GPS Signal Propagation

Tropospheric refraction
Ionospheric refraction
Clock errors
Antenna phase center biases

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[Map showing geographic distribution]
GPS signal propagation

- GPS signal (= carrier phase modulated by satellite PRN code) sent by satellite.

- About 66 msec (20,000 km) later signal arrives at GPS receiver, which:
  - Decodes propagation time by correlating incoming signal with internal replica of the code.
  - Counts carrier phases.

- Resulting observables:
  - Propagation $\times c =$ pseudorange.
  - Carrier phase count.

- During propagation, signal passes through:
  - Ionosphere (10-100 m of delay)
  - Neutral atmosphere (2.3-30 m delay, depending on elevation angle).

- To estimate an accurate position from range data, one needs to account for all these propagation effects and time offsets.
GPS signal propagation

- L1 and L2 frequencies are affected by \textit{atmospheric refraction}:
  - Ray bending (negligible)
  - Propagation velocity decrease (w.r.t. vacuum) \Rightarrow propagation delay

- In the \textit{troposphere}:
  - Delay is a function of (P, T, H), 1 to 5 m
  - Largest effect due to pressure

- In the \textit{ionosphere}: delay function of the electronic density, 0 to 50 m

- This \textit{refractive delay} biases the satellite-receiver range measurements, and, consequently the estimated positions: essentially in the vertical.
GPS signal propagation

- Velocity of electromagnetic waves:
  - In a vacuum = \( c \)
  - In the atmosphere = \( v \) (with \( v < c \))
  - Dimensionless ratio \( n = \frac{c}{v} \) = refractive index

- Consequently, GPS signals in the atmosphere experience a delay compared to propagation in a vacuum.

- This delay is the difference between the actual path of the carrier \( S \) and the straight-line path \( L \) in a vacuum:
  \[
  dt = \int_S \frac{dS}{v} - \int_L \frac{dL}{c}
  \]

- In terms of distance, after multiplying by \( c \):
  \[
  cdt = \int_S ndS - \int_L dL = \int_L (n - 1)dL + \left( \int_S ndS - \int_L ndL \right)
  \]

  Change of refractive delay along path length  Change of path length
Tropospheric refraction

- Total tropospheric delay $\Delta L$ in terms of the equivalent increase in path length ($n(l) = \text{index of refraction, Fermat’s principle}$):

$$\Delta L = \int_{\text{pathL}} [n(l) - 1]dl$$

- Refractivity $N$ used instead of refraction $n$:

$$N = (n - 1) \times 10^6$$

- Refractivity $N$ is a function of temperature $T$, partial pressure of dry air $P_d$, and partial pressure of water vapor $e$ ($k_1$, $k_2$, and $k_3$ are constants determined experimentally):

$$N = k_1 \frac{P_d}{T} + k_2 \frac{e}{T} + k_3 \frac{e}{T^2}$$

- The delay for a zenith path is the integral of the refractivity over altitude in the atmosphere:

$$\Delta L^{\text{zen}} = 10^{-6} \int N dz$$

$$\Delta L^{\text{zen}} = 10^{-6} \left[ \int k_1 \frac{P_d}{T} + k_2 \frac{e}{T} + k_3 \frac{e}{T^2} dz \right]$$
Tropospheric refraction

It is convenient to consider separately the **hydrostatic delay** and the **wet delay**:

\[
\Delta L_{\text{zen}} = \Delta L_{\text{hydro}} + \Delta L_{\text{wet}}
\]

**Hydrostatic or “dry” delay:**
- Molecular constituents of the atmosphere in hydrostatic equilibrium.
- **Can be modeled** with a simple dependence on surface pressure \(P_0 = \text{surface pressure in mbar}, \lambda = \text{latitude}, H = \text{height above the ellipsoid})

\[
\Delta L_{\text{hydro}}^{\text{zen}} = \left(2.2768 \pm 0.0024 \times 10^{-7}\right) \frac{P_0}{f(\lambda,H)}
\]

\[
f(\lambda,H) = 1 - 0.00266 \cos(2\lambda) - 0.00028 H
\]
- Standard deviation of current modeled estimates of this delay ~0.5 mm.

**Non-hydrostatic or “wet” delay:**
- Associated with water vapor that is not in hydrostatic equilibrium.
- **Very difficult to model** because the quantity of atmospheric water vapor is highly variable in space and time:

\[
\Delta L_{\text{wet}}^{\text{zen}} = 10^{-6} \left[ k_2 - \frac{M_w}{M_d} k_1 \right] \int \frac{e}{T} \, dz + k_3 \int \frac{e}{T^2} \, dz
\]

\((M_w \text{ and } M_d = \text{molar masses of dry air and water vapor})\)
- Standard deviation of current modeled estimates of this delay ~2 cm.
Tropospheric refraction

• Range error:
  – Hydrostatic delay ~ 200 to 230 cm at zenith at sea level
  – Wet delay typically 30 cm at zenith at sea level

• Tropospheric delays increase with decreasing satellite elevation angle

• This increase in delay as a function of elevation angle must be accounted for: mapping functions
Tropospheric mapping functions

- For a flat homogeneous atmosphere:
  - Measurement includes for slant delay
  - Many slant delays at a given time => many unknowns
  - To reduce number of unknowns: project all slant delays onto zenith => one single zenith delay

- From diagram to the right: \( \sin \varepsilon = \frac{H_z}{R} \)

- Proportionality factor between slant and zenith delay is:
  \[
  \frac{R}{H_z} = \frac{1}{\sin \varepsilon} = m(\varepsilon)
  \]

- \( m(\varepsilon) = \) mapping function, one for dry and one for wet delays

\[
\Delta L_{tropo} = m_h(\varepsilon)\Delta L_{zen}^{\text{hydro}} + m_w(\varepsilon)\Delta L_{zen}^{\text{wet}}
\]
Tropospheric mapping functions

- For a spherically symmetric atmosphere, the $1/\sin(\varepsilon)$ term is “tempered” by curvature effects:

$$m(\varepsilon) = \frac{1 + \frac{a}{b}}{a + \frac{1 + c}{1 + c}}$$

$$= \frac{\sin(\varepsilon) + \frac{a}{b}}{\sin(\varepsilon) + \frac{b}{\sin(\varepsilon) + c}}$$

$$m(\varepsilon) = 1 \quad \text{when} \quad \varepsilon = 90$$

- Several different parameterizations have been proposed:
  - Marini (original one): $a$, $b$, $c$ constant
  - Niell mapping function uses $a$, $b$, $c$ that are latitude, height and time of year dependent.
Tropospheric mapping functions

- Tropospheric delay is not homogeneous vertically: constantly varies with latitude, longitude, time
- Niell mapping functions (NMF; Niell, 1996): latitude and time-of-year dependence
- Isobaric mapping functions (IMF; Niell, 2001): derived from numerical weather model.
- Vienna mapping functions (VMF1; Boehm et al., 2006): derived at 6-hour intervals by ray-tracing across numerical weather models, highest accuracy
- Global mapping functions (GMF; Boehm et al., 2006): average VMF using spherical harmonics (degree 9 order 9)
Tropospheric mapping functions

Difference between GPS height estimates using VMF1 and NMF mapping functions

Scatter in GPS height estimates as a function of the hydrostatic mapping function used

Tropospheric refraction

• How to handle the range error introduced by tropospheric refraction?
  – **Correct**: using a priori knowledge of the zenith delay (total or wet) from met. model, WVR, radiosonde (not from surface met data…)
  – **Filter**:…?
  – **Model**: ok for dry delay, not for wet…
  – **Estimate**:
    → Introduce an additional unknown = zenith total delay
    → Solve for it together with station position and time offset
    → Even better: also estimate lateral gradients because of deviations from spherical symmetry

• If tropospheric delay is estimated, then GPS is also an atmospheric remote sensing tool!
“GPS meteorology”

- GPS data can be used to estimate Zenith Total Delay (ZTD)
- ZTD can be converted to ZWD by removing hydrostatic component if ground pressure is known
- ZWD is related to (integrated) Precipitable Water Vapor (PWV) by:

\[ PWV = \Pi(T_m)\Delta L_{\text{zen}}^{\text{wet}} \]

- \( P \) is a function of the mean surface temperature, \( \sim 0.15 \).
- Trade-off between (vertical) position and ZTD

Red: GPS estimates
Yellow: water vapor radiometer measurements
Green stars: radiosonde measurements
Tropospheric refraction – summary

- Atmospheric delays are one of the limiting error sources in GPS positioning
- Delays are nearly always estimated:
  - Using accurate mapping functions is key
  - At low elevation angles there can be problems with mapping functions…
  - … therefore cutoff angle has impact on position.
  - Lateral inhomogeneity of atmospheric delays still unsolved problem even with gradient estimates.
  - Estimated delays used for weather forecast (if latency <2 hrs).
Ionospheric refraction

- The ionospheric index of refraction is a function of the wave frequency $f$ and of the plasma resonant frequency $f_p$ of the ionosphere. It is slightly different from unity and can be approximated (neglecting higher order terms in $f$) by:

$$n_{ion} = 1 - \frac{f_p^2}{2f^2}$$

- The plasma frequency $f_p$ has typical values between 10-20 MHz and represents the characteristic vibration frequency between the ionosphere and electromagnetic signals.

- The GPS carrier frequencies have been chosen to minimize attenuation by taking $f_1$ and $f_2 >> f_p$. Since:

$$f_p^2 = \frac{N(z)q_e^2}{\pi m_e}$$

where $N(z)$ is the electron density (a function of the altitude $z$), and $q_e$ and $m_e$ are the electron charge and mass respectively, $n_{ion}$ can be written as:

$$n(z) = 1 - \frac{N(z)q_e^2}{2\pi m_e f^2}$$
Ionospheric refraction

- The total propagation time at velocity \( v(z) = c/n(z) \), where \( c \) is the speed of light in vacuum, is:

\[
T(f,z) = \int_{rec}^{sat} \frac{dz}{v(f,z)} = \int_{rec}^{sat} \frac{n(z)}{c} \, dz = \int_{rec}^{sat} \frac{dz}{c} - \int_{rec}^{sat} \frac{N(z)q_e^2}{2\pi m_e f^2 c} \, dz
\]

- Substituting in previous equations and replacing \( q_e \) and \( m_e \) by their numerical values, we obtain, for a given frequency \( f \):

\[
\Delta t(f,z) = \int_{rec}^{sat} \frac{N(z)q_e^2}{2\pi m_e f^2 c} \, dz = \frac{A}{cf^2} \int_{rec}^{sat} N(z) \, dz = \frac{A}{cf^2} IEC
\]

with the constant \( A = 40.3 \, m^3 \cdot s^{-2} \). \( IEC \) is the Integrated Electron Content along the line-of-sight between the satellite and the receiver.

- In other words, the ionospheric delay is proportional to the electron density along the GPS ray path.
Ionospheric refraction

- The ionospheric delay is given by:
  
  \[ I_1 = \frac{A}{cf_1^2} IEC \]
  \[ I_2 = \frac{A}{cf_2^2} IEC \]

- Note that:
  \[ I_2 - I_1 = \frac{A(f_1^2 - f_2^2)}{f_1^2 f_2^2} IEC \]

- And:
  \[ \frac{I_1}{I_2} = \frac{f_2^2}{f_1^2} \]
Ionospheric refraction

- The phase equations can be written as:

\[
\phi_1 = \frac{f_1}{c} \rho + f_1 \Delta t + f_1 I_1 + f_1 T + N_1
\]

\[
\phi_2 = \frac{f_2}{c} \rho + f_2 \Delta t + f_2 I_2 + f_2 T + N_2
\]

- Let us write the following linear combination:

\[
\phi_{LC} = \frac{f_1^2}{f_1^2 - f_2^2} \phi_1 - \frac{f_1 f_2}{f_1^2 - f_2^2} \phi_2 \implies \phi_{LC} = \frac{f_1^2 f_2}{f_1^2 - f_2^2} I_1 - \frac{f_1 f_2 f_2}{f_1^2 - f_2^2} I_2 + \ldots
\]

\[
\Leftrightarrow \phi_{LC} = \frac{f_1^2 f_1}{f_1^2 - f_2^2} I_2 - \frac{f_1 f_2 f_2}{f_1^2 - f_2^2} I_2 + \ldots
\]

\[
\Leftrightarrow \phi_{LC} = \frac{f_1 f_2^2}{f_1^2 - f_2^2} I_2 - \frac{f_1 f_2^2}{f_1^2 - f_2^2} I_2 + \ldots
\]

Recall that:

\[
\frac{I_1}{I_2} = \frac{f_2^2}{f_1^2}
\]

\[
= 0
\]
Ionospheric refraction

• Therefore ionospheric delay cancels out in $\varphi_{LC}$ …

• We have a new observable $\varphi_{LC}$:

$$\varphi_{LC} = \frac{f_1^2}{f_1^2 - f_2^2} \varphi_1 - \frac{f_1 f_2}{f_1^2 - f_2^2} \varphi_2$$

$\Rightarrow \varphi_{LC} = 2.546 \times \varphi_1 - 1.984 \times \varphi_2$

– Linear combination of L1 and L2 phase observables
– Independent of the ionospheric delay
– Unfortunately $\varphi_{LC}$ is ~3 times noisier than L1 or L2

Ionospheric refraction

- **Dual-frequency receivers:**
  - Ionosphere-free observable $\phi_{LC}$ can be formed
  - Ionospheric propagation delays cancel
  - Note that ambiguities are not integers anymore
  - Note that model corrects for first-order only

- **Single-frequency receivers:**
  - Broadcast message:
    - Contains ionospheric model data: 8 coefficients for computing the group (pseudorange) delay
    - Efficiency: 50-60% of the delay is corrected
  - Differential corrections.
Ionospheric refraction

- From the phase equations, one can write:
  \[ \varphi_2 - \frac{f_2}{f_1} \varphi_1 = \frac{f_2}{c} (I_{2\varphi} - I_{1\varphi}) \quad (+N) \]

- We can plug this in the relationship between differential ionospheric delay and IEC and get:
  \[ \varphi_2 - \frac{f_2}{f_1} \varphi_1 = \frac{f_2}{c} A(f_1^2 - f_2^2) IEC \]
  \[ \Rightarrow IEC = \left( \varphi_2 - \frac{f_2}{f_1} \varphi_1 \right) \times \frac{c f_1^2 f_2}{A(f_1^2 - f_2^2)} \]

- We can solve for IEC using GPS data (note \( N \ldots \)).
Ionospheric refraction
GPS clock errors

- GPS satellites move at about 1 km/sec => 1 msec time error results in 1 m range error:
  - For pseudo-range positioning, 1 msec errors OK.
  - For phase positioning (1 mm), time accuracy needed to 1 msec.

- 1 msec ~ 300 m of range => pseudorange accuracy of a few meters is sufficient for a time accuracy of 1 msec.
Satellite clock errors

- Under selective availability (S/A) => ~200 ns (60 m)
- Currently ~5 ns = 1.5 m
- IGS orbits contains precise satellite clock corrections
Receiver clock errors

- Can reach kilometers...
- Sometimes well-behaved $\Rightarrow$ can be modeled using linear polynomials.
- Usually not the case...
- Estimate receiver clocks at every measurement epoch (can be tricky with bad clocks)
- Cancelled clock errors using a “trick”: double differencing
Double differences

- Combination of phase observables between 2 sats \((k,l)\) and 2 rcvs \((i,j)\):
  \[
  \Phi_{ij}^{kl} = (\Phi_i^k - \Phi_i^l) - (\Phi_j^k - \Phi_j^l)
  \Rightarrow \Phi_{ij}^{kl} = (\rho_i^k - \rho_i^l + \rho_j^k - \rho_j^l)f/c(h^k - h^l + h_i - h^k + h_j - h^l - h^k - h^l) - (N_i^k - N_i^l + N_j^k - N_j^l)
  \Rightarrow \Phi_{ij}^{kl} = (\rho_i^k - \rho_i^l + \rho_j^k - \rho_j^l)f/cN_{ij}^{kl}
  \]

⇒ Clock errors \(h_s(t)\) et \(h_r(t)\) eliminated (but number of observations has decreased)

⇒ Any error common to receivers \(i\) and \(j\) will also cancel…!
  - Atmospheric propagation errors cancel if receivers close enough to each other.
  - Therefore, short baselines provide greater precision than long ones.
GPS antennas are very diverse: shapes, radomes, etc.
Antenna phase center

- **Antenna phase center:**
  - Point where the radio signal measurement is referred to.
  - Does not coincide with geometric antenna center.
  - Varies with direction and elevation of incoming signal.

- **No direct access to the antenna phase center:**
  - We setup the antenna using its Antenna Reference Point = ARP.
  - Need to correct for offset between ARP and phase center (1-2 cm).

- **Corrections must accounted for:**
  - Mean phase center offset to
  - Elevation- and azimuth-dependent variations of the phase center

- **Provided by IGS:**
  ftp://igscb.jpl.nasa.gov/igscb/station/general/igs_01.pcv
Antenna phase center

Example of two different Leica antennas
(from Rotacher)
Satellite phase center
Multipath

- GPS signal may be reflected by surfaces near the receiver => superposition of direct and reflected signals
- Multipath errors:
  - Code measurements: up to 50 m
  - Phase measurements: up to 5 cm
- Multipath repeats daily because of repeat time of GPS constellation: can be used to filter it out.
- Most critical at low elevation and for short observation sessions
- Mitigation:
  - Antenna design (choke ring)
  - Site selection (free horizon)
  - Long observation sessions (averaging)
Error budget

- **Satellite:**
  - Clocks
  - Orbits

- **Signal propagation:**
  - Ionospheric refraction
  - Tropospheric refraction

- **Receiver/antenna:**
  - Ant. phase center variations
  - Multipath
  - Clock
  - Electronic noise
  - Operator errors: up to several km…

- **User Equivalent Range Error:**
  - UERE ~ 11 m if SA on
  - UERE ~ 5 m if SA off

- **In terms of position:**
  - Standard deviation = UERE x DOP
  - SA on: HDOP = 5 => $\sigma_{e,n} = 55$ m
  - SA off: HDOP = 5 => $\sigma_{e,n} = 25$ m

- **Dominant error sources:**
  - S/A
  - Ionospheric refraction

- SV clock ~ 1 m
- SV ephemeris ~ 1 m
- S/A ~ 100 m

- Troposphere ~ 1 m
- Ionosphere ~ 5 m

- Phase center variations ~ 1 cm
- Multipath ~ 0.5 m

- Pseudorange noise ~ 1 m
- Phase noise < 1 mm