GPS Geodesy - LAB 2

Gravity measurements

Start by downloading and reading:
http://web.ics.purdue.edu/~ecalais/teaching/ce511_eas591/gravimeter.pdf

1. Starting from the equation for Earth's gravitational acceleration, derive the equation that gives the change of gravity with height (spherical non-rotating Earth).

2. The Civil Engineering Building is about 30 meters high (?). Calculate the expected change of gravity between the basement and the 4^{th} floor. Is this detectable with the type of gravimeter used for this lab? Justify your answer using the characteristics of the Lacoste&Romberg gravimeter.

3. Split in groups of *n* students, $2<n<5$. Each group will:

- Make *n* measurements of gravity in the CIVL building (each group should measure at a different elevation, each student in a given group should make an independent measurement so that we can have some statistics on the measurement precision). **Between each reading: turn gravimeter off, lock the arrestment knob, and take the gravimeter off its leveling plate.**
- Convert each gravity reading to mGals. Make sure you calculate an uncertainty as well (use the standard deviation of the 4 measurements).
- Estimate the height (w.r.t. basement) at which your measurement was made (e.g. measure the height of a stair step and multiply by the number of steps from basement to your measurement point). This does not need to be very precise.
- Provide your gravity value, associated uncertainty, and height of the measurement to the TA, who will compile the measurements for all groups.

[As one group makes their measurements, the rest of the class works on the rest of the lab problems. They switch with another group when they return from their measurements. IF YOUR GROUP DID NOT HAVE TIME TO COMPLETE THEIR MEASUREMENTS, SCHEDULE WITH THE TA A TIME WHEN YOU CAN BORROW THE GRAVIMETER DURING THE WEEK.]

At home, once the TA has complied all measurements in a table:

4. Plot on a graph the measured gravity and its uncertainty (error bars) as a function of height. For each height, plot the theoretical gravity value. Do the measurements agree with the theory? Comment.

5. In general, how do the measurements compare, in terms of precision, with the specifications given by the manufacturer? Comment.

<u>Deliverables:</u>

- Derivation for question 1

- Numerical value for question 2 and brief comment on gravimeter precision.
- Question 3, step 1: provide your data (incl. uncertainties) to the TA.
- Question 3, step 2: once the TA has compiled all the data, provide graph and brief comment.

Gravity on the reference ellipsoid:

1. From the equation for the potential of a rotating ellipsoid given in the lecture handouts, derive an expression for gravity at the surface of the reference ellipsoid (neglect the tangential term $\partial V / \partial \theta$). WARNING: potential has opposite sign in geodesy (cf lectures) and geophysics… Make sure you use the "geophysics" convention.

2. Using the GRS80 values recalled below, plot the value of gravity on the ellipsoid as a function of latitude (for 0<latitude<90) assuming:
   - A spherical non-rotating Earth
   - A spherical rotating Earth
   - An ellipsoidal rotating Earth

   GRS80 values:
   a = 6,378,137 m (semi-major axis)
   GM = 398,600.5 x 10^9 m^3s^{-2}
   $J2$ = 1,082.63 x 10^{-6}
   ω = 7,292,115 x 10^{-11} rad s^{-1}
   f = 1/298.257223563 (flattening)

3. Plot the data provided in columns 2 and 3 of the table below on top of the previous 3 curves.
   Complete the table.
   Comment on values of residual gravity (any physical reason why some are larger than others?)

Location	Lat.	Measured g ($m.s^{-2}$)	Spherical non-rotating Earth	Rotation component	Flattening component	Total gravity	Residual gravity
North Pole	90N	9.83245					
Reykjavik	64N	9.8227					
Paris	49N	9.8094					
Mid-Atlantic ocean	46N	9.804782					
Chamonix (French Alps)	46N	9.803329					
San Francisco	38N	9.7996					
Morovia	6N	9.7816					
Equator	0	9.7803					
Jakarta	6S	9.7818					
Melbourne	38S	9.7999					

Deliverables:

- Derivation for question 1
- One plot for question 2
- Completed table and short paragraph of comments on residual values.

Potential and Geoid:

Use http://www.ngs.noaa.gov/cgi-bin/GEOID_STUFF/geoid99_prompt1.prl

- Your GPS gives you an ellipsoidal height of 300 m in Indiana (latitude 40N, longitude 87W). What is the geoid height for that location? What is the height with respect to mean sea level?
- Same question for a 300 m ellipsoidal height reading on your GPS in northeastern Wyoming (latitude 45N, longitude 105W).

Deliverables: Numerical answers to the questions above.

Extra credit:

1. Let us approximate the Earth as a non-rotating homogeneous sphere. A 1 kg mass falls toward the Earth from a very large distance. What is its velocity as it crashes on the Earth's surface?

2. A geostationary (also called geosynchronous) satellite is a satellite located on an orbit such that it remains above the same point on the Earth's surface.
   - What is the angular velocity of such a satellite?
   - What is the altitude of such a satellite?