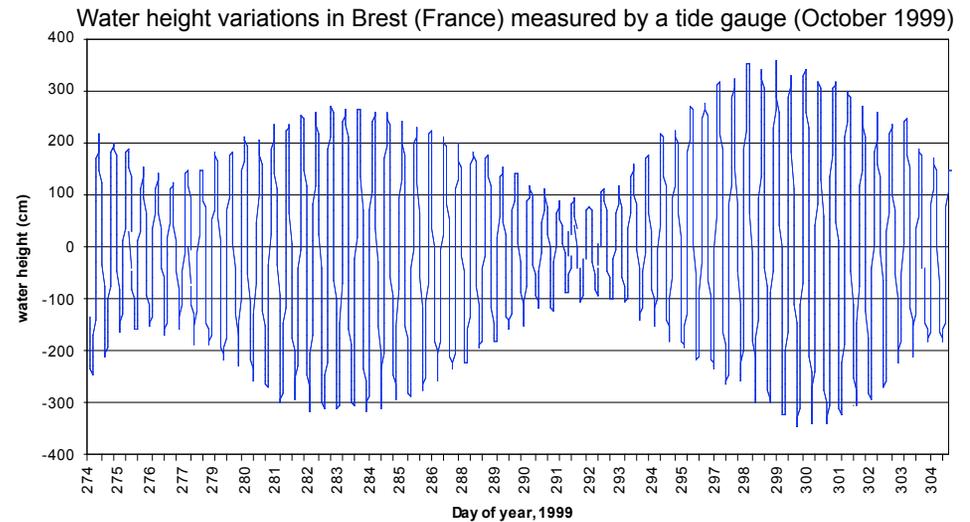


3 sites in Greenland, vertical position

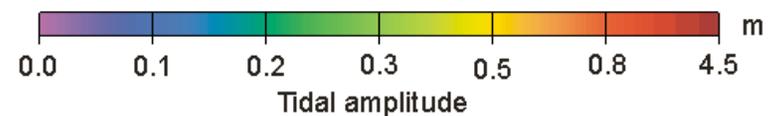


Ocean tides

- Tidal force results in displacements of Earth's constituents:
 - Oceans
 - Solid Earth
 - Atmosphere
- Ocean tides:
 - Measured with tide gauges
 - Complex pattern, local amplifications.
- Once tides are removed, what do tide gauges measure?



The ocean tides for harmonic M2 (period of 12 hours and 25 minutes) . The color represent the amplitude and the contour lines indicate the phase lag of the tides with a spacing of 60 degrees. (Doc. H.G. Scherneck)



Loading effects

- Earth ~elastic solid => deforms under load, as a function of:
 - Load (=source) characteristics (spatial and temporal evolution of mass)
 - Elastic properties: quantified by loading Love numbers: h_n', l_n', k_n'

- Radial deformation of elastic Earth:

$$u_r(\lambda, \varphi) = \frac{3}{\rho_e} \sum_{n=0}^{\infty} \frac{h_n'}{2n+1} q_n(\lambda, \varphi)$$

ρ_e = mean Earth density

h_n' = load Love number (function of assumed rheology)

N = degree of spherical harmonic series

$q_n(\lambda, \varphi)$ = spherical harmonic expansio of surface load

- Ocean tides are one of the major sources of load:
 - Result is centimeters of radial motion
 - Higher power at diurnal and semi-diurnal periods (K1, M2)
 - Loading effect larger in coastal regions, decreases land-ward

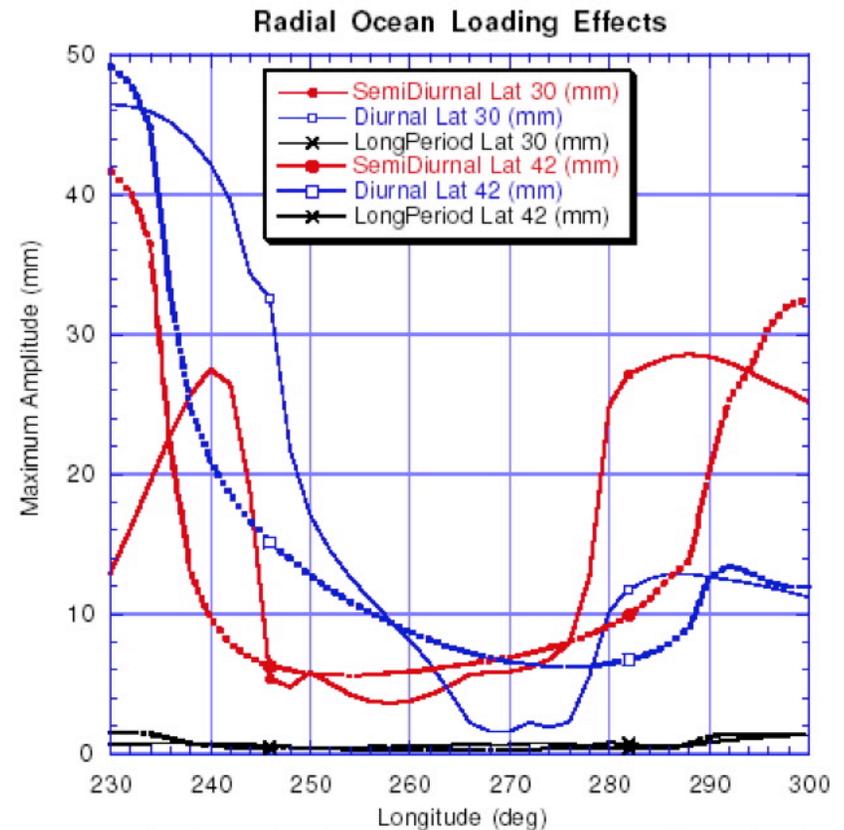


Fig. T. Herring

Loading effects

- Additional loading sources include
 - Atmospheric pressure:
 - Centimeters: ~ 0.5 mm/mbar
 - Amplitude related to weather patterns => mostly short period
 - Ground water:
 - Rivers + soil + snow
 - Centimeters: ~ 0.5 mm/cm of water
 - Higher power at seasonal period
- Ultimate GPS precision requires correcting for loading effects:
 - Ocean loading:
 - Routine
 - Ocean tidal models accurate (except some local areas)
 - Atm and hydro:
 - Still experimental
 - Must rely on pressure and continental water measurements (and models...)

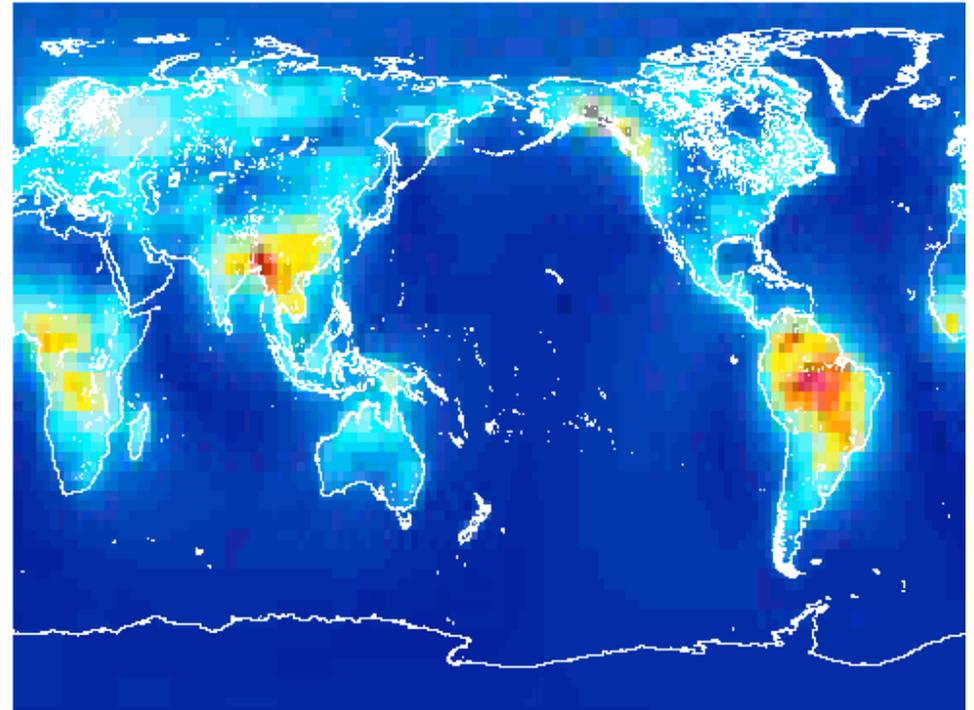


Fig. T. vanDam

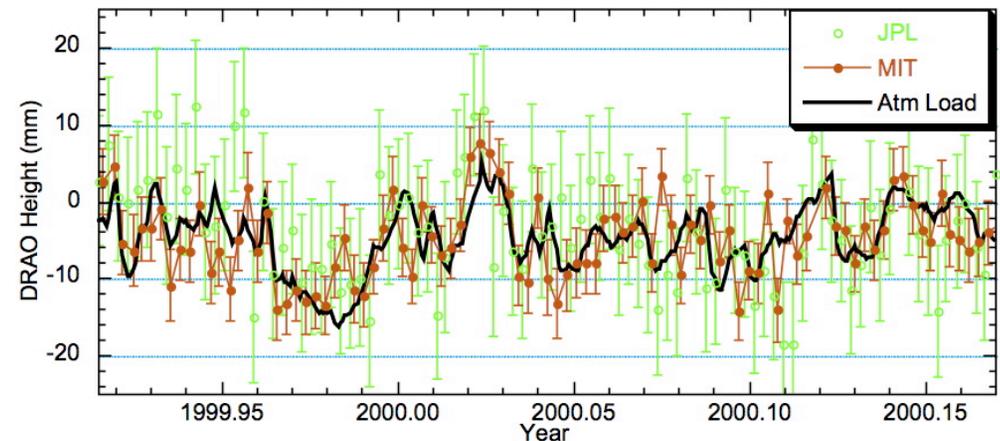
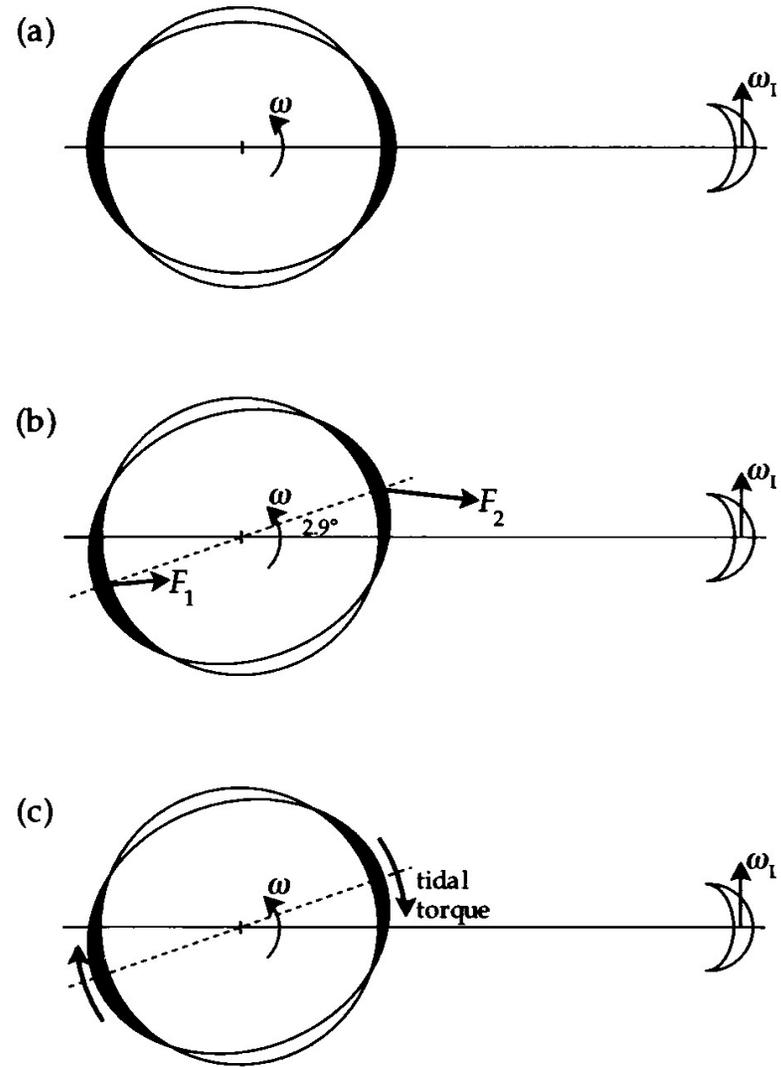


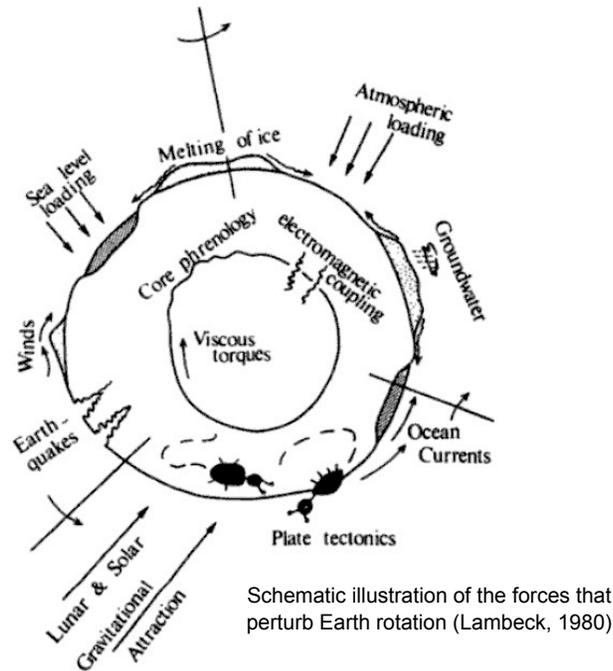
Fig. T. Herring

Tidal friction and LOD

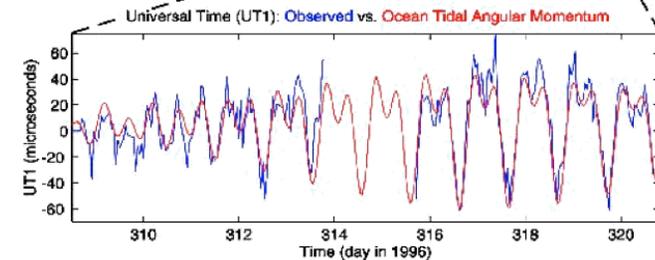
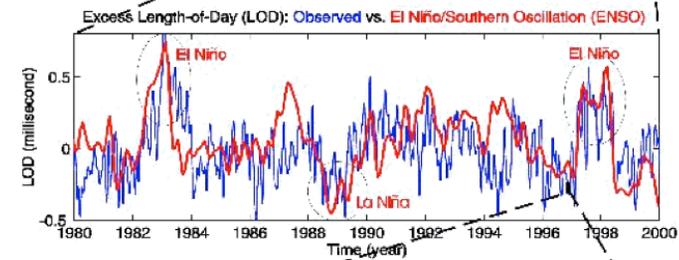
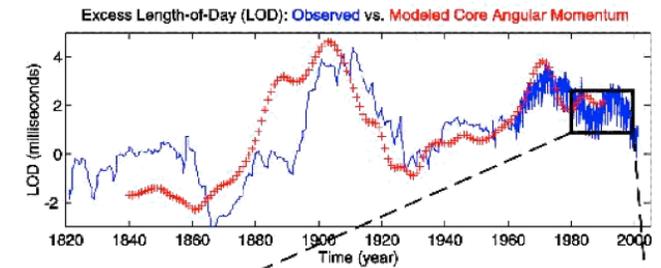
- If the Earth was purely elastic => tidal bulge aligned with the Moon (a)
- But Earth tidal response is not instantaneous because of Earth anelasticity
 - => slight delay between high tide and Moon alignment (~12 min)
 - => creates torque that tends to bring the the tidal bulge axis back into the Earth-Moon direction, in a direction opposite to the Earth's rotation
 - Deceleration of the Earth's rotation => increase of length-of-day: ~2 s/100,000 yr
 - Analysis of growth rings of fossil corals, 350 Ma old => 1 day = 22 hr, 1 year = 400 days
- The Earth tidal bulge creates a similar torque on the Moon, in opposite direction (conservation of Earth-Moon angular momentum) => deceleration of the Moon revolution
- Because of Kepler's third Law ($\text{period}^2/a^3 = \text{const}$), the Moon-Earth distance increases (~3.8 cm/yr)



Geophysical Fluids and Earth Rotation



- Geophysical fluids act on Earth rotation ω in 2 ways:
 - By the torques (τ) they exert on the planet
 - By the modifying the Earth moment of inertia (I)
- Since angular momentum of the whole Earth system has to be conserved (recall that $L=I\omega$ and $\tau=dL/dt$): torques and changes in moment of inertia result in changes in ω .
- Any geophysical process involving mass transport will trigger Earth rotation variations with spatial and temporal characteristics function of the triggering process:
 - Atmosphere: pressure systems = air mass moving around the planet
 - Oceans: water displacements due to tides, wind, thermohaline fluxes
 - Whole earth: body tide, mantle flow (e.g. post-glacial rebound), tectonic plate motion, earthquakes
 - Liquid core: the “geodynamo”
 - Continents: snow and groundwater



Chao, 2004: LOD/UT1 variations with respect to various geophysical excitation sources ranging from core flow, to El Ninos, to ocean tides

Geophysical Fluids and J_2 dot

- Recall that the dynamic oblateness of the Earth J_2 relates to the Earth moment of inertia via:

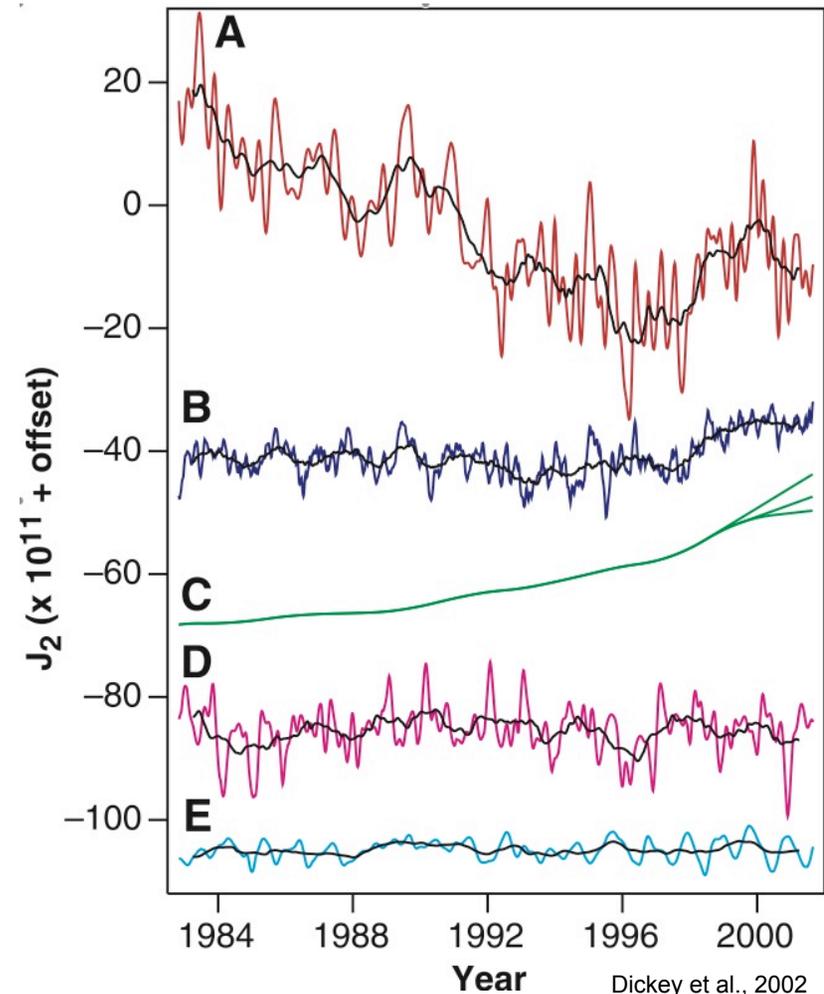
$$J_2 = \frac{C - A}{Ma^2}$$

- Similar to tidal torque exerted by moon and sun on Earth that cause precession, Earth exerts a torque on artificial satellites => precession of the satellite orbit = “regression of the nodes”
- In the simplified case of a circular orbit, one can show that the angular change in the position of the node is given by:

$$\frac{\Delta\Omega}{2\pi} = -\frac{3}{2} \frac{a^2}{r^2} J_2 \cos i$$

with i = inclination of satellite’s orbit plane w.r.t. equatorial plane, r = orbital radius

- Since one can track the position of satellites from Earth-based stations, $\Delta\Omega$ can be measured => J_2 can be determined using geodetic observations.
- Actually, higher-order coefficients as well => determination of the “satellite geoid”.
- In practice, this is more involved since orbits are elliptical.



A = geodetic observations of J_2
 B = theoretical effect of oceans on J_2
 C = theoretical effect of subpolar ice melt
 D = theoretical effect of atmosphere
 E = theoretical effects of groundwater

- ⇒ Until 1996: J_2 decreases, a result of postglacial rebound (1,000 years scale)
- ⇒ 1997-1998: J_2 increases because of surge in ice melt and mass shift in oceans (also strong El Nino event, 1998 warmest global mean temperature on record).