GPS Signal Propagation

Tropospheric refraction
Ionospheric refraction
Clock errors
Antenna phase center biases

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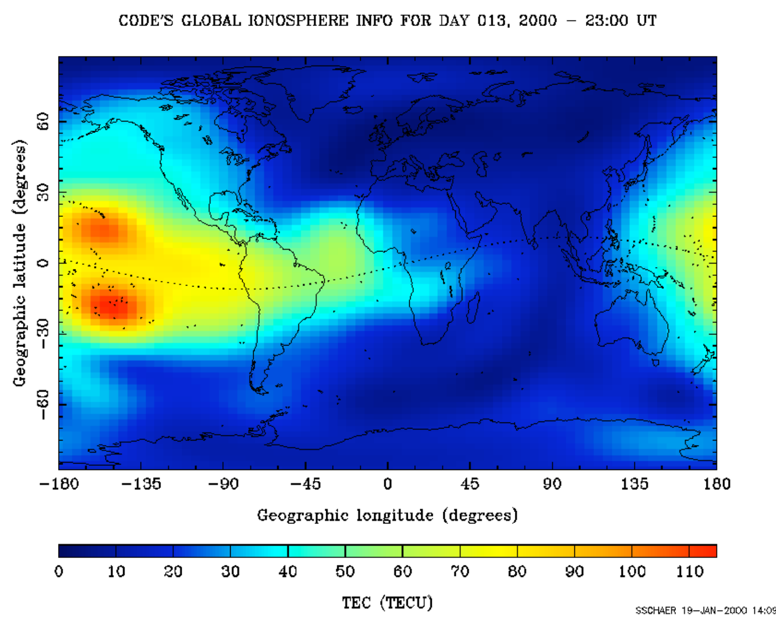


GPS signal propagation

- GPS signal (= carrier phase modulated by satellite PRN code) sent by satellite.

- About 66 msec (20,000 km) later signal arrives at GPS receiver, which:
  - Decodes propagation time by correlating incoming signal with internal replica of the code.
  - Counts carrier phases.

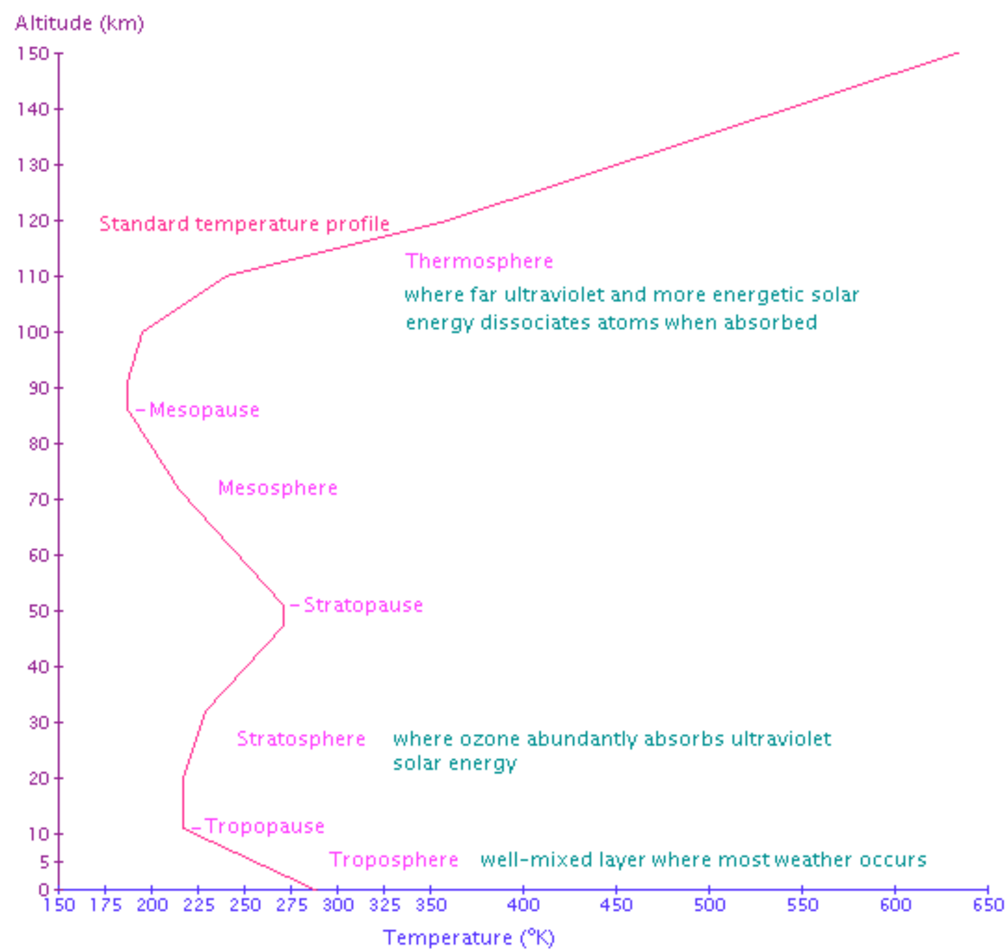
- Resulting observables:
  - Propagation $\times$ c = pseudorange.
  - Carrier phase count.

- During propagation, signal passes through:
  - Ionosphere (10-100 m of delay)
  - Neutral atmosphere (2.3-30 m delay, depending on elevation angle).

- To estimate an accurate position from range data, one needs to account for all these propagation effects and time offsets.

GPS signal propagation

- L1 and L2 frequencies are affected by **atmospheric refraction**:
  - ⇒ Ray bending (negligible)
  - ⇒ Propagation velocity decrease (w.r.t. vacuum) ⇒ propagation delay
- In the **troposphere**:
  - – Delay is a function of (P, T, H), 1 to 5 m
  - – Largest effect due to pressure
- In the **ionosphere**: delay function of the electronic density, 0 to 50 m
- This **refractive delay** biases the satellite-receiver range measurements, and, consequently the estimated positions: essentially in the vertical.



GPS signal propagation

- Velocity of electromagnetic waves:
  - In a vacuum = c
  - In the atmosphere = v (with $v < c$)
  - Dimensionless ratio $n = c/v$ = refractive index
- Consequently, GPS signals in the atmosphere experience a **delay** compared to propagation in a vacuum.
- This delay is the difference between the actual path of the carrier S and the straight-line path L in a vacuum:

$$dt = \int_S \frac{dS}{v} - \int_L \frac{dL}{c}$$

- In terms of distance, after multiplying by c:

$$cdt = \int_S ndS - \int_L dL = \boxed{\int_L (n-1)dL} + \boxed{\left(\int_S ndS - \int_L ndL\right)}$$

Change of refractive delay along path length

Change of path length

Tropospheric refraction

- Total tropospheric delay ΔL in terms of the equivalent increase in path length $(n(l)$ = index of refraction, Fermat's principle):

$$\Delta L = \int_{pathL} \left[n(l) - 1 \right] dl$$

- Refractivity N used instead of refraction n:

$$N = (n-1) \times 10^6$$

- Refractivity N is a function of temperature T, partial pressure of dry air P_d, and partial pressure of water vapor e (k_1, k_2, and k_3 are constants determined experimentally):

$$N = k_1 \frac{P_d}{T} + k_2 \frac{e}{T} + k_3 \frac{e}{T^2}$$

- The delay for a **zenith** path is the integral of the refractivity over altitude in the atmosphere:

$$\Delta L^{zen} = 10^{-6} \int N dz$$

$$\Delta L^{zen} = 10^{-6} \left[\int k_1 \frac{P_d}{T} + k_2 \frac{e}{T} + k_3 \frac{e}{T^2} dz \right]$$

Tropospheric refraction

It is convenient to consider separately the **hydrostatic delay** and the **wet delay**:

$$\Delta L^{zen} = \Delta L^{zen}_{hydro} + \Delta L^{zen}_{wet}$$

- **Hydrostatic or "dry" delay:**
  - Molecular constituents of the atmosphere in hydrostatic equilibrium.
  - **Can be modeled** with a simple dependence on surface pressure (P_0 = surface pressure in mbar, λ = latitude, H = height above the ellipsoid)

$$\Delta L^{zen}_{hydro} = \left(2.2768 \pm 0.0024 \times 10^{-7}\right)\frac{P_0}{f(\lambda,H)} \qquad f(\lambda,H) = 1 - 0.00266\cos(2\lambda) - 0.00028H$$

  - Standard deviation of current modeled estimates of this delay ~0.5 mm.

- **Non-hydrostatic or "wet" delay:**
  - Associated with water vapor that is not in hydrostatic equilibrium.
  - **Very difficult to model** because the quantity of atmospheric water vapor is highly variable in space and time:

$$\Delta L^{zen}_{wet} = 10^{-6}\left[\left(k_2 - \frac{M_w}{M_d}k_1\right)\int \frac{e}{T}\,dz + k_3\int \frac{e}{T^2}\,dz\right]$$

(M_w and M_d = molar masses of dry air and water vapor)

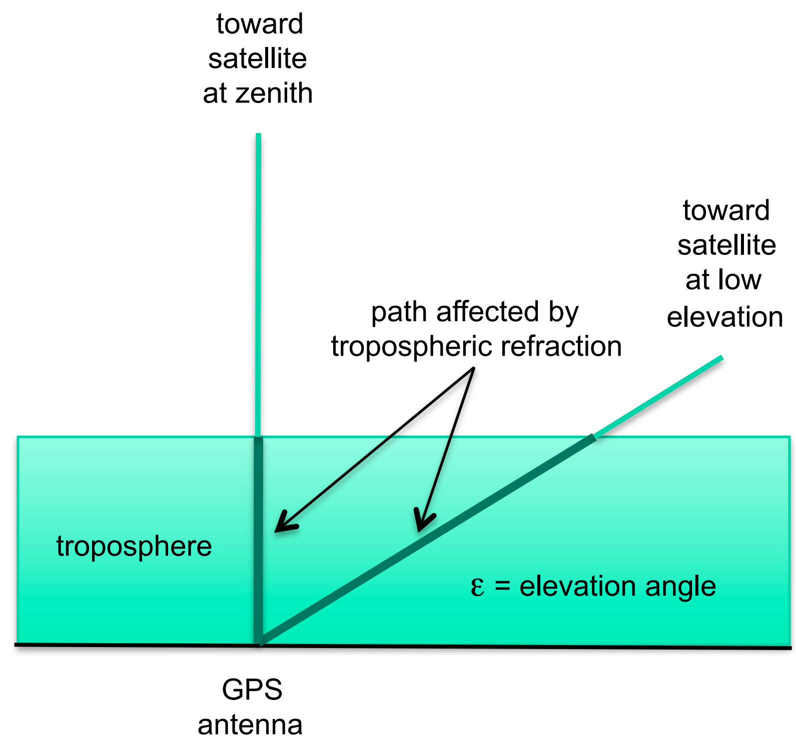
  - Standard deviation of current modeled estimates of this delay ~2 cm.

Tropospheric refraction

- Range error:
  - Hydrostatic delay ~ 200 to 230 cm at zenith at sea level
  - Wet delay typically 30 cm at zenith at sea level

- Tropospheric delays increase with decreasing satellite elevation angle

- This increase in delay as a function of elevation angle must be accounted for: mapping functions

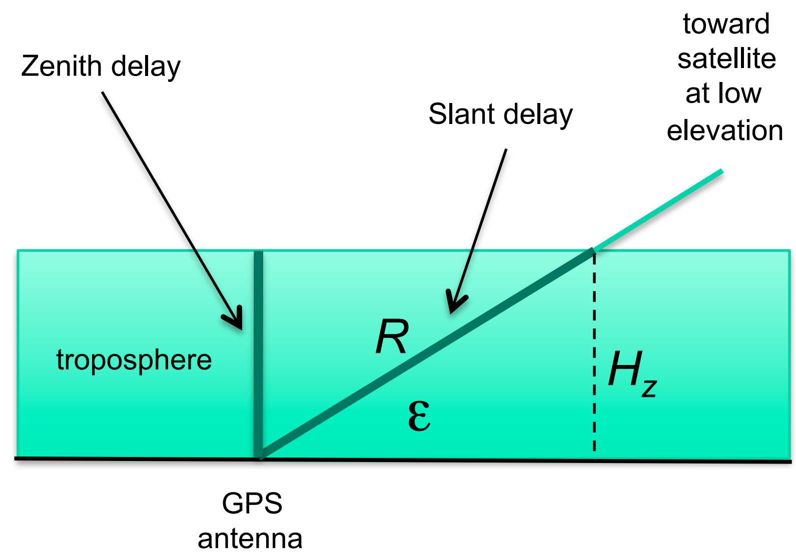


Tropospheric mapping functions

- For a flat homogeneous atmosphere:
  - Measurement includes for slant delay
  - Many slant delays at a given time => many unknowns
  - To reduce number of unknowns: project all slant delays onto zenith => one single zenith delay
- From diagram to the right: $\sin\varepsilon = \dfrac{H_z}{R}$
- Proportionnality factor between slant and zenith delay is:

$$\frac{R}{H_z} = \frac{1}{\sin\varepsilon} = m(\varepsilon)$$

- $m(\varepsilon)$ = mapping function, one for dry and one for wet delays



$$\Delta L_{tropo} = m_h(\varepsilon)\Delta L_{hydro}^{zen} + m_w(\varepsilon)\Delta L_{wet}^{zen}$$

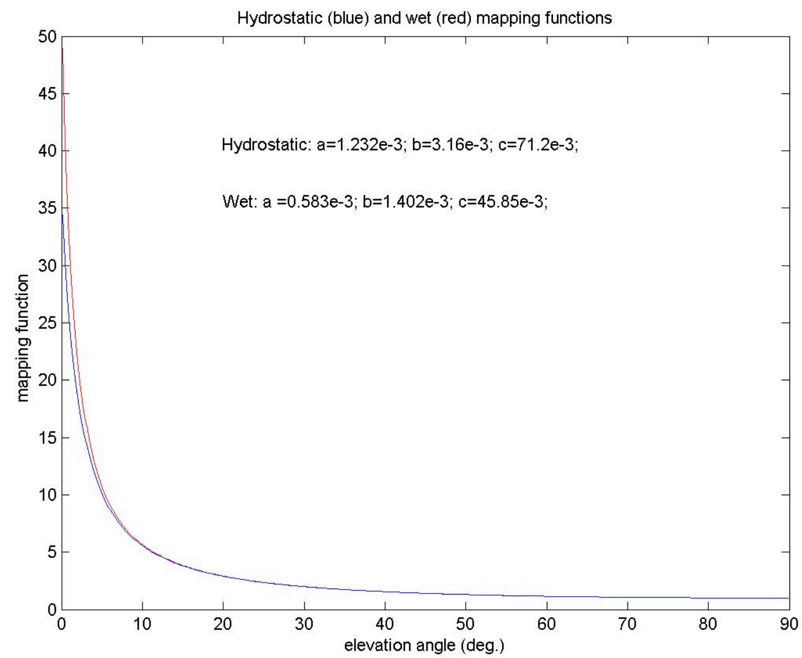
Tropospheric mapping functions

- For a spherically symmetric atmosphere, the 1/sin(ε) term is "tempered" by curvature effects:

$$m(\varepsilon) = \frac{1 + \cfrac{a}{a + \cfrac{b}{1 + c}}}{\sin(\varepsilon) + \cfrac{a}{\sin(\varepsilon) + \cfrac{b}{\sin(\varepsilon) + c}}}$$

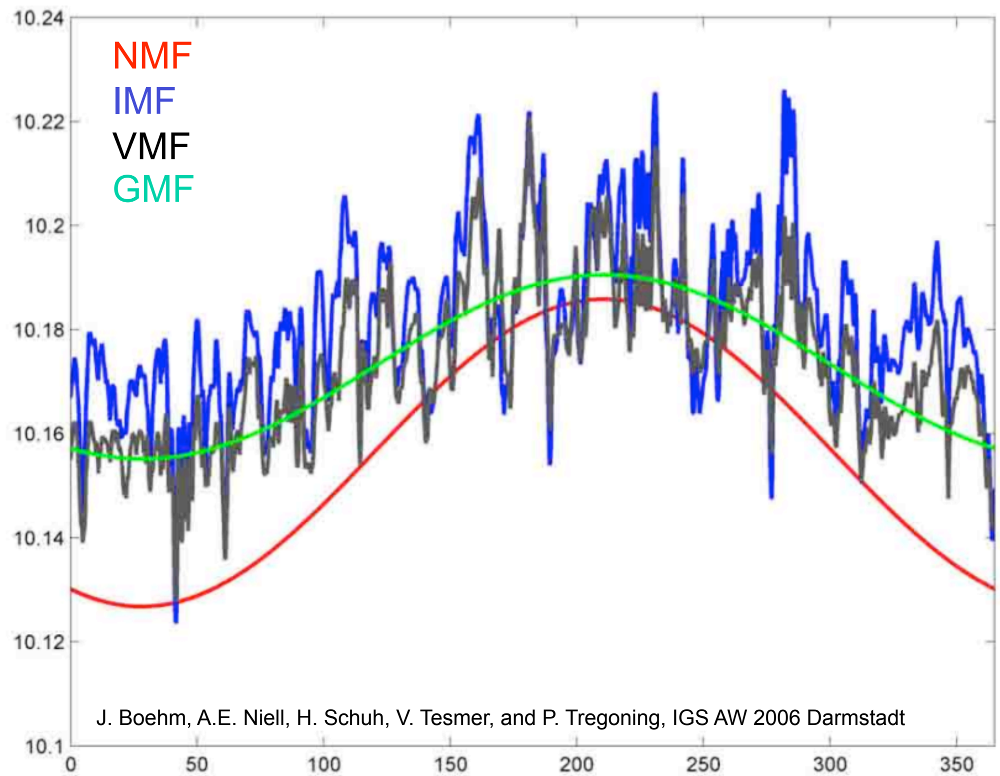
$$m(\varepsilon) = 1 \quad when \quad \varepsilon = 90$$

- Several different parameterizations have been proposed:
  - Marini (original one): a, b, c constant
  - Niell mapping function uses a, b, c that are latitude, height and time of year dependent.



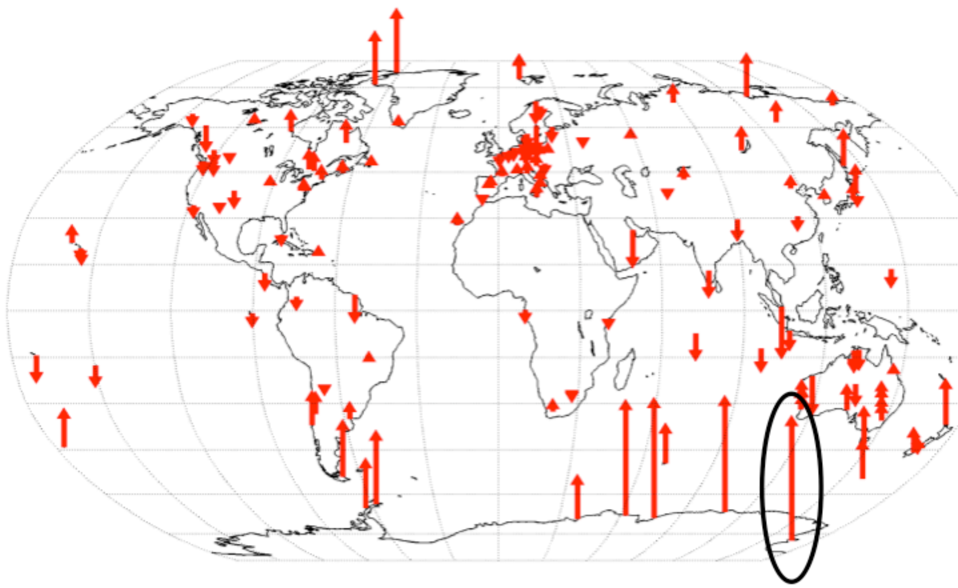
Tropospheric mapping functions

- Tropospheric delay is not homogeneous vertically: constantly varies with latitude, longitude, time
- Niell mapping functions (**NMF**; Niell, 1996): latitude and time-of-year dependence
- Isobaric mapping functions (**IMF**; Niell, 2001): derived from numerical weather model.
- Vienna mapping functions (**VMF1**; Boehm et al., 2006): derived at 6-hour intervals by ray-tracing across numerical weather models, highest accuracy
- Global mapping functions (**GMF**; Boehm et al., 2006): average VMF using spherical harmonics (degree 9 order 9)

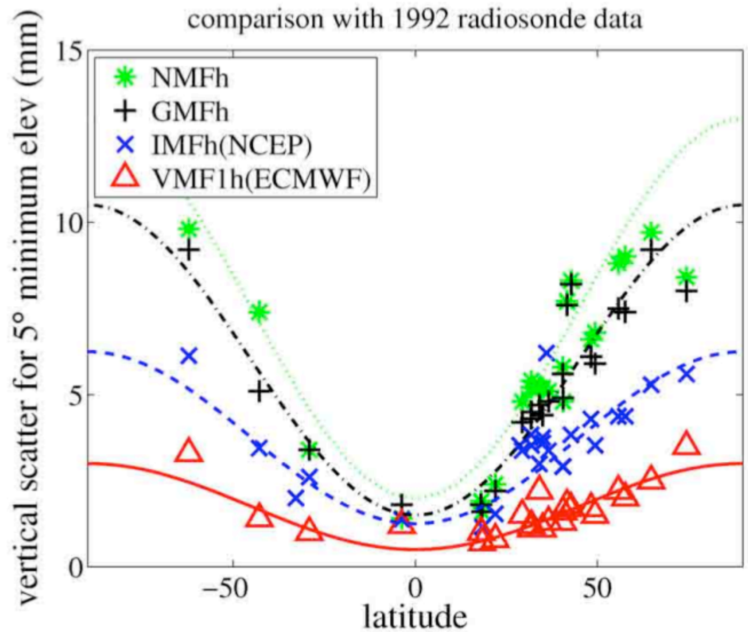


Hydrostatic mapping function at 5° elevation at O‘Higgins in 2005

Tropospheric mapping functions



Difference between GPS height estimates using VMF1 and NMF mapping functions



Scatter in GPS height estimates as a function of the hydrostatic mapping function used

ftp://igscb.jpl.nasa.gov/pub/resource/pubs/06_darmstadt/IGS%20Presentations%20PDF/11_8_Boehm.pdf

Tropospheric refraction

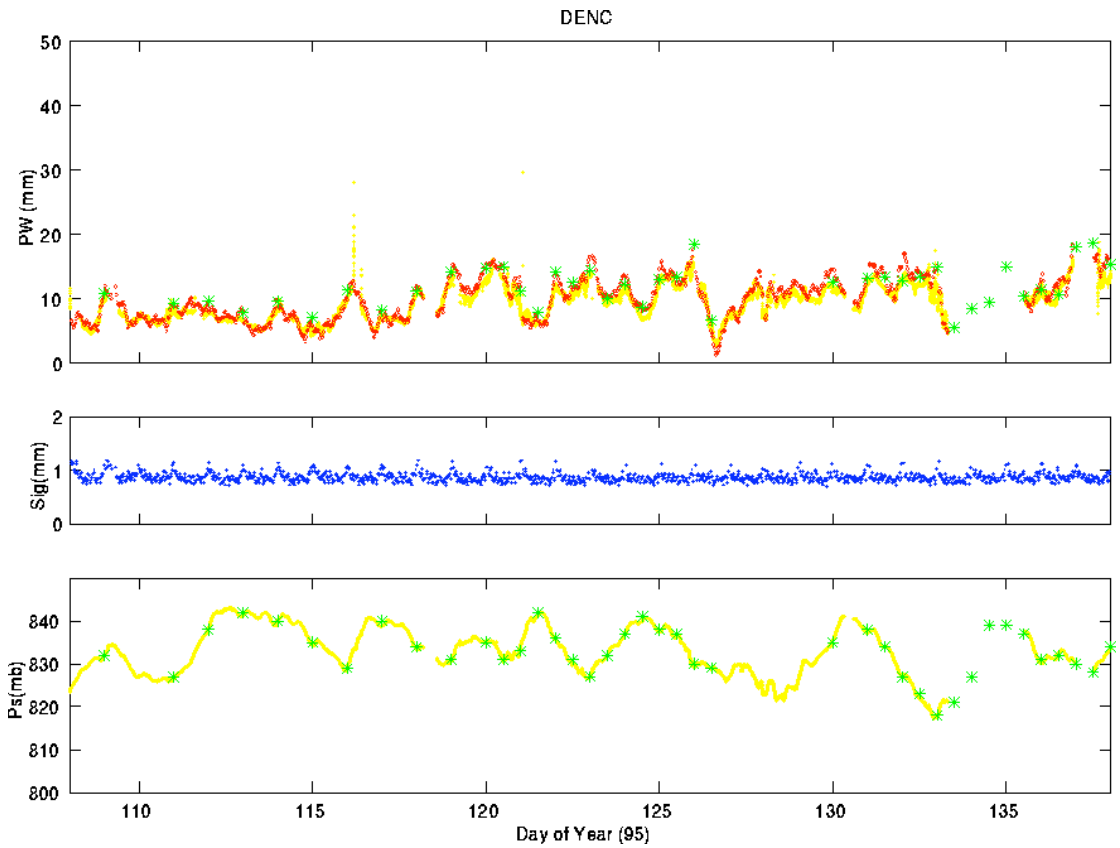
- How to handle the range error introduced by tropospheric refraction?
  - Correct: using a priori knowledge of the zenith delay (total or wet) from met. model, WVR, radiosonde (not from surface met data…)
  - Filter:…?
  - Model: ok for dry delay, not for wet…
  - Estimate:
    - → Introduce an additional unknown = zenith total delay
    - → Solve for it together with station position and time offset
    - → Even better: also estimate lateral gradients because of deviations from spherical symmetry
- If tropospheric delay is estimated, then GPS is also an atmospheric remote sensing tool!

“GPS meteorology”

- GPS data can be used to estimate Zenith Total Delay (ZTD)
- ZTD can be converted to ZWD by removing hydrostatic component if ground pressure is known
- ZWD is related to (integrated) Precipitable Water Vapor (PWV) by:

$$PWV = \Pi(T_m)\Delta L_{wet}^{zen}$$

- P is a function of the mean surface temperature, ~0.15.
- Trade-off between (vertical) position and ZTD



Red: GPS estimates
Yellow: water vapor radiometer measurements
Green stars: radiosonde measurements

Tropospheric refraction – summary

- Atmospheric delays are one of the limiting error sources in GPS positioning
- Delays are nearly always estimated:
  - Using accurate mapping functions is key
  - At low elevation angles there can be problems with mapping functions…
  - … therefore cutoff angle has impact on position.
  - Lateral inhomogeneity of atmospheric delays still unsolved problem even with gradient estimates.
  - Estimated delays used for weather forecast (if latency <2 hrs).

Ionospheric refraction

- The ionospheric index of refraction is a function of the wave frequency f and of the plasma resonant frequency f_p of the ionosphere. It is slightly different from unity and can be approximated (neglecting higher order terms in f) by:

$$n_{ion} = 1 - f_p^2 / 2f^2$$

- The plasma frequency f_p has typical values between 10-20 MHz and represents the characteristic vibration frequency between the ionosphere and electromagnetic signals.

- The GPS carrier frequencies have been chosen to minimize attenuation by taking f_1 and $f_2 >> fp$. Since:

$$f_p^2 = N(z) q_e^2 / \pi m_e$$

where $N(z)$ is the electron density (a function of the altitude z), and and m_e are the electron charge and mass respectively, n_{ion} can be written as:

$$n(z) = 1 - \frac{N(z) q_e^2}{2\pi m_e f^2}$$

Ionospheric refraction

- The total propagation time at velocity $v(z)=c/n(z)$, where c is the speed of light in vacuum, is:

$$T(f,z) = \int_{rec}^{sat} \frac{dz}{v(f,z)} = \int_{rec}^{sat} \frac{n(z)}{c} dz = \int_{rec}^{sat} \frac{dz}{c} - \int_{rec}^{sat} \frac{N(z) q_e^2}{2\pi m_e f^2 c} dz$$

- Substituting in previous equations and replacing q_e and m_e by their numerical values, we obtain, for a given frequency f:

$$\Delta t(f,z) = \int_{rec}^{sat} \frac{N(z) q_e^2}{2\pi m_e f^2 c} dz = \frac{A}{cf^2} \int_{rec}^{sat} N(z) dz = \frac{A}{cf^2} IEC$$

with the constant $A = 40.3\ m^3.s^{-2}$. IEC is the Integrated Electron Content along the line-of-sight between the satellite and the receiver.

- In other words, the ionospheric delay is proportional to the electron density along the GPS ray path.

Ionospheric refraction

- The ionospheric delay is given by:

$$I_1 = \frac{A}{cf_1^2} IEC$$

$$I_2 = \frac{A}{cf_2^2} IEC$$

- Note that: $$I_2 - I_1 = \frac{A(f_1^2 - f_2^2)}{f_1^2 f_2^2} IEC$$

- And: $$\frac{I_1}{I_2} = \frac{f_2^2}{f_1^2}$$

Ionospheric refraction

- The phase equations can be written as:

$$\varphi_1 = \frac{f_1}{c}\rho + f_1\Delta t + f_1 I_1 + f_1 T + N_1$$

$$\varphi_2 = \frac{f_2}{c}\rho + f_2\Delta t + f_2 I_2 + f_2 T + N_2$$

- Let us write the following linear combination:

$$\varphi_{LC} = \frac{f_1^2}{f_1^2 - f_2^2}\varphi_1 - \frac{f_1\, f_2}{f_1^2 - f_2^2{}_2}\,\varphi_2 \Rightarrow \varphi_{LC} = \frac{f_1^2 f_1}{f_1^2 - f_2^2} I_1 - \frac{f_1\, f_2 f_2}{f_1^2 - f_2^2} I_2 + \ldots$$

$$\Leftrightarrow \varphi_{LC} = \frac{f_1^2 f_1}{f_1^2 - f_2^2}\frac{f_2^2}{f_1^2} I_2 - \frac{f_1\, f_2 f_2}{f_1^2 - f_2^2} I_2 + \ldots$$

Recall that: $\frac{I_1}{I_2} = \frac{f_2^2}{f_1^2}$

$$\Leftrightarrow \varphi_{LC} = \underbrace{\frac{f_1\, f_2^2}{f_1^2 - f_2^2} I_2 - \frac{f_1\, f_2^2}{f_1^2 - f_2^2} I_2}_{=0} + \ldots$$

Ionospheric refraction

- Therefore ionospheric delay cancels out in φ_{LC} …
- We have a new observable φ_{LC}:

$$\varphi_{LC} = \frac{f_1^{\,2}}{f_1^{\,2} - f_2^{\,2}}\varphi_1 - \frac{f_1\, f_2}{f_1^{\,2} - f_2^{\,2}}{}_2\, \varphi_2$$

$$\Rightarrow \varphi_{LC} = 2.546 \times \varphi_1 - 1.984 \times \varphi_2$$

  - Linear combination of L1 and L2 phase observables
  - Independent of the ionospheric delay
  - Unfortunately φ_{LC} is ~3 times noisier than L1 or L2

See also: http://www.gmat.unsw.edu.au/snap/gps/gps_survey/chap6/642.htm

Ionospheric refraction

- Dual-frequency receivers:
  - Ionosphere-free observable φ_{LC} can be formed
  - Ionospheric propagation delays cancel
  - Note that ambiguities are not integers anymore
  - Note that model corrects for first-order only

- Single-frequency receivers:
  - Broadcast message:
    - Contains ionospheric model data: 8 coefficients for computing the group (pseudorange) delay
    - Efficiency: 50-60% of the delay is corrected
  - Differential corrections.

Ionospheric refraction

- From the phase equations, one can write:

$$\varphi_2 - \frac{f_2}{f_1}\varphi_1 = \frac{f_2}{c}(I_{2,\varphi} - I_{1,\varphi}) \qquad (+N)$$

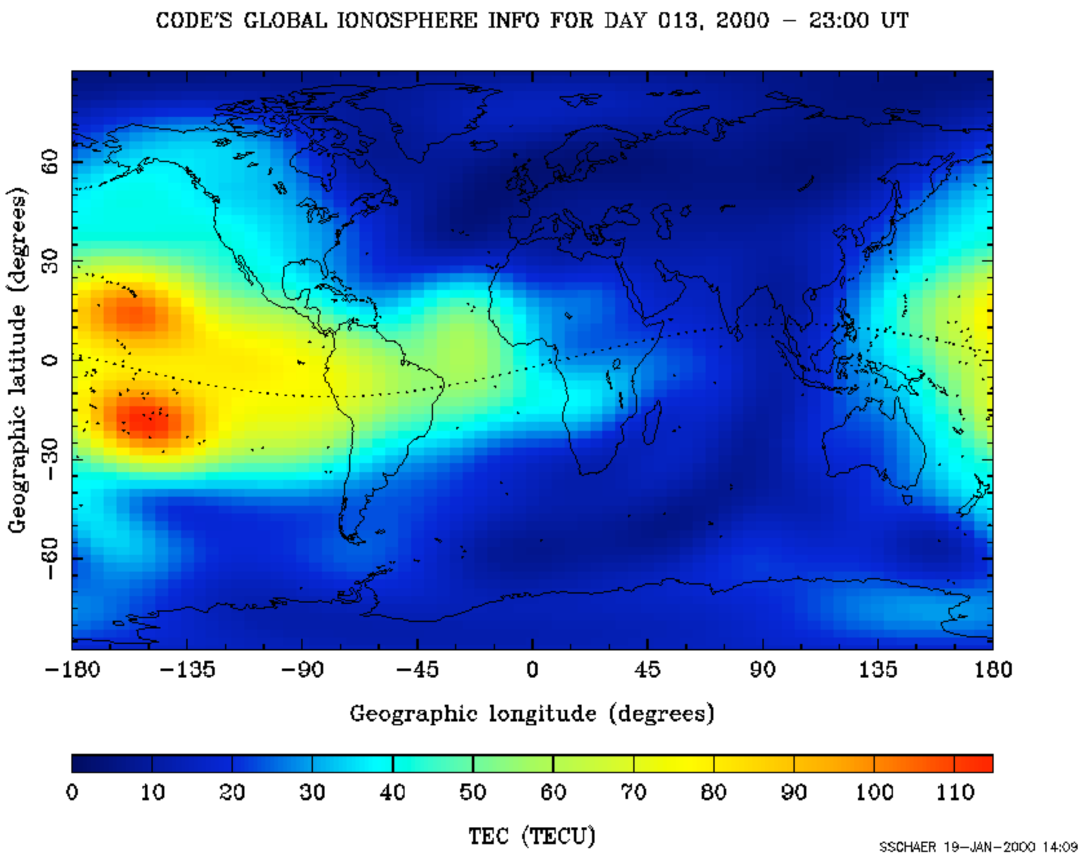
- We can plug this in the relationship between differential ionospheric delay and IEC and get:

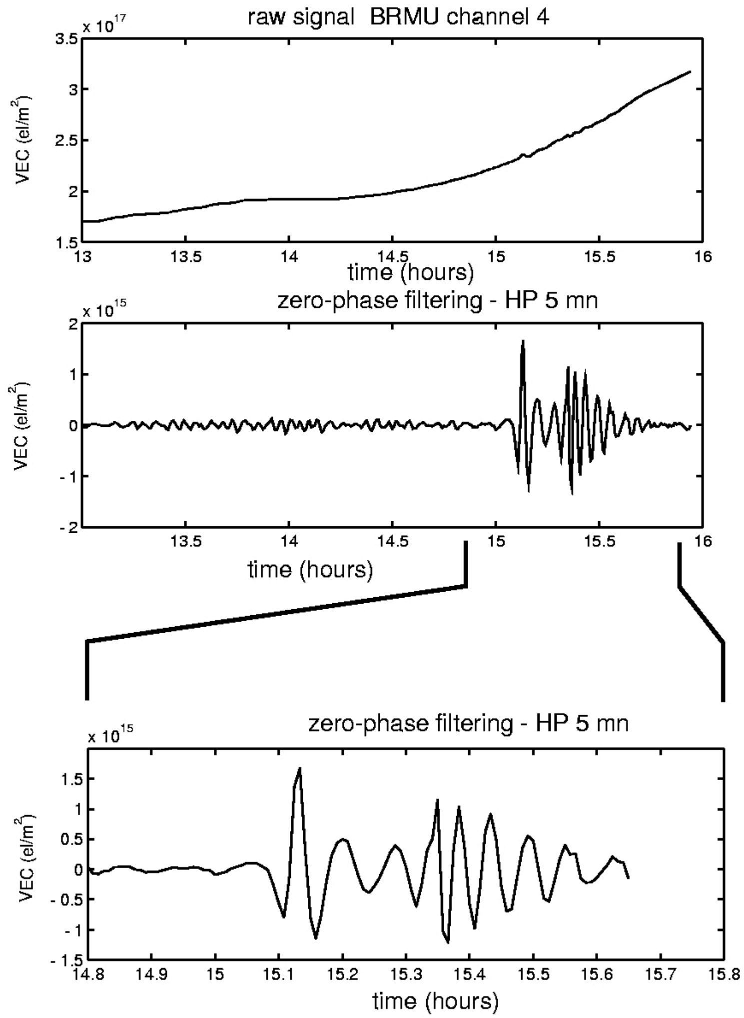
$$\varphi_2 - \frac{f_2}{f_1}\varphi_1 = \frac{f_2}{c}\frac{A(f_1^2 - f_2^2)}{f_1^2 f_2^2} IEC$$

$$\Rightarrow IEC = \underbrace{\left(\varphi_2 - \frac{f_2}{f_1}\varphi_1\right)}_{L_G} \times \frac{cf_1^2 f_2}{A(f_1^2 - f_2^2)}$$

- We can solve for IEC using GPS data (note N…).

Ionospheric refraction

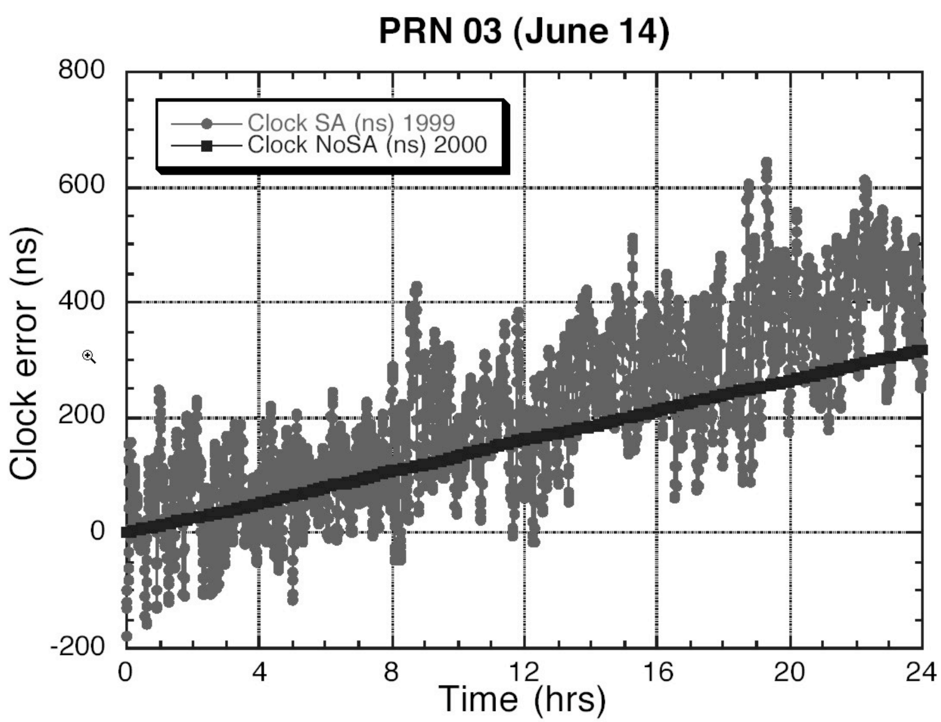


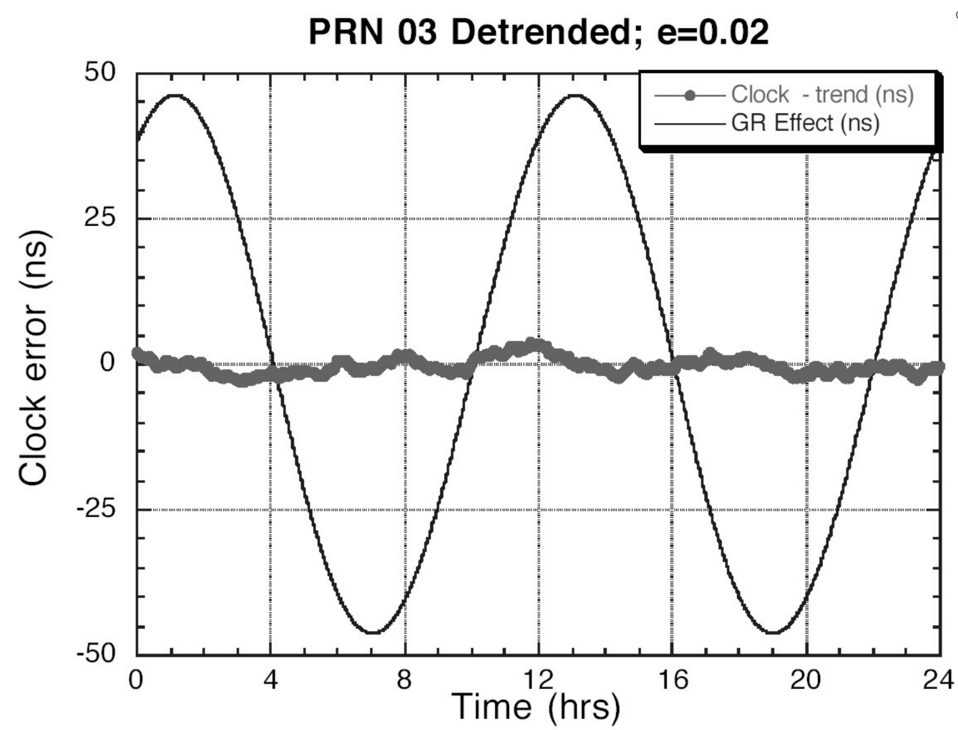


GPS clock errors

- GPS satellites move at about 1 km/sec => 1 msec time error results in 1 m range error :
  - For pseudo-range positioning, 1 msec errors OK.
  - For phase positioning (1 mm), time accuracy needed to 1 msec.
- 1 msec ~ 300 m of range => pseudorange accuracy of a few meters is sufficient for a time accuracy of 1 msec.

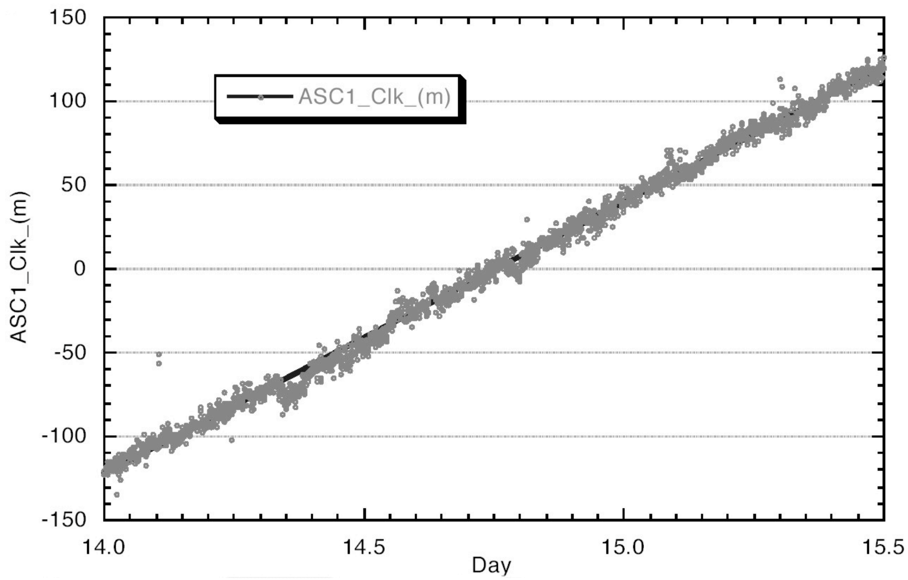
Satellite clock errors

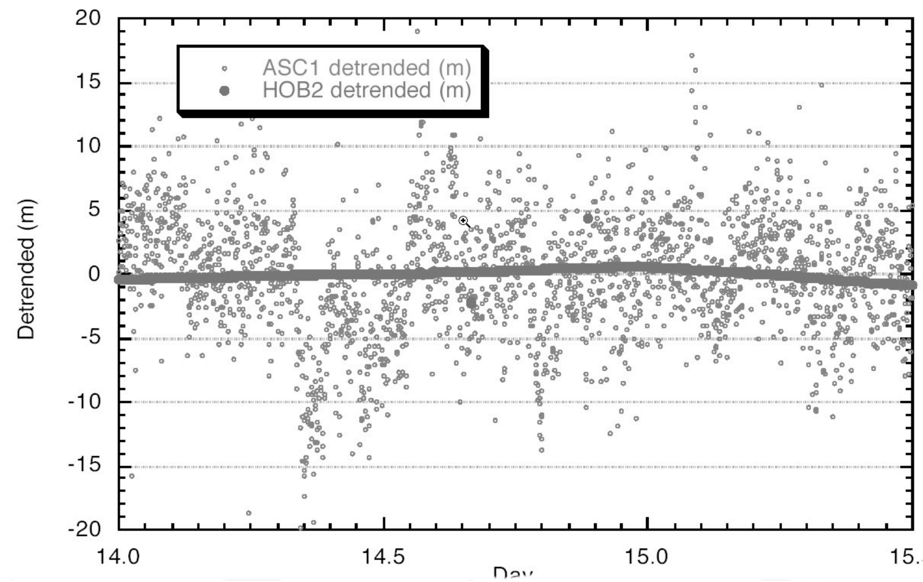




- Under selective availability (S/A) => ~200 ns (60 m)
- Currently ~5 ns = 1.5 m
- IGS orbits contains precise satellite clock corrections

Receiver clock errors





- Can reach kilometers…
- Sometimes well-behaved ⇒ can be modeled using linear polynomials.
- Usually not the case…
- Estimate receiver clocks at every measurement epoch (can be tricky with bad clocks)
- Cancelled clock errors using a “trick”: double differencing

Double differences

- Combination of phase observables between 2 sats (k,l) and 2 rcvs (i,j):

$$\Phi_{ij}^{kl} = (\Phi_i^k - \Phi_i^l) - (\Phi_j^k - \Phi_j^l)$$

$$\Rightarrow \Phi_{ij}^{kl} = (\rho_i^k - \rho_i^l + \rho_j^k - \rho_j^l)*f/c - (h^k - h_i - h^l + h_i - h^k + h_j + h^l - h_j) - (N_i^k - N_i^l + N_j^k - N_j^l)$$

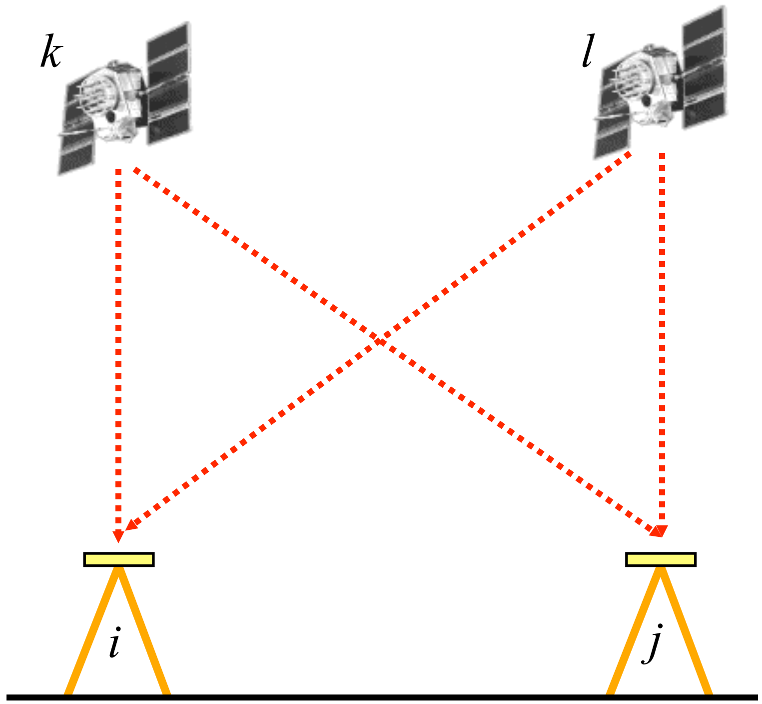
$$\Rightarrow \Phi_{ij}^{kl} = (\rho_i^k - \rho_i^l + \rho_j^k - \rho_j^l)*f/c - N_{ij}^{kl}$$

$\Rightarrow$ Clock errors $h_s(t)$ et $h_r(t)$ eliminated (but number of observations has decreased)

$\Rightarrow$ Any error common to receivers i and j will also cancel…!

– Atmospheric propagation errors cancel if receivers close enough to each other.

– Therefore, short baselines provide greater precision than long ones.



Antenna phase center

Ashtech 700936 mit Radom

Dorne Margolin T (JPL)

Leica SR399

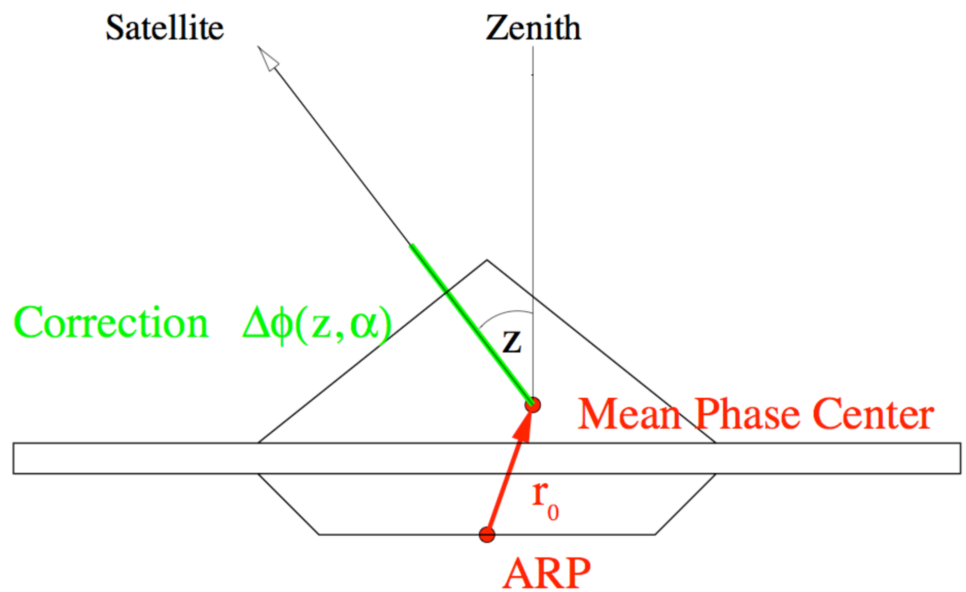
Trimble 22020 (Compact L1/L2)

GPS antennas are very diverse: shapes, radomes, etc.

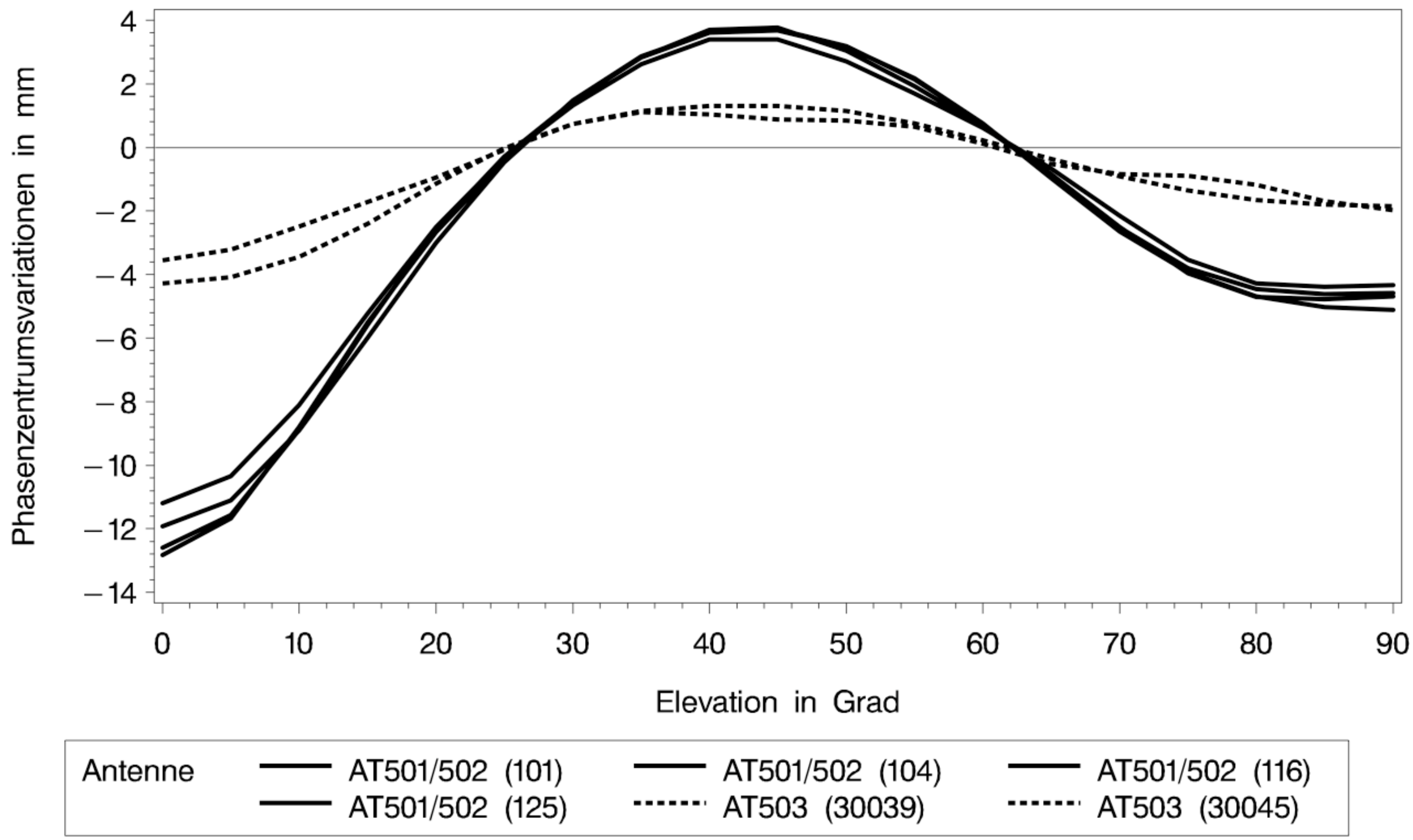
Antenna phase center

- Antenna phase center:
  - Point where the radio signal measurement is referred to.
  - Does not coincide with geometric antenna center.
  - Varies with direction and elevation of incoming signal.
- No direct access to the antenna phase center:
  - We setup the antenna using its Antenna Reference Point = ARP.
  - Need to correct for offset between ARP and phase center (1-2 cm).
- Corrections must accounted for:
  - Mean phase center offset to
  - Elevation- and azimuth-dependent variations of the phase center
- Provided by IGS:

ftp://igscb.jpl.nasa.gov/igscb/station/general/igs_01.pcv



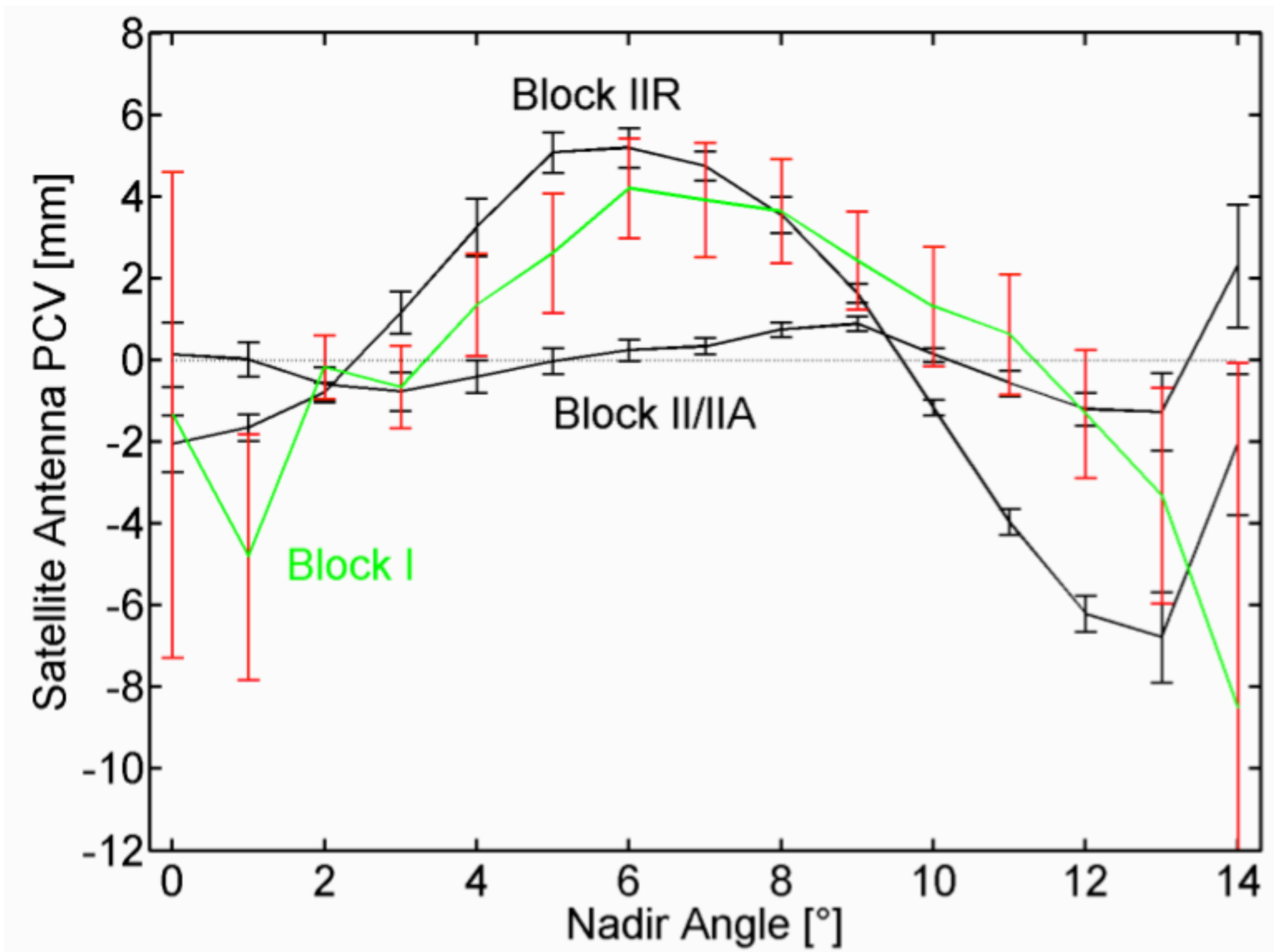
Antenna phase center



Example of two different Leica antennas

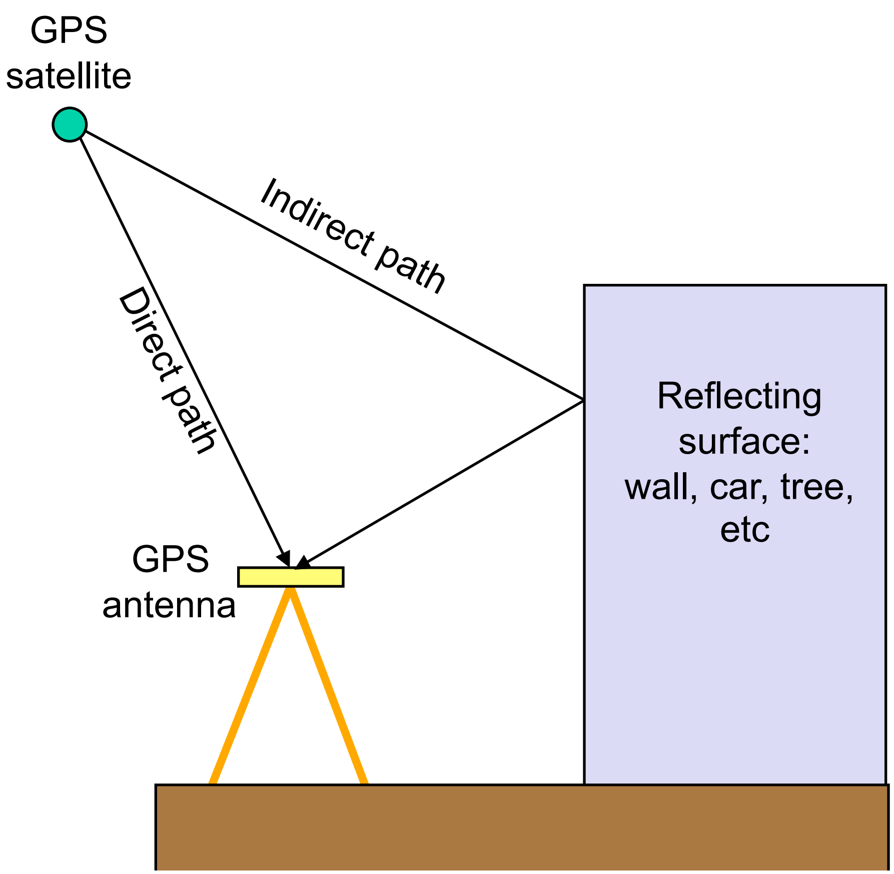
(from Rotacher)

Satellite phase center



Multipath

- GPS signal may be reflected by surfaces near the receiver => superposition of direct and reflected signals
- Multipath errors:
  - Code measurements: up to 50 m
  - Phase measurements: up to 5 cm
- Multipath repeats daily because of repeat time of GPS constellation: can be used to filter it out.
- Most critical at low elecation and for short observation sessions
- Mitigation:
  - Antenna design (choke ring)
  - Site selection (free horizon)
  - Long observation sessions (averaging)



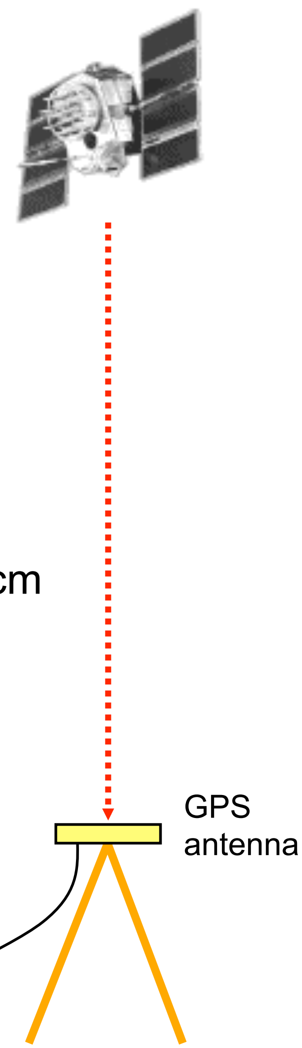
Error budget

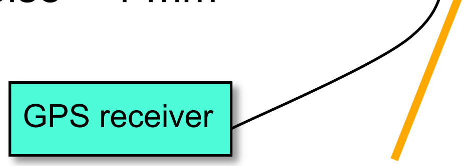
- SV clock ~ 1 m
- SV ephemeris ~ 1 m
- S/A ~ 100 m

- Troposphere ~ 1 m
- Ionosphere ~ 5 m

- Phase center variations ~ 1 cm
- Multipath ~ 0.5 m

- Pseudorange noise ~ 1 m
- Phase noise < 1 mm





- **Satellite:**
  - Clocks
  - Orbits
- **Signal propagation:**
  - Ionospheric refraction
  - Tropospheric refraction
- **Receiver/antenna:**
  - Ant. phase center variations
  - Multipath
  - Clock
  - Electronic noise
  - Operator errors: up to several km…
- **User Equivalent Range Error:**
  - UERE ~ 11 m if SA on
  - UERE ~ 5 m if SA off
- **In terms of position:**
  - Standard deviation = UERE x DOP
  - SA on: HDOP = 5 => $\sigma_{e,n}$ = 55 m
  - SA off: HDOP = 5 => $\sigma_{e,n}$ = 25 m
- **Dominant error sources:**
  - S/A
  - Ionospheric refraction