Reference Frames

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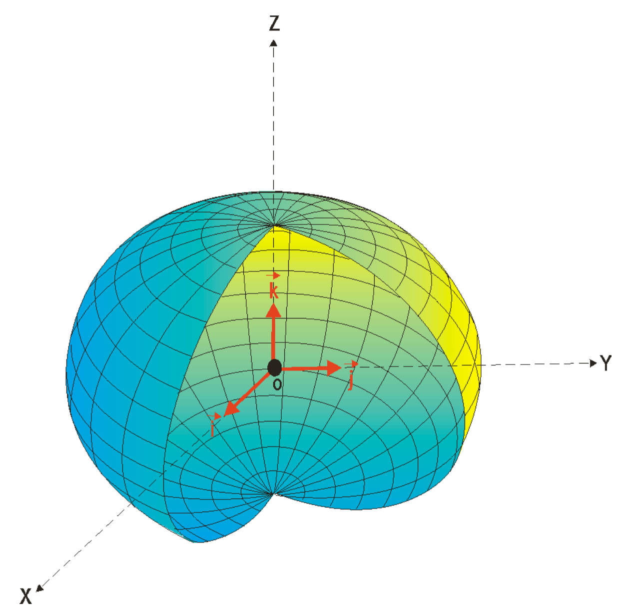
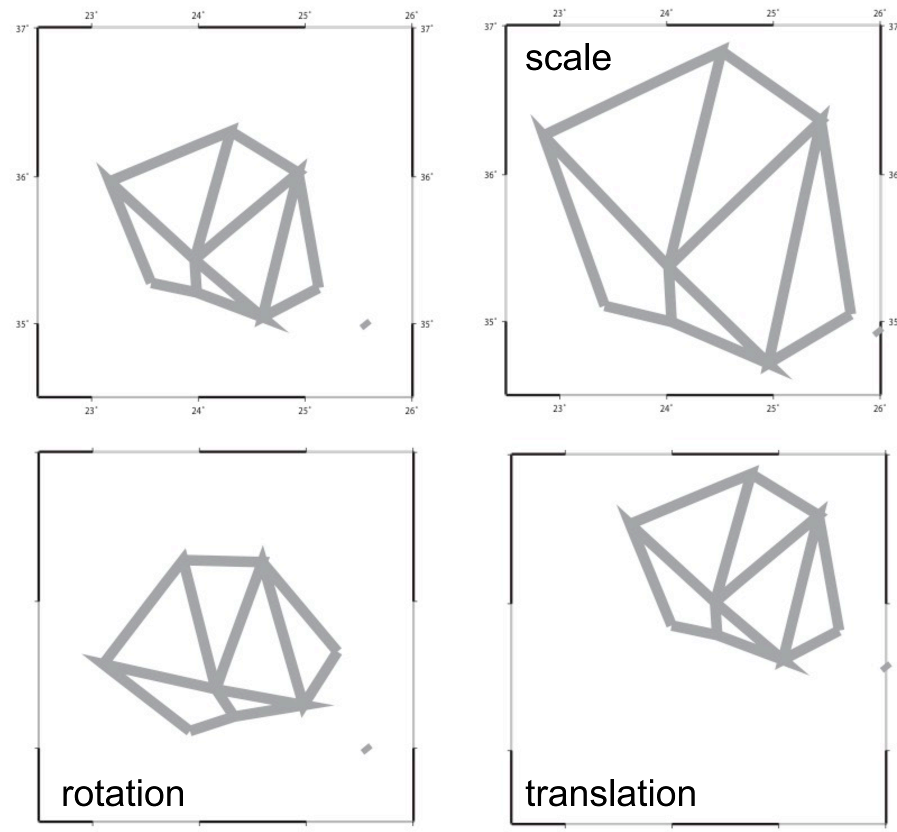


Need for a Reference Frame

1. Positions and velocities from geodetic measurements:
   - Are not direct observations, but estimated quantities
   - Are not absolute quantities
   - Need for a **"Terrestrial Reference"** in which (or relative to which) positions and velocities can be expressed.

2. Geodetic data are not sufficient by themselves to calculate coordinates…!
   - Ex. of triangulation data (angle measurements): origin, orientation, and scale need to be fixed
   - Ex. of distance measurements: origin and orientation need to be fixed, scale is given by the data
   - Need to fix some quantities => define a frame



4 equivalent figures derived from angle measurements

Mathematically: the Datum Defect problem

- Assume terrestrial measurements at 3 sites (in 3D):
  - 6 independent data:
    - 2 independent distance measurements
    - 2 independent angle measurements
    - 2 independent height difference measurements
  - 9 unknowns: *[X,Y,Z]* (or lat, lon, elev) at each site
- For 4 sites: 12 unknowns, 9 independent data

⇒ **Datum defect = rank deficiency of the matrix that relates the observations to the unknowns**

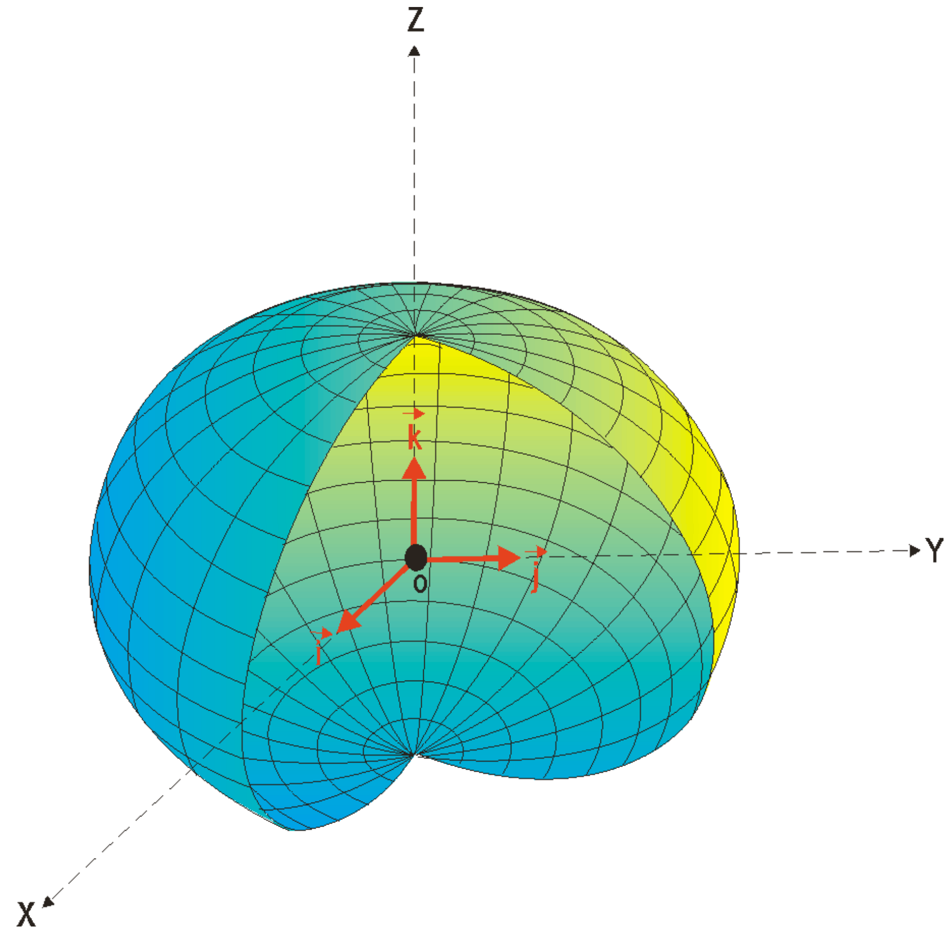
⇒ Solution: define a frame!
  - Fix or constrain a number of coordinates
  - Minimum 3 coordinates at 2 sites to determine scale, orientation, origin
  - A! *a priori* variance of site positions will impact the final uncertainties (*e.g.,* over-constraining typically results in artificially small uncertainties)

System vs. Frame

- Terrestrial Reference **System** (TRS):
  - Mathematical definition of the reference in which positions and velocities will be expressed.
  - Therefore invariable but “inaccessible” to users in practice.
- Terrestrial Reference **Frame** (TRF):
  - Physical materialization of the reference system by way of geodetic sites.
  - Therefore accessible but perfectible.

The ideal TRS

- Tri-dimensional right-handed orthogonal (X,Y,Z) Euclidian affine frame.
- Base vectors have same length = define the **scale**
- Geocentric: **origin** close to the Earth's center of mass (including oceans and atmosphere)
- Equatorial **orientation**: Z-axis is direction of the Earth's rotation axis
- Rotating with the Earth.



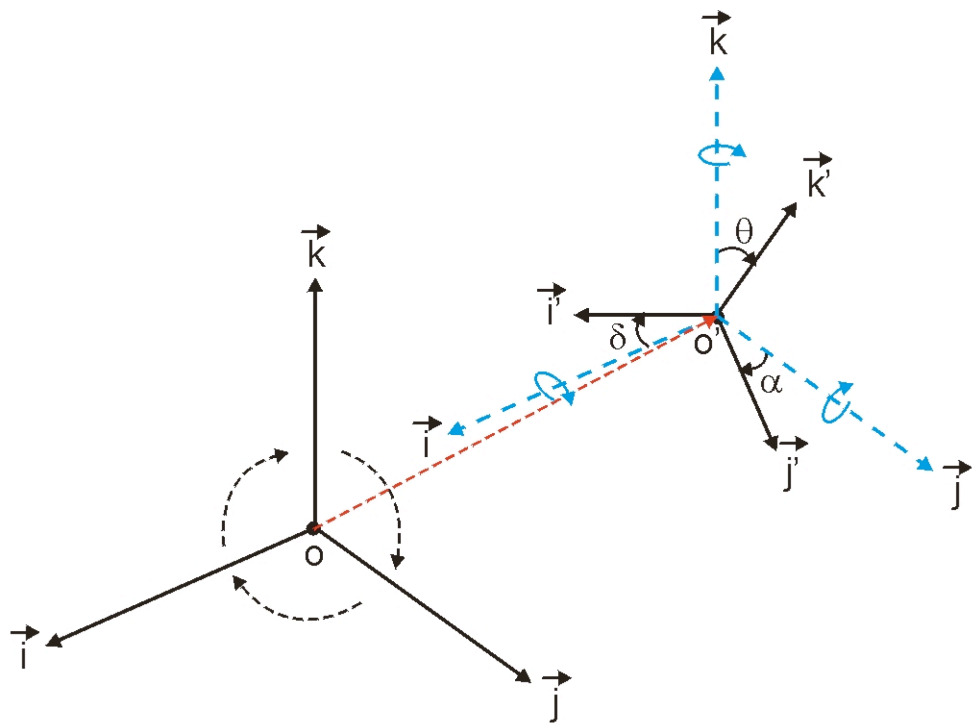
3D similarity

- Under these conditions, the transformation of Cartesian coordinates of any point between 2 TRSs (1) and (2) is given by a 3D similarity:

$$X^{(2)} = T_{1,2} + \lambda_{1,2} R_{1,2} X^{(1)}$$

$X^{(1)}$ and $X^{(2)}$ = position vectors in TRS(1) and TRS(2)
$T_{1,2}$ = translation vector
$\lambda_{1,2}$ = scale factor
$R_{1,2}$ = rotation matrix

- Also called a Helmert, or 7-parameter, transformation:
  - If translation (3 parameters), scale (1 parameter) and rotation (3 parameters) are known, then one can convert between TRSs
  - If there are common points between 2 TRSs, one can solve for T, λ, R: minimum of 3 points.



3-D Similarity

- 3D similarity between TRS1,X_1 and TRS2, X_2 can be linearized as:

$$X_2 = X_1 + T + DX_1 + RX_1 \qquad D = \text{scale factor} \quad R = \begin{pmatrix} 0 & -R_3 & R_2 \\ R_3 & 0 & -R_1 \\ -R_2 & R_1 & 0 \end{pmatrix} \quad T = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix}$$

- X_1, X_2, T, D, R are generally functions of time (plate motions, Earth's deformation) => differentiation w.r.t. time gives:

$$\dot{X}_2 = \dot{X}_1 + \dot{T} + \dot{D}X_1 + D\dot{X}_1 + \dot{R}X_1 + R\dot{X}_1$$

- D and R ~ 10^{-5} and Xdot ~ 10 cm/yr $\Rightarrow$ DXdot and RXdot negligible, ~ 0.1 mm/100 years, therefore:

$$\dot{X}_2 = \dot{X}_1 + \dot{T} + \dot{D}X_1 + \dot{R}X_1$$

Estimation

- The above equations can be written as:

$$X_2 = X_1 + T + DX_1 + RX_1 \Leftrightarrow X_2 = X_1 + A\theta$$

$$\dot{X}_2 = \dot{X}_1 + \dot{T} + \dot{D}X_1 + \dot{R}X_1 \Leftrightarrow \dot{X}_2 = \dot{X}_1 + A\dot{\theta}$$

- with:

$$\theta = \left[T_1, T_2, T_3, D, R_1, R_2, R_3\right]$$

$$\dot{\theta} = \left[\dot{T}_1, \dot{T}_2, \dot{T}_3, \dot{D}, \dot{R}_1, \dot{R}_2, \dot{R}_3\right]$$

$$A = \begin{pmatrix} . & . & . & . & . & . & . \\ 1 & 0 & 0 & x & 0 & z & -y \\ 0 & 1 & 0 & y & -z & 0 & x \\ 0 & 0 & 1 & z & y & -x & 0 \\ . & . & . & . & . & . & . \end{pmatrix}$$

- Assuming X_1 and X_2 are known, the least-squares solutions are:

$$\theta = (A^T P_x A)^{-1} A^T P_x (X_2 - X_1)$$

$$\dot{\theta} = (A^T P_v A)^{-1} A^T P_v (\dot{X}_2 - \dot{X}_1)$$

where P_x and P_v are the weight matrix for station positions and velocities, respectively

Problem when defining a frame…

- Unknowns = positions in frame 2 + 7 Helmert parameters => more unknowns than data = datum defect
- Not enough data from space geodetic observations to estimate all frame parameters
- Solution: additional information
  - Tight constraints: estimated station positions/velocities are constrained to a priori values within 10^{-5} m and a few mm/yr.
  - Loose constraints: same, with 1 m for position and 10 cm/yr for velocities.
  - Minimal constraints.

Mathematically…

- The estimation of the coordinates of a network of GPS sites is often done by solving for the linear system:

$$AX = Obs \qquad \left(\Sigma_{Obs}^{-1}\right)$$

  A = linearized model design matrix (partial derivatives) between the GPS observations *Obs* and the parameters to estimate *X*. $\Sigma^{-1}{}_{Obs}$ is the weight matrix associated to *Obs* (inverse of its covariance matrix).

- Solution is:

$$X = (A^T \Sigma_{Obs}^{-1} A)^{-1} A^T Obs$$

- But normal matrix $N = A^T \Sigma_{Obs} A$ usually rank-defficient and not invertible.

Constraint equation

- To make *N* invertible, one usually add constraints by using a condition equation.
- E.g., forcing the coordinates of a subset of sites to tightly follow values of a given reference frame:

$$X_{cons} = X_o \qquad \left(\Sigma_{apriori}^{-1}\right)$$

($\Sigma_{\text{a priori}}$ defines the constraint level, e.g. 1 cm in NE and 5 cm in U)

- The resulting equation system becomes:

$$\begin{pmatrix} A \\ I \end{pmatrix} X_{cons} = \begin{pmatrix} Obs \\ X_o \end{pmatrix} \qquad \begin{pmatrix} \Sigma_{obs}^{-1} & 0 \\ 0 & \Sigma_{apriori}^{-1} \end{pmatrix}$$

- And the solution:

$$X_{cons} = \left(A^T \Sigma_{Obs}^{-1} A + \Sigma_{apriori}^{-1}\right)^{-1} \left(A^T \Sigma_{Obs}^{-1} Obs + \Sigma_{apriori}^{-1}\right) X_o$$

Constrained solution

- The covariance matrix of the constrained solution is given by:

$$\Sigma_{cons}^{-1} = A^T \Sigma_{Obs}^{-1} A + \Sigma_{apriori}^{-1} = \Sigma_{unc}^{-1} + \Sigma_{apriori}^{-1}$$

- This can cause artificial deformations of the network if the constraint level is too tight, given the actual accuracy of X_0 => errors propagate to the whole network.
- Also, the equation above modifies the variance of the result (and its structure). E.g., if constraint level very tight, the variance of estimated parameters becomes artificially small.
- To avoid these problems, constraints have to be removed from individual solutions before they can be combined: suboptimal
- Better solution = minimal constraints.

Minimal constraints

- Same basic idea, use a condition equation to the system: impose the estimated coordinates to be expressed in the same frame as a subset of reference sites.
- But instead of tightly constraining a subset of sites to a priori positions, impose that their positions are expressed in a known frame through a similarity transformation (see previous slides):

$$X = X_o + T + DX_o + RX_o \Leftrightarrow X = X_o + E\theta$$

- Least squares solution is:

$$\theta = (E^T \Sigma_X^{-1} E)^{-1} E^T \Sigma_X^{-1} (X - X_o)$$

- "Estimated positions expressed in the same frame as the reference frame chosen" ⇔ transformation parameters between the 2 frames is zero, i.e. θ = 0. Therefore:

$$B\left(X - X_o\right) = 0 \qquad \left(\Sigma_\theta^{-1}\right) \qquad B = (E^T \Sigma_X^{-1} E)^{-1} E^T \Sigma_X^{-1}$$

Minimal constraints

- Resulting equation system (with the condition equation) becomes:

$$\begin{pmatrix} A \\ B \end{pmatrix} X_{mc} = \begin{pmatrix} Obs \\ BX_o \end{pmatrix} \qquad \begin{pmatrix} \Sigma_{obs}^{-1} & 0 \\ 0 & \Sigma_\theta^{-1} \end{pmatrix}$$

- Solution is:

$$X_{mc} = \left(A^T \Sigma_{Obs}^{-1} A + B^T \Sigma_\theta^{-1} B\right)^{-1} \left(A^T \Sigma_{Obs}^{-1} Obs + B^T \Sigma_\theta^{-1} B\right) X_o$$

- With covariance: $\Sigma_{mc}^{-1} = A^T \Sigma_{Obs}^{-1} A + B^T \Sigma_\theta^{-1} B = \Sigma_{unc}^{-1} + B^T \Sigma_\theta^{-1} B$

- Covariance: reflects data noise + reference frame effect (via *B*)
- Minimal constraints = algebraic expression on the covariance matrix that the reference frame implementation is performed through a similarity transformation.

The combination model

- For each site *i* in solution *s* (*s* = regional or global for instance), simultaneously estimate position X^i_{comb} at epoch t_0 (epoch of the combination), velocity X^i_{comb}, and a 14-parameter transformation between the individual and the combined solution using:

$$
\begin{aligned}
& X_s^i = X_{comb}^i + \left(t_{comb} - t_s\right)\widehat{X}_{comb}^i \\
& +T_k + D_k X_{comb}^i + R_k X_{comb}^i \\
& +\left(t_{comb} - t_s\right)\left[\widehat{T}_k + \widehat{D}_k X_{comb}^i + \widehat{R}_k X_{comb}^i\right]
\end{aligned}
$$

X^i_s = position of site *i* in solution *s* at epoch t_s

X^i_{comb} = estimated position of site *i* at epoch t_{comb}

X^i_{comb} = estimated velocity in the combination

T_k, D_k, R_k and {T_k, D_k, R_k}hat = transformation parameters between individual solutions *s* and the combined solution and their time derivatives.

- Combination = solve for one T_k, D_k, R_k, {T_k, D_k, R_k}hat per solution and one X^i_{comb} per site.

In practice

- Constrained solution can be done in globk (or glred) by tightly constraining some sites (+ orbits) to a priori positions: ok for small networks (= local solution)
- Minimally constrained solution computed in a 2-step manner:
  - Combine regional + global solutions in globk:
    - Globk reads each solution sequentially and combines it to the previous one
    - Loose constraints applied to all estimated parameters
    - Chi2 change should be small is data consistent with model from previous slide
    - Output = loosely constrained solution
  - Compute minimally constrained solution in glorg:
    - Matrix A comes from globk
    - Minimal constraints matrix B formed using sites that define frame
- Choice of reference sites:
  - Global distribution
  - Position and velocity precise and accurate
  - Error on their position/velocity and correlations well known

The international Terrestrial Reference System: ITRS

- Definition adopted by the IUGG and IAG: see http://tai.bipm.org/iers/conv2003/conv2003.html
- Tri-dimensional orthogonal (X,Y,Z), equatorial (Z-axis coincides with Earth's rotation axis)
- Non-rotating (actually, rotates with the Earth)
- Geocentric: origin = Earth's center of mass, including oceans and atmosphere.
- Units = meter and second S.I.
- Orientation given by BIH at 1984.0.
- Time evolution of the orientation ensured by imposing a **no-net-rotation** condition for horizontal motions.

The no-net-rotation (NNR) condition

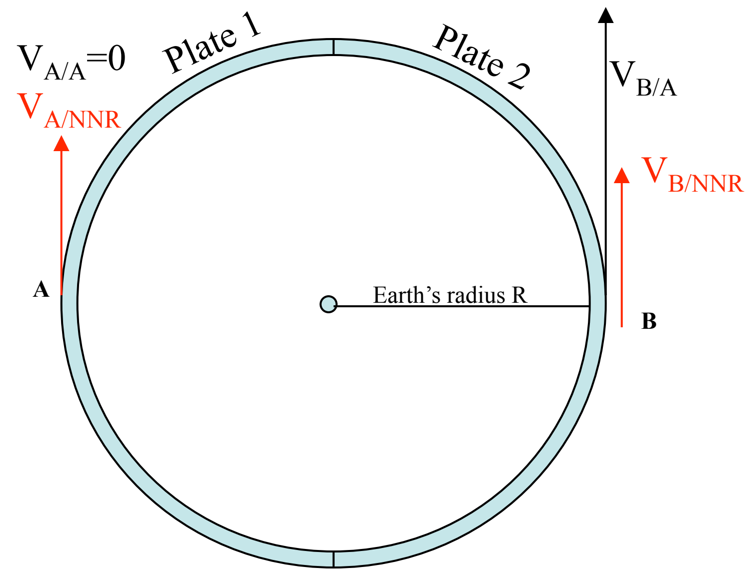
- Objective:
  - Representing velocities without referring to a particular plate.
  - Solve a datum defect problem: ex. of 2 plates ⇒ 1 relative velocity to solve for 2 "absolute" velocities… (what about 3 plates?)

- The no-net-rotation condition states that the total angular momentum of all tectonic plates should be zero.

- See figure for the simple (and theoretical) case of 2 plates on a circle.

- The NNR condition has no impact on relative plate velocities.

- It is an additional condition used to define a reference for plate motions that is not attached to any particular plate.



$$L_A = \int_A \vec{R} \times \vec{V}_{A/NNR}\, dm$$

$$L_B = \int_B \vec{R} \times \vec{V}_{B/NNR}\, dm$$

$$\sum L = 0 \Rightarrow \vec{V}_{A/NNR} = \vec{V}_{B/NNR} = \frac{\vec{V}_{B/A}}{2}$$

The Tisserand reference system

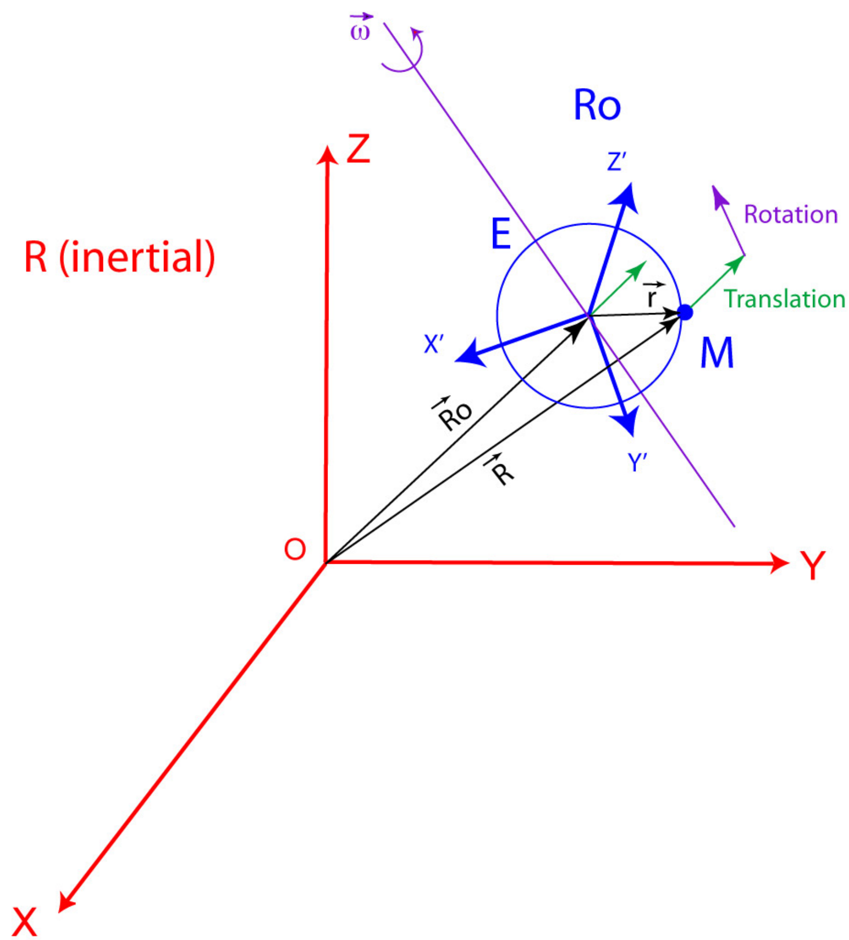
- "Mean" coordinate system in which deformations of the Earth do not contribute to the global angular momentum (important in Earth rotation theory)

- Let us assume two systems R (inertial) and Ro (translates and rotates w.r.t. R). Body E is attached to Ro. At point M, one can write:

$$\begin{cases} \vec{R} = \vec{R}_o + \vec{r} \\ \vec{V} = \vec{V}_o + \vec{v} + \vec{\omega} \times \vec{r} \end{cases}$$

- One can show that the Tisserand condition is equivalent to:

$$\begin{cases} \int_E \vec{v} \quad dm = \vec{0} & \text{No translation condition} \\ \int_E \vec{v} \times \vec{r} \quad dm = \vec{0} & \text{No rotation condition} \end{cases}$$



The Tisserand reference system

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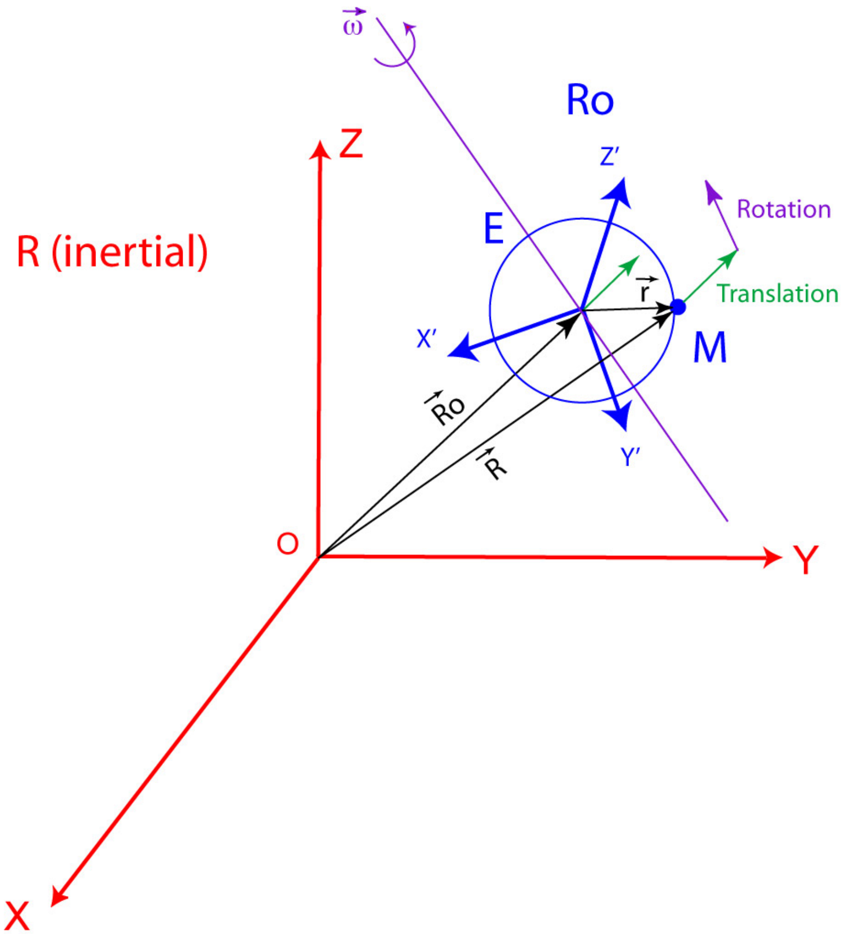


- The system of axis defined by the above conditions is called "Tisserand system".
- Integration domain:
  - Should be entire Earth volume
  - But velocities at surface only => integration over surface only
- With hypothesis of spherical Earth + uniform density, volume integral becomes a surface integral

The NNR reference system

- The Tisserand no-rotation condition is also called “no-net-rotation” condition (NNR).

- For a spherical Earth of unit radius and uniform density, the NNR conditions writes:

$$\int_S \vec{r} \times \vec{v} \; dA = \vec{0}$$

- The integral can be broken into a sum to account for discrete plates:

$$\int_S \vec{r} \times \vec{v} \; dA = \sum_P \int_P \vec{r} \times \vec{v} \; dA$$

- With, for a given plate: $L_P = \int_P \vec{r} \times \vec{v} \; dA$

The NNR reference system

- Assuming rigid plates, velocity at point *M* (position vector *r* in NNR) on plate *P* is given by:

$$\vec{v}(\vec{r}) = \vec{\omega}_P \times \vec{r} \qquad \Rightarrow L_P = \int_P \vec{r} \times (\vec{\omega}_P \times \vec{r})\ dA$$

- Developing the vector product with the triple product expansion gives:

$$L_P = \int_P ((\vec{r}.\vec{r})\vec{\omega}_P - (\vec{r}.\vec{\omega}_P)\vec{r})\ dA = \int_P (\vec{r}.\vec{r})\vec{\omega}_P\ dA - \int_P (\vec{r}.\vec{\omega}_P)\vec{r}\ dA$$

- Assuming a spherical Earth of unit radius (*r* = *1*), the first term introduces the plate area A_P:

$$\int_P (\vec{r}.\vec{r})\vec{\omega}_P\ dA = r^2\vec{\omega}_P \int_P dA = \vec{\omega}_P A_P$$

- Dealing with the second term is a bit more involved, see next.

The NNR reference system

$$(\vec{r}.\vec{\omega}_P)\vec{r} = (x_1\omega_1 + x_2\omega_2 + x_3\omega_3)\vec{r}$$

$$= \begin{bmatrix} x_1^2\omega_1 + x_1x_2\omega_2 + x_1x_3\omega_3 \\ x_1x_2\omega_1 + x_2^2\omega_2 + x_2x_3\omega_3 \\ x_1x_3\omega_1 + x_2x_3\omega_2 + x_3^2\omega_3 \end{bmatrix}$$

$$= \begin{bmatrix} x_1^2 & x_1x_2 & x_1x_3 \\ x_1x_2 & x_2^2 & x_2x_3 \\ x_1x_3 & x_2x_3 & x_3^2 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

Therefore:

$$\int_P (\vec{r}.\vec{\omega}_P)\vec{r}\, dA =$$

$$\begin{bmatrix} \int x_1^2 & \int x_1x_2 & \int x_1x_3 \\ \int x_1x_2 & \int x_2^2 & \int x_2x_3 \\ \int x_1x_3 & \int x_2x_3 & \int x_3^2 \end{bmatrix} \vec{\omega}_P\, .dA$$

We introduce a 3x3 symmetric matrix S_p with elements defined by: $S_{Pij} = \int_P (x_i . x_j)\ dA$

Therefore the integral becomes: $\int_P (\vec{r}.\vec{\omega}_P)\vec{r}\, dA = S_P\ \vec{\omega}_P$

The NNR reference system

- Finally: $L_P = \int_P (\vec{r}.\vec{r})\vec{\omega}_P \; dA - \int_P (\vec{r}.\vec{\omega}_P)\vec{r} \, dA$
- Reduces to: $$\begin{aligned} L_P &= \vec{\omega}_P A_P - S_P \vec{\omega}_P \\ &= \left(A_p I - S_p\right)\vec{\omega}_P \\ &= Q_P \vec{\omega}_P \end{aligned}$$
- With: $Q_P = A_P \; I - S_P$

- Q_p is a 3x3 matrix that only depends on the plate geometry, with its components defined by:

$$Q_{Pij} = \int_P \left(\delta_{ij} - x_i x_j\right) dA$$

Kronecker delta: $\delta_{ij} = \begin{cases} 1 & if \; i = j \\ 0 & if \; i \neq j \end{cases}$

The NNR reference system

- The non-rotation condition: $\int_S \vec{r} \times \vec{v} \; dA = \sum_P \int_P \vec{r} \times \vec{v} \; dA = \vec{0}$
- Becomes: $\sum_P Q_P \vec{\omega}_P = \vec{0}$
- Now, observations are relative plate motions, for instance plate *P* w.r.t. Pacific plate. Angular velocities are additive, one can then write:

$$\vec{\omega}_{P/NNR} = \vec{\omega}_{P/Pacific} + \vec{\omega}_{Pacific/NNR}$$

- Therefore:

$$\sum_P Q_P \left(\vec{\omega}_{P/Pacific} + \vec{\omega}_{Pacific/NNR} \right) = \vec{0}$$

$$\Rightarrow \sum_P Q_P \; \vec{\omega}_{P/Pacific} + \sum_P Q_P \; \vec{\omega}_{Pacific/NNR} = \vec{0}$$

$$\Rightarrow \sum_P Q_P \; \vec{\omega}_{P/Pacific} + \frac{8\pi}{3} I \; \vec{\omega}_{Pacific/NNR} = \vec{0}$$

(because on a unit radius sphere: $\sum_P Q_P = \frac{8\pi}{3} I$)

The NNR reference system

- Finally, the angular velocity of the Pacific plate w.r.t. NNR can be calculated using:

$$\vec{\omega}_{Pacific/NNR} = -\frac{3}{8\pi}\sum_{P} Q_P \ \vec{\omega}_{P/Pacific} \qquad \text{with} \qquad Q_P = \int_P \left(\delta_{ij} - x_i x_j\right) dA$$

($\omega_{p/\mathrm{Pacific}}$ are known from a relative plate model, Q_p are 3x3 matrices computed for each plate from its geometry: δ is Kronecker delta, x is a position vector, A is the plate area)

- Once the angular velocity of the Pacific plate in NNR is found, the angular velocity of any plate P can be computed using:

$$\vec{\omega}_{P/NNR} = \vec{\omega}_{P/Pacific} + \vec{\omega}_{Pacific/NNR}$$

- This method is the one used to compute the NNR-NUVEL1A model (Argus and Gordon, 1991).

The no-net-rotation (NNR) condition

- “Mean” coordinate system in which deformations of the Earth do not contribute to the global angular momentum => used as a constraint to solve datum defect problem, but has a “dynamic” origin.
- First proposed by Lliboutry (1977) as an approximation of a reference frame where moment of forces acting on lower mantle is zero, which implies:
  - Rigid lower mantle
  - Uniform thickness lithosphere
  - No lateral viscosity variations in upper mantle

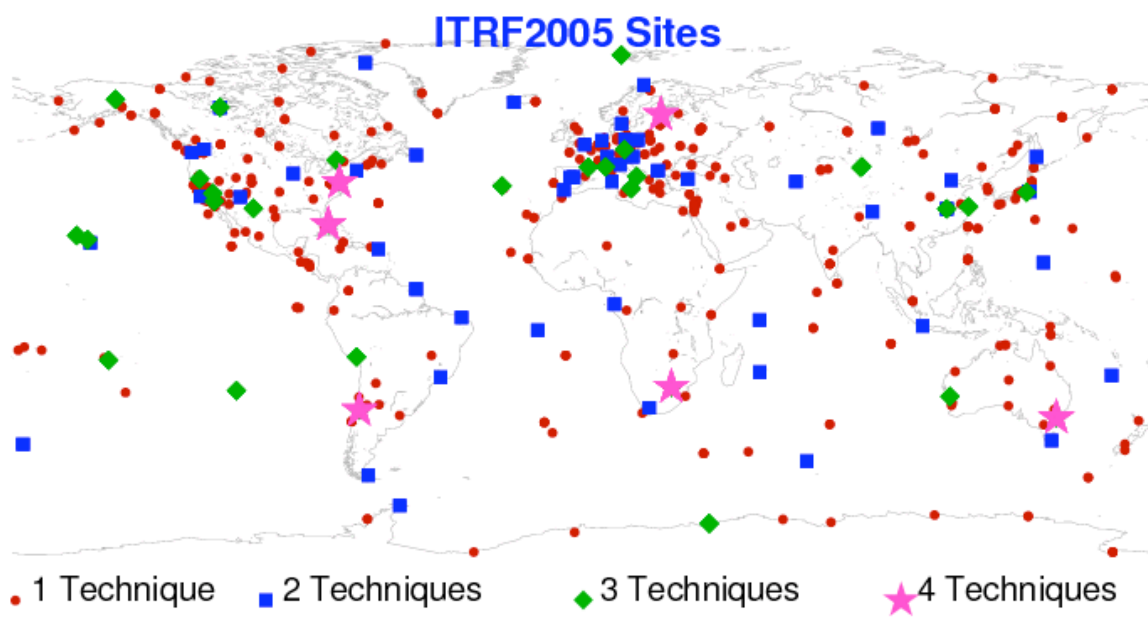
  $\Rightarrow$ NNR is a frame in which the internal dynamics of the mantle is null.
- These conditions are not realistic geophysically, in particular because of slabs in upper and lower mantle, that contribute greatly to driving plate motions (Lithgow-Bertelloni and Richards, 1995)
- But that’s ok, as long as NNR is simply used as a **<u>conventional</u>** reference.

The international Terrestrial Reference Frame: ITRF

- Positions (at a given epoch) and velocities of a set of geodetic sites (+ associated covariance information) = <u>dynamic</u> datum

- Positions and velocities estimated by combining independent geodetic solutions and techniques.

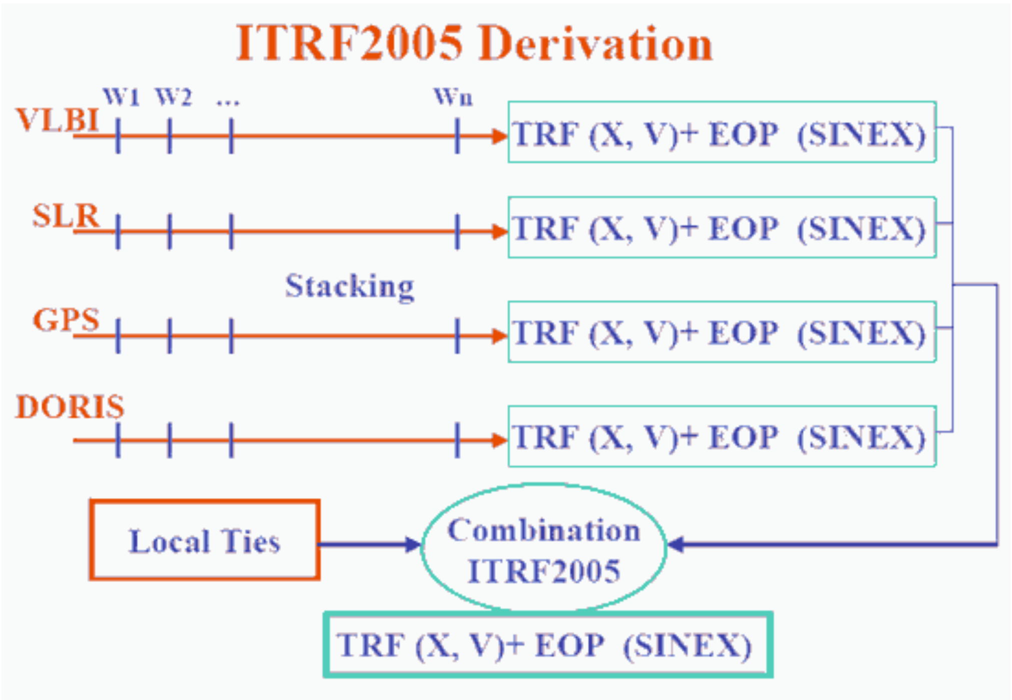
- Combination:
  - “Randomizes” systematic errors associated with each individual solutions
  - Provides a way of detecting blunders in individual solutions
  - Accuracy is equally important as precision



- 1984: VLBI, SLR, LLR, Transit
- 1988: TRF activity becomes part of the IERS => first ITRF = ITRF88
- Since then: ITRF89, ITRF90, ITRF92, ITRF93, ITRF94, ITRF96, ITRF97, ITRF2000
- **Current = ITRF2005:**
  - Up to 25 years of data
  - GPS sites defining the ITRF are all IGS sites
  - Wrms on velocities in the combination: 1 mm/yr VLBI, 1-3 mm/yr SLR and GPS
  - Solutions used: 3 VLBI, 1 LLR, 7 SLR, 6 GPS, 2 DORIS
- ITRF improves as:
  - Number of sites with long time series increases
  - New techniques appear
  - Estimation procedures are improved

The international Terrestrial Reference Frame: ITRF

- Apply minimum constraints equally to all loosely constrained solutions: this is the case of SLR and DORIS solutions
- Apply No-Net-Translation and No-Net-Rotation condition to IVS solutions provided under the form of Normal Equation
- Use as they are minimally constrained solutions: this is the case of IGS weekly solutions
- Form per-technique combinations (TRF + EOP), by rigorously staking the time series, solving for station positions, velocities, EOPs and 7 transformation parameters for each weekly (daily in case of VLBI) solution w.r.t the per-technique cumulative solution.
- Identify and reject/de-weight outliers and properly handle discontinuities using piece-wise approach.
- Combine if necessary cumulative solutions of a given technique into a unique solution: this is the case of the two DORIS solutions.
- Combine the per-technique combinations adding local ties in co-location sites.



The international Terrestrial Reference Frame: ITRF

- Origin: The ITRF2005 origin is defined in such a way that there are null translation parameters at epoch 2000.0 and null translation rates between the ITRF2005 and the ILRS SLR time series.

- Scale: The ITRF2005 scale is defined in such a way that there are null scale factor at epoch 2000.0 and null scale rate between the ITRF2005 and IVS VLBI time series.

- Orientation: The ITRF2005 orientation is defined in such a way that there are null rotation parameters at epoch 2000.0 and null rotation rates between the ITRF2005 and ITRF2000. These two conditions are applied over a core network.

ITRF in practice

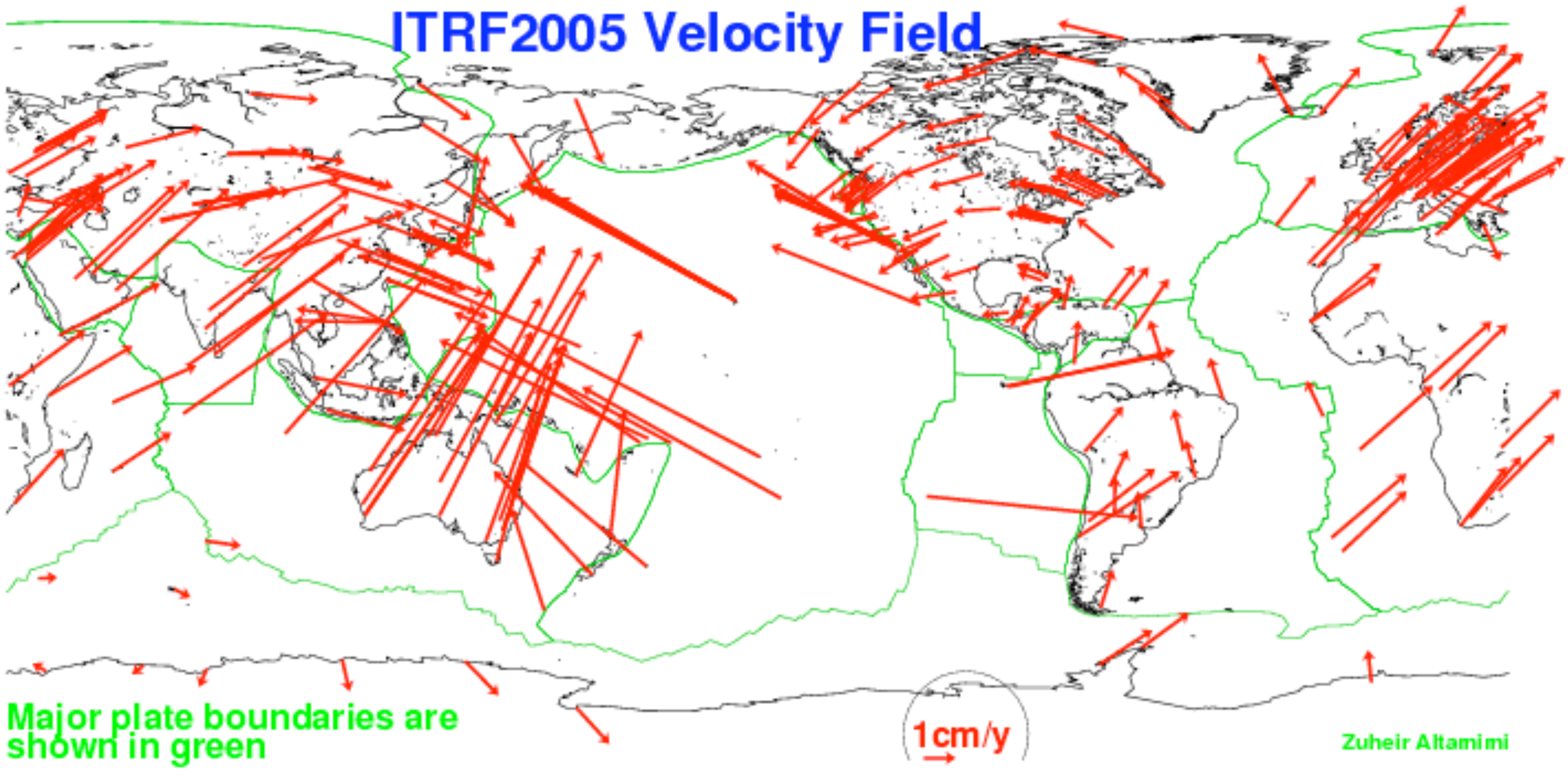
- Multi-technique combination.
- Origin = SLR, scale = VLBI, orientation = all.
- Position/velocity solution.
- Velocities expressed in no-net-rotation frame:
  - ITRF2000: minimize global rotation w.r.t. NNR-NUVEL1A using 50 high-quality sites far from plate boundaries
  - Subtlety: ITRF does not exactly fulfill a NNR condition because Nuvel1A is biased…
- Provided as tables (position, velocities, uncertainties)
- Full description provided as SINEX file (Solution Indepent Exchange format): ancillary information + vector of unknowns + full variance-covariance matrix (i.e. with correlations).

ITRF in practice

ITRF2005 STATION POSITIONS AT EPOCH 2000.0 AND VELOCITIES
GPS STATIONS

DOMES NB.	SITE NAME	TECH.	ID.	X/Vx	Y/Vy	Z/Vz	Sigmas			SOLN	DATA_START	DATA_END
				m/m/y								
10001S006	PARIS	GPS	OPMT	4202777.434	171367.913	4778660.147	0.005	0.002	0.006			
10001S006				-.0118	0.0170	0.0111	.0011	.0004	.0012			
10002M006	GRASSE	GPS	GRAS	4581690.969	556114.738	4389360.731	0.001	0.000	0.001	1	00:000:00000	03:113:00000
10002M006				-.0139	0.0186	0.0116	.0001	.0001	.0001			
10002M006	GRASSE	GPS	GRAS	4581690.975	556114.741	4389360.734	0.001	0.000	0.001	2	03:113:00000	04:295:43200
10002M006				-.0139	0.0186	0.0116	.0001	.0001	.0001			
10002M006	GRASSE	GPS	GRAS	4581690.974	556114.744	4389360.739	0.001	0.001	0.001	3	04:295:43200	00:000:00000
10002M006				-.0139	0.0186	0.0116	.0001	.0001	.0001			
10003M004	TOULOUSE	GPS	TOUL	4627846.086	119629.236	4372999.754	0.001	0.000	0.001			
10003M004				-.0111	0.0191	0.0117	.0003	.0001	.0003			
10003M009	TOULOUSE	GPS	TLSE	4627851.889	119639.921	4372993.492	0.001	0.001	0.001			
10003M009				-.0111	0.0191	0.0117	.0003	.0001	.0003			
10004M004	BREST	GPS	BRST	4231162.638	-332746.764	4745130.859	0.004	0.001	0.004			
10004M004				-.0111	0.0162	0.0134	.0009	.0003	.0009			
10023M001	La Rochelle	GPS	LROC	4424632.623	-94175.321	4577544.022	0.003	0.001	0.003			
10023M001				-.0106	0.0183	0.0123	.0006	.0002	.0006			
10090M001	SAINT JEAN DES	GPS	SJDV	4433469.919	362672.729	4556211.652	0.002	0.001	0.002	1	00:000:00000	99:071:57600
10090M001				-.0118	0.0186	0.0121	.0008	.0002	.0008			
10090M001	SAINT JEAN DES	GPS	SJDV	4433469.921	362672.729	4556211.656	0.001	0.000	0.001	2	99:071:57600	00:000:00000
10090M001				-.0118	0.0186	0.0121	.0008	.0002	.0008			
10202M001	REYKJAVIK	GPS	REYK	2587384.422	-1043033.508	5716563.995	0.001	0.000	0.001	1	00:000:00000	00:169:56460
10202M001				-.0216	-.0028	0.0059	.0001	.0001	.0002			
10202M001	REYKJAVIK	GPS	REYK	2587384.410	-1043033.501	5716563.980	0.006	0.003	0.012	2	00:169:56460	00:173:03120
10202M001				-.0216	-.0028	0.0059	.0001	.0001	.0002			
10202M001	REYKJAVIK	GPS	REYK	2587384.415	-1043033.509	5716564.003	0.001	0.000	0.001	3	00:173:03120	00:000:00000
10202M001				-.0216	-.0028	0.0059	.0001	.0001	.0002			
10202M003	REYKJAVIK	GPS	REYZ	2587383.736	-1043032.722	5716564.472	0.001	0.001	0.001			
10202M003				-.0216	-.0028	0.0059	.0001	.0001	.0002			

ITRF in practice



Summary

- Geodetic observations face datum defect problem => need for a reference frame.
- ITRF (currently 2005) = multitechnique combination, provides positions + velocities at reference sites:
  - Include some of these sites in processing to tie a regional solution to ITRF.
  - Combine regional solution with global solutions – better.
- Reference frame can be implemented by:
  - Constraining positions/velocities of a subset of sites to a priori values
  - Using minimal constraints – better.
- When using ITRF, velocities are expressed in a no-net-rotation frame (derived from Tisserand system) => frame independent from any plate.