GEOPHYSICAL RESEARCH LETTERS, VOL. 20, NO. 2, PAGES 93-96, JANUARY 22, 1993

BACKGROUND VELOCITY INVERSION WITH A GENETIC ALGORITHM

Side Jin and Raul Madariaga

Département de Sismologie, Institut de Physique du Globe de Paris et Université Paris 7

Abstract. We propose a method for the non-linear inversion of the velocity field from reflection profiles. The inverse problem is separated into a linear and a nonlinear domain. Linearized inversion is applied to the retrieval of the short wavelength features of the velocity or impedance field. This problem has a huge number of degrees of freedom but it can be solved by an efficient asymptotic migration-inversion method. The low frequency part of the velocity field — the background — is inverted using a non-linear genetic algorithm applied to an objective functional defined in migrated data space. Computer time is significantly reduced using this objective function instead of straightforward waveform fitting. We apply our method to the inversion of a 1-D background velocity model from a reflexion profile of the North Sea. For this problem, we found that the genetic algorithm is more reliable and efficient than other velocity analysis methods.

Introduction

A fundamental problem facing seismic imaging is the determination of the background or reference velocity field. This is the long wavelength, smooth part of the seismic velocities. Prestack depth migration/inversion of seismic data can construct an accurate image of rather complicated structures, if an accurate background velocity model has been previously determined. Although the theory of full prestack, depth migration is well known, the determination of the background velocity model is an area of active research (see, e.g., Tarantola et al., 1988; Landa et al., 1989). The main problem comes from the fact that the background velocity affects only the travel times and can not be retrieved in a reasonable time by iterative linearized inversion of waveforms.

Several methods for the determination of background velocity fields have been applied with varying degrees of success, e. g. CMP stacking velocity analysis and traveltime tomography. These approaches have a number of serious drawbacks and require extensive operator intervention. One of the most practical method is iterative migration velocity analysis (Yilmas and Chambers, 1984; Al-Yahya, 1989), but this technique still requires human intervention at each iteration step. In order to make iterative migration automatic, Jin and Madariaga (1992) separated the inversion of the velocity model into a linear and a non-linear part and proposed a two-step non-linear Monte-Carlo inversion method. They used a fast ray modeling technique for linearized inversion, and an objective functional defined in data space so that the random search method for the non-linear inversion of the 2D background velocity model was possible using a work-station.

Paper number 92GL02781
0094-8534/93/92GL-02781$03.00

Recently two alternatives to Monte Carlo inversion have been proposed for geophysical problems: Simulated annealing and genetic algorithms (GA). GA were well investigated by many authors using classical waveform fitting objective functions (see, e.g. Stoffa and Sen, 1991; Gallagher et al., 1991; Wilson, 1991 and Sambridge and Drijkoningen, 1992). The algorithm has not been applied, to our knowledge, to the nonlinear inversion of seismic reflection profiles with large amounts of real data. In practice the waveform fitting methods proposed by the previous authors are too expensive because they require extensive forward modeling in order to evaluate the cost functionals. The purpose of this note is to improve our two-step inversion method using a GA for the non-linear part of the inversion.

The Objective Functional

The main question for background velocity estimation is how do we decide that a velocity model is better than another. A direct way would be to use the difference between observed and calculated waveforms, as used by Stoffa and Sen (1991) and Sambrigde and Drijkoningen (1992). However, since the velocity information is contained only in the traveltimes, the waveform misfit functional is not the most suitable for velocity inversion. The reason is simple, waveform fitting requires extensive forward modeling. Computer time for the calculation of synthetic seismograms make this approach utterly unrealistic, at least in present day computers. For this reason nonlinear inversion using a single waveform misfit objective functional is difficult to envisage in practical situations.

Actually, fitting the waveforms is not the only way to retrieve the background velocity information. Working directly with migrated data rather than with original data, we can avoid expensive forward modeling. in fact, most practical background velocity determination methods (for example, migration velocity analysis) are based on image gather analysis. In an image gather each trace represents a migrated image of the subsurface at the same horizontal position. The principle is that reflexion events in an image gather should be horizontally aligned if the background velocity model is correct.

Many criteria exist for measuring the horizontal alignment of the reflection events in an image gather. Here we use the one proposed by Jin and Madariaga (1992). We express the velocity field as the sum of a smooth background velocity field c and a small-scale (small characteristic wavelength) perturbation δc. Using the current background velocity model, for every source s_i, we obtain a prestack depth-migrated image by linearized inversion of the surface data that we designate $f(x, z|s_i, c)$. As shown by Lambaré et al. (1992) the imaged function $f = -2\delta c/c^3$ is a scaled version δc.

For the non-linear inversion of the reference velocity, we define the following objective function directly in depth-migrated image space:

$$J(\mathbf{c}|x) = \sum_z | \sum_i f(x, z|s_i, \mathbf{c}) |, \tag{1}$$

here x is the surface location where we analyze the coherency of the models inverted from different shot gathers. $J(\mathbf{c}|x)$ is the sum over depth z and sources s_i of the images obtained at the horizontal location x. If the velocity $\mathbf{c}$ was correct, the traces for different values of s_i should be horizontally aligned, and $J(\mathbf{c}|x)$ would be at a maximum. As defined in (1), J is an $\mathcal{L}_1$ norm that can be easily calculated using the fast asymptotic prestack depth inversion method proposed by Lambaré et al. (1992). Other prestack inversion methods can probably be used without difficulty. In fact, our inversion is performed in two-steps: a linearized inversion-migration for the small scale velocity perturbations f using the current velocity model $\mathbf{c}$; and a non-linear search for the model $\mathbf{c}$ using the objective functional J.

The Genetic Algorithm

The goal of the inversion method is to choose a smooth velocity model which maximizes the objective functional $J(\mathbf{c}|x)$ defined in (1). This goal can be achieved by many methods, for instance by Monte Carlo (MC), as used by Jin and Madariaga (1992).

For many optimization problems, genetic algorithms (GA) have been shown to be more efficient nonlinear optimization methods than strict MC (see the references in the Introduction). Since our emphasis here is the application of GA, we briefly describe the particular implementation that we used and we refer the reader to previous publications on the subject for the theory of GA. In a GA the model parameters are represented by simple binary strings. Initially a starting set of M models is randomly chosen and the value of the objective functional J_k is calculated for each of them, where k is an index over the models of the starting set. A GA iteration consists in three steps: reproduction, crossover and mutation. In the reproduction step the members of the current population are selected with a probability function $P_r(J_k)$. We followed the suggestion of Stoffa and Sen (1991) and used the Boltzmann distribution $P_r(J_k) = Z^{-1}\exp(-J_k/T)$, where $Z = \sum_i \exp(-J_i/T)$ is the partition function, and T is a parameter that plays the role of temperature in statistical mechanics. The larger is T, the wider is the probability distribution P_r. In the crossover step a new generation of models is created from the parent population, by mixing, or cross-over of the bit strings from two parents. The cross-over of the low-order bits beyond a certain position is made with probability P_c. Finally, in the mutation step we randomly perturb with small probability P_m the model parameters of a child model in order to introduce some diversity in the current population of models. The new generation of models is compared to the old one and those models are selected for the new generation with probability P_r calculated as from the objective function J_k as explained above. The result of the three steps is a new population of models of the same size as the previous one. The iterative procedure is stopped after a few iterations in which the objective functional stops increasing.

Application to a seismic profile from the North Sea

We apply our non-linear inversion method to a marine reflection profile from the Nord Sea that was previously studied by Lambaré et al (1992) and Jin and Madariaga (1992). These studies showed that the background velocity of the surveyed region had weak lateral variation, so that we can locally approximate the background model by a 1-D model in which background velocity depends only on depth.

For the numerical test, we chose 24 common shot gathers of 48 receivers each. One of the gathers is presented in Figure 1. The complete migrated and inverted profiles were shown in Lambare et al. (1992). Sources and receivers are regularly distributed along the profile with a spacing of 50 m. Sampling rate is 4 ms and the record length is 4 s. The first and last offsets are 183 m and 2533 m, respectively.

In order to parametrize the smooth background velocity model, we used cubic splines. We define the velocity model at 15 knots located at the surface, at 800 m and at 14 equidistant depths located every 200 m. The last nodal point is at 3600 m. The velocity on the surface is fixed at 1800 m/s. This velocity can not be well determined by inversion. The model space is limited by hard bounds, so that velocities are searched only between 1600 m/s and 4000 m/s at all depths.

We have a total of 15 velocity knots so that our space has 15 degrees of freedom. Each unknown was coded with a 6 bit binary string, defining 64 possible velocity values in the range defined by the hard bounds. Every model is characterized by a 90 bit string, so that the total number of possible models (the statistical universe) is therefore of size 2^{90}. By trial and error the "temperature" T of the Boltzman distribution was fixed at 0.1.

In Figure 2, the performance of the GA is displayed for different values of the free parameters P_c, P_m and M. Each curve is an average over 50 separate trials with different random sequences. The objective functions are normalized with respect to its maximum value in an experiment. Figure 2a shows the variation of objective function as a function of generation number using 3 different crossover probabilities P_c. The mutation probabilities P_m for all these cases was fixed at 0.01. The result shows that the convergence rate with high crossover probability is faster. Figure 2b demonstrates the effect of the numbers of models M in the population on the convergence rate. We

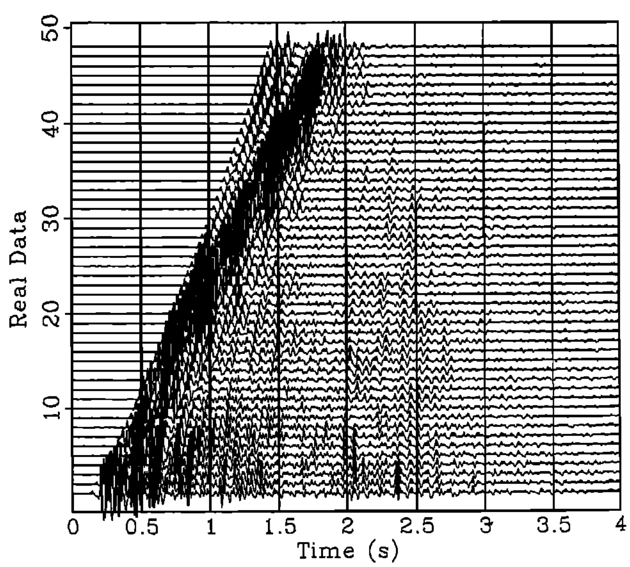

Fig. 1. An example of the 24 shot gathers used for nonlinear velocity inversion. Linear dynamic correction is applied for display.

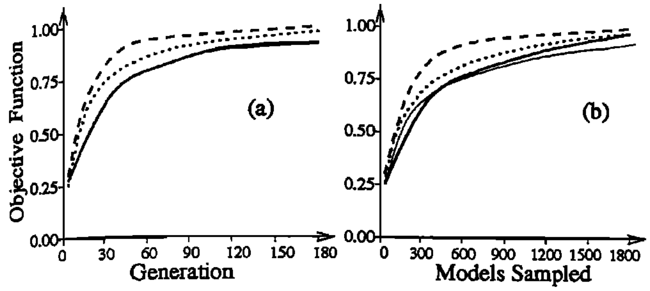

Fig. 2. (a): Comparison of convergence of GA with different crossover propabilities p_c. $p_c = 0.95$ for dashed line, $p_c = 0.5$ for dotted line and $p_c = 0.3$ for solid line. The mutation probability is 0.01 for all the cases. The number of models in a generation is 20. (b): Comparison of convergence of GA with different population sizes M. $M = 20$ for dashed line, $M = 10$ for dotted line and $M = 50$ for solid line. $p_c = 0.95$, p_m=0.01 are used in the comparison. To compare the performance of GA with MC, the objective function calculated by MC is depicted by fine solid line.

have fixed $P_c = 0.95$ and $P_m = 0.01$. The convergence rate with population sizes of 10, 20 and 50 models indicates that the convergence is fastest for the test with 20 models. This comparison suggests that the performance is not always improved by increasing the size of the population. This reason is that the larger the population the longer it takes for information to transmit across the population. In order to compare the performance of MC with respect to GA, we present also in Figure 2b the variation of the objective function as a function of the number of models. In our experiments the objective function for MC (thin solid line in Figure 2b) goes rarely above 0.9 within 1800 sampled models. The results show an improvement in performance for GA relative to MC for our problem.

In order to see how the image gather changes with the improvement in objective function, we show in Figure 3 the image gathers for four different velocity models. The four models are shown in Figure 4. We observe that the reflection events in the image gathers are visually well aligned for values of $J = 0.6$ (Figure 3b). Conventional migration velocity analysis would stop at this point, yielding an inaccurate velocity model. We continue the exploration for better velocity models with even higher values of the objective function J. As shown by Figure 2b after 1800 iterations, strict Monte Carlo would find models with values of the objective function of the order of 0.9 (see Figure 2b). A typical model found by the MC method is shown in Figure 3c. This model is not entirely satisfactory because it does not model well the two reflectors at depths of 2.1 km and 2.2 km. With the GA algorithm, on the other hand, the objective function increases rapidly to values closer to $J = 1$. The final model obtained with the GA is shown in Figure 3d. It successfully models these two reflectors. Thus our results indicate that the GA yields more accurate results than MC for a similar computer time. It is remarkable that the aspect of the image gathers changes dramatically with the value of the objective function. The amplitude of the deeper reflectors increases significantly as the value of objective function increases. The image gathers of Figure 3 show that the maximum value of the objective function results not only from the alignment of events in the image gathers, but also from the increase in amplitude of the reflectors. The corresponding velocity models shown Figure 4 show that the algorithm

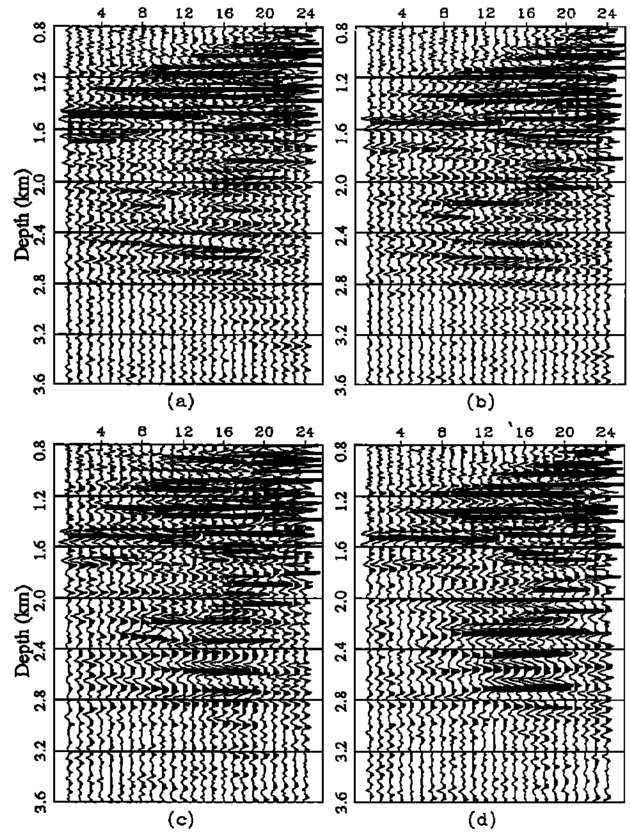

Fig. 3. Image gathers obtained using velocity models that give very different values of the objective function. $J = 0.3$ for (a), $J = 0.6$ for (b), $J = 0.9$ for (c) and $J = 1.0$ for (d).

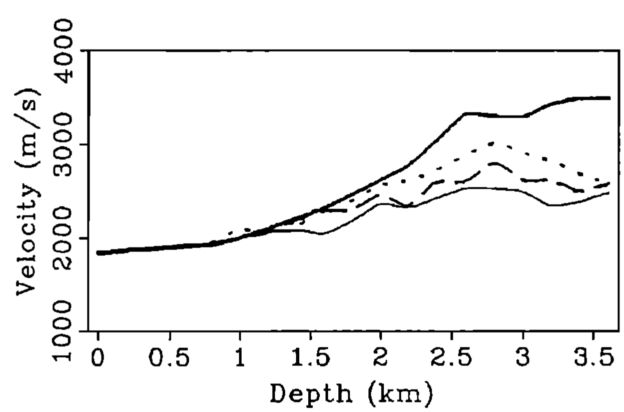

Fig. 4. Velocity models corresponding to the common image gathers in Figure 3. Fine solid line corresponding to Figure 3a, dashed line to Figure 3b, dotted line to Figure 3c and thik solid line to Figure 3d.

improves the velocities from top to bottom. This observation is in agreement with layer-stripping methods used in velocity estimation. The set of the best 100 models found by the GA, for which $J > 0.95$, is shown in Figure 5 in order to give an idea of the resolution of the velocity model. We observe that resolution is very good at shallow depths but that it degrades rapid when the depth is greater than the maximum offset of data (2553 m).

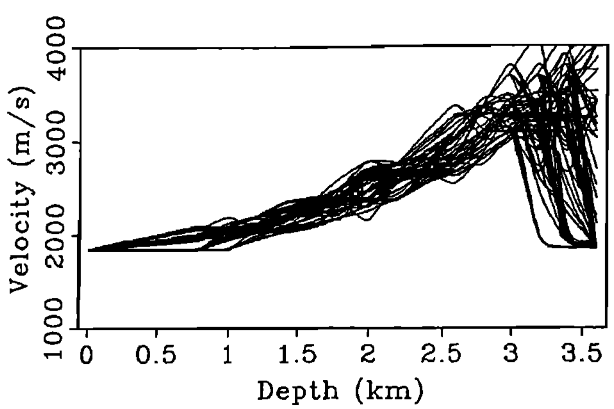

Fig. 5. Plot of 100 velocity models for which the objective function is within 95 % of the maximum. This figure gives an idea of the resolution of the inverse problem.

Discussion and Conclusions

We proposed a method for the inversion of background velocity based on a non-linear functional defined in migrated data space. Following our recent work on Monte Carlo methods, we defined a cost functional that measures the alignment of reflection events in image gathers. Defining the objective function in this space is much more economical than using waveform fitting because it avoids the computation of numerous and expensive forward models.

Following Jin and Madariaga (1992) we separated the inversion into two parts: linear migration-inversion for the small scale components of the velocity model and nonlinear inversion for the background velocity. The interest of this separation is that nonlinear inversion can be formulated with a small number of parameters (15 in the example presented above). Random search techniques become realistic in this case. Although the linear inversion still has a large number of parameters, less time-consuming gradient-methods can handle this part of the problem efficiently. A genetic algorithm similar to that of Stoffa and Sen (1991) and Sambridge and Drijkoningen (1992) was used for non-linear inversion. We showed that genetic algorithms are superior in performance to Monte-Carlo methods, even if our implementation of the GA is far from being optimal.

We presented only an 1-D example but the method can well be applied to 2-D problems. In spite of the assumption that the background velocity model is 1-D, our approach is still more powerful than stacking velocity analysis because: (1) We do not require that reflectors be flat. Because we use a 2-D prestack migration method, we can also handle dipping reflectors. In this way, the important drawback of CMP stacking, the so-called dip move out (DMO) problem, is avoided by the prestack depth processing. (2) Like migration velocity analysis, our method is model-based. The advantage of model-based method is that a priori information about the model can be included in the inversion. (3) Our method is automatic. The objective function can be computed and compared without human intervention at each iteration. (4) Our method gives more accurate result than migration velocity analysis because it uses numerical information rather than a visual observation of reflector alignments.

Acknowledgements. This work is a contribution of Groupement de Recherche Sismique sponsored by Centre National de la Recherche Scientifique, Elf Aquitaine Company, and Institut Français du Pétrole. The North-Sea data set was kindly provided to us by SNEAP. We thank J. Virieux and G. Lambaré for their advice and some essential computer software.

References

Al-Yahya, K. M., Velocity analysis by iterative profile migration: *Geophysics, 54*, 718-729, 1989

Gallagher, K., Sambridge, M., Drijkoningen, G., Genetic algorithms: An evolution from Monte-Carlo methods for strongly nonlinear geophysical optimization problems: *Geophys. Res. Lett., 18*, 2177-2180, 1991.

Jin, S., and Madariaga, R., Nonlinear velocity inversion by a two step Monte-Carlo method: in press *Geophysics*, 1992.

Lambaré, G., Virieux, J., Madariaga, R., and Jin, S., Iterative asymptotic inversion of seismic profiles in the acoustic approximation: *Geophysics 57*, 1138-1154, 1992.

Landa, E., Beydoun, W., and Tarantola, A., Reference velocity model estimation from prestack waveforms: Coherency optimization by simulated annealing, *Geophysics, 54*, 984-990, 1989.

Sambridge, M., and Drijkoningen, G., Genetic algorithms in seismic waveform inversion, *Geophys. J. Int., 109*, 323-342, 1992.

Stoffa, P. L., and Sen, M. K., Nonlinear multiparameter optimization using genetic algorithms: Inversion of plan-wave seismograms: *Geophysics, 56*, 1794-1810, 1991.

Tarantola, A., Jobert, G., Trezeguet, D., and Denelle, E., The non-linear inversion of seismic waveforms can be performed either by time extrapolation or by depth extrapolation, *Geophys. Prospecting, 36*, 383-416, 1988.

Wilson, W., and K. Vasudevan, Application of the genetic algorithm to residual statics estimation:*Geophys. Res. Lett., 18*, 2181-2184, 1991.

Yilmaz, O., and Chambers, R., E., Migration velocity analysis by wave-field extrapolation: *Geophysics, 49*, 1664-1674, 1984.

S. Jin and R. Madariaga, Département de Sismologie, IPG de Paris, 4, Place Jussieu, Box 89, 75252 Paris Cedex 05, France.

(Received May 22, 1992;
revised November 5, 1992;
accepted November 23, 1992.)