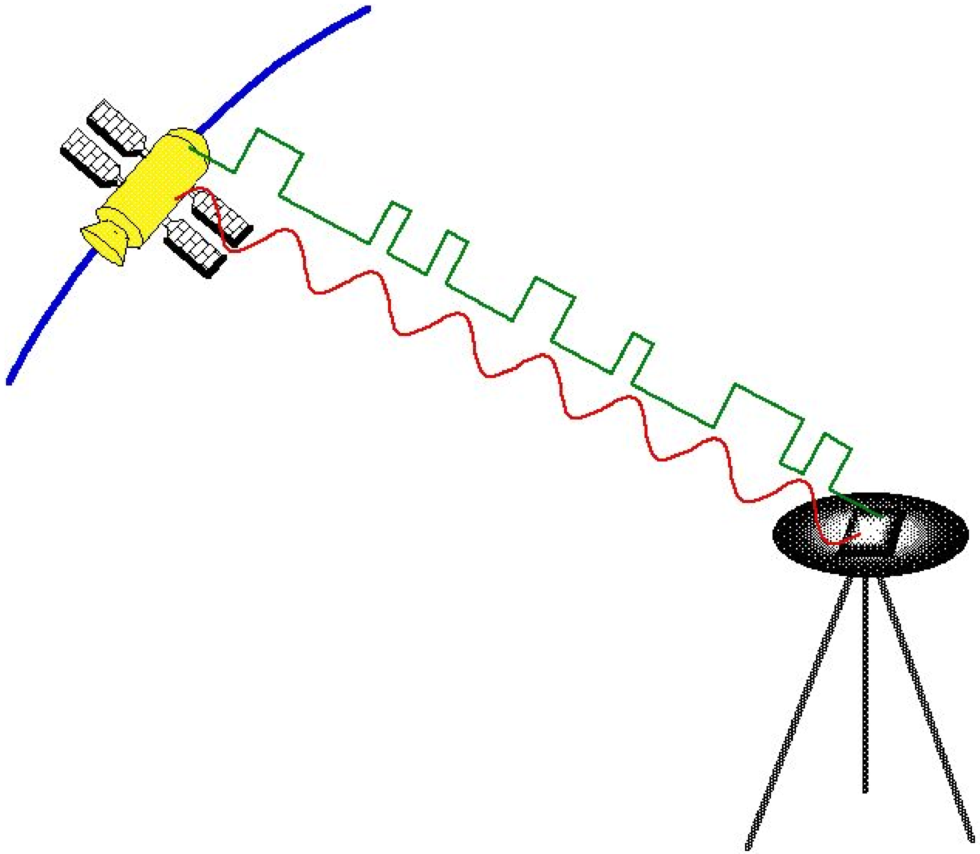
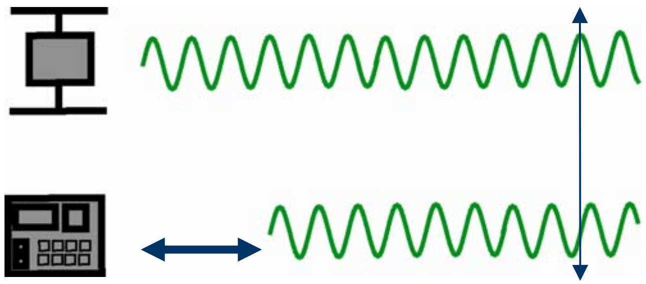
Fundamentals of GPS



Basics of GPS uncertainties/precision/errors

1. what kind of noise/errors/uncertainties/unmodeled things affect the GPS precision : AS, SA, orbits, clocks, ionosphere, troposphere, antenna phase centers, centering (tribrachs), etc...
2. how do we evaluate GPS uncertainties ? (difference between formal and a posteriori) can we trust them ?
3. difference between precision and accuracy (internal consistency like repeatability is assertion of precision, comparison with other method affected by different biases is accuracy)
4. reference frames : how do we map ? with what precision ? What influence on results ?

Phase measurement precision



station **j**, at time **tj** (defined by the station clock) receives a signal coming from satellite **i**.

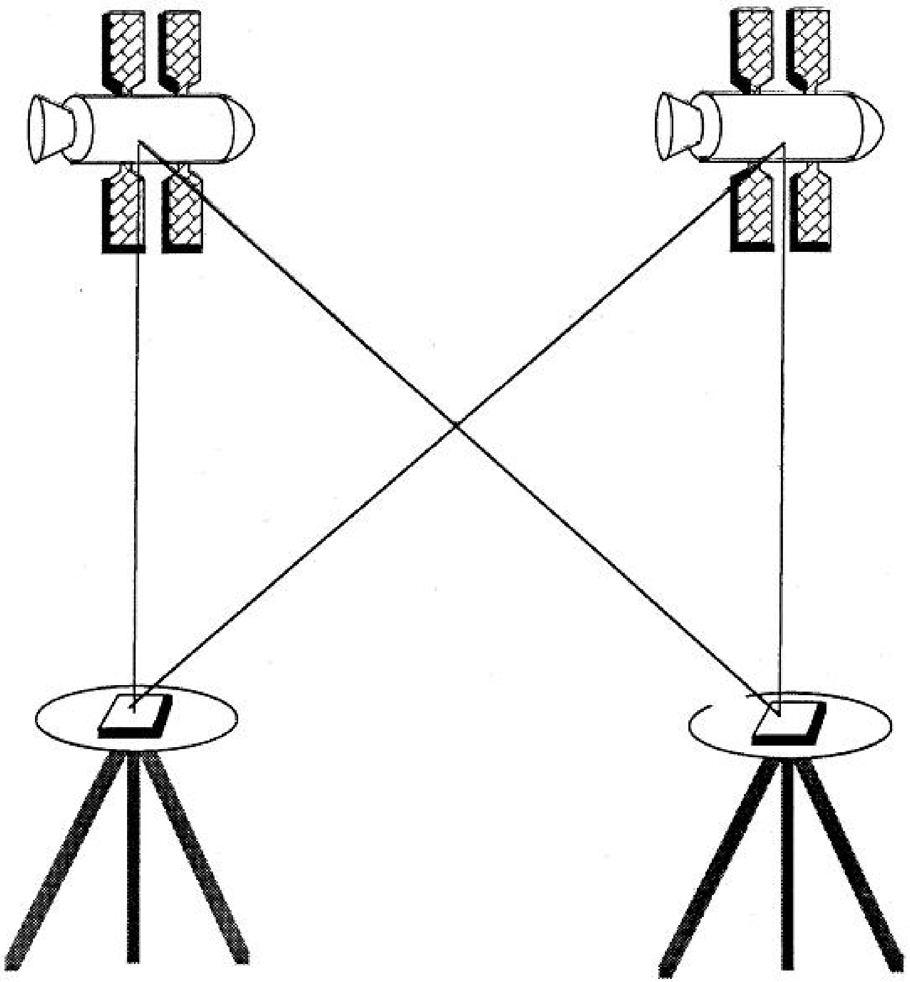
The phase offset $\Phi_{ij}(t_j)$ of this signal is :

$$\Phi_{ij}(t_j) = \Phi_{ij}^{received}(t_j) - \Phi_{ij}^{local}(t_j) + n_{ij} + \Phi noise$$

- t_j : time of signal reception at station j
- $\Phi_{ij}^{received}$: phase of signal received at station j coming from satellite i
- Φ_j^{local}: phase of receiver j oscillator
- $\Phi noise$: random noise of phase measurement
- n_{ij} : integer number (n-cycles) representing phase ambiguity

The precision of the phase measurement is affected by the quality of the electronic (Φnoise) but mostly by the the precision of the time tj.
To get a measurement precision of **3 cm**, we need a precision of **10^{-10} seconds**. Which is never achieved in standard receivers clocks…….

Double differences



One way phases are affected by **stations and satellites** clock uncertainties

Single differences are affected by

stations clock uncertainties

Or

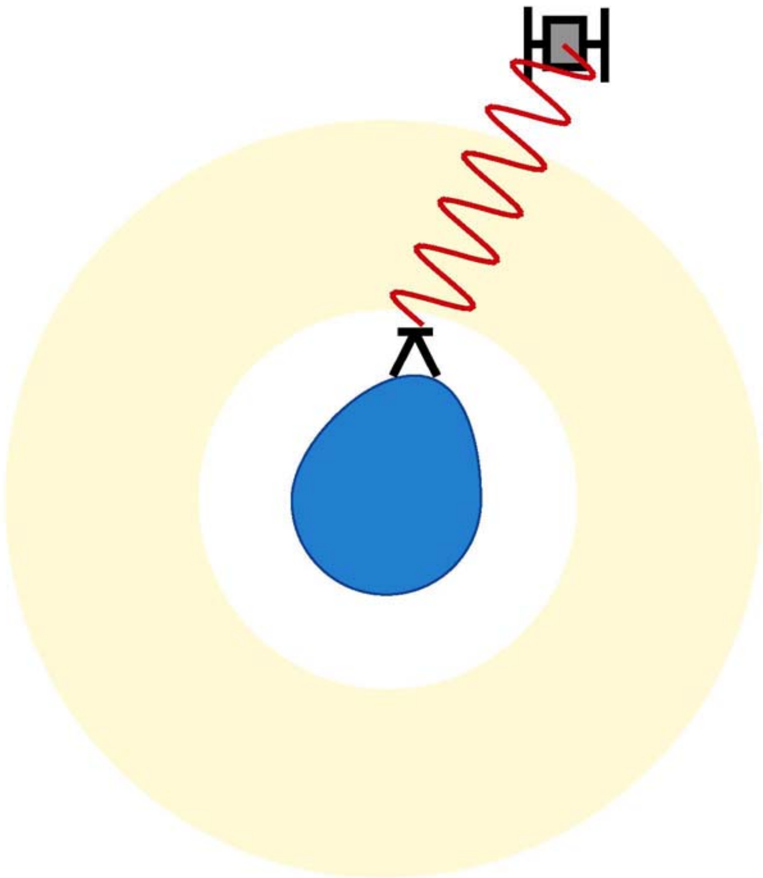
satellites clock uncertainties

Double differences Are free from all clock uncertainties **but**

=> Measurement of distances between points **(= baselines**)

=> **Relative positioning**

Other perturbation : The Ionosphere



Correct measurement

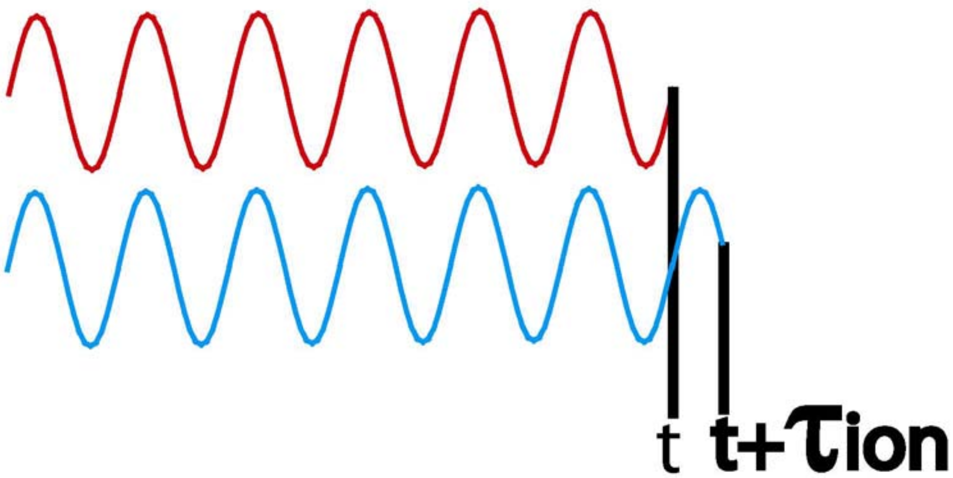
in an empty space

But the ionosphere perturbates propagation of electric wavelength …..

… and corrupts the measured distance

… and the inferred station position

Ionosphere theory



Ionospheric delay τion depends on :

- ionosphere contains in charged particules (ions and electrons) : **Ne**
- Frequency of the wave going through the ionosphere : f

$$\tau_{ion} = 1.35\ 10^{-7}\ Ne / f^2$$

Ionosphere : solution = dual frequency

Problem : **Ne** changes with time and is never known

solution : sample the ionosphere with 2 frequencies

$\tau ion_1 = 1.35\ 10^{-7}\ Ne / f_1^2$ $\tau ion_2 = 1.35\ 10^{-7}\ Ne / f_2^2$

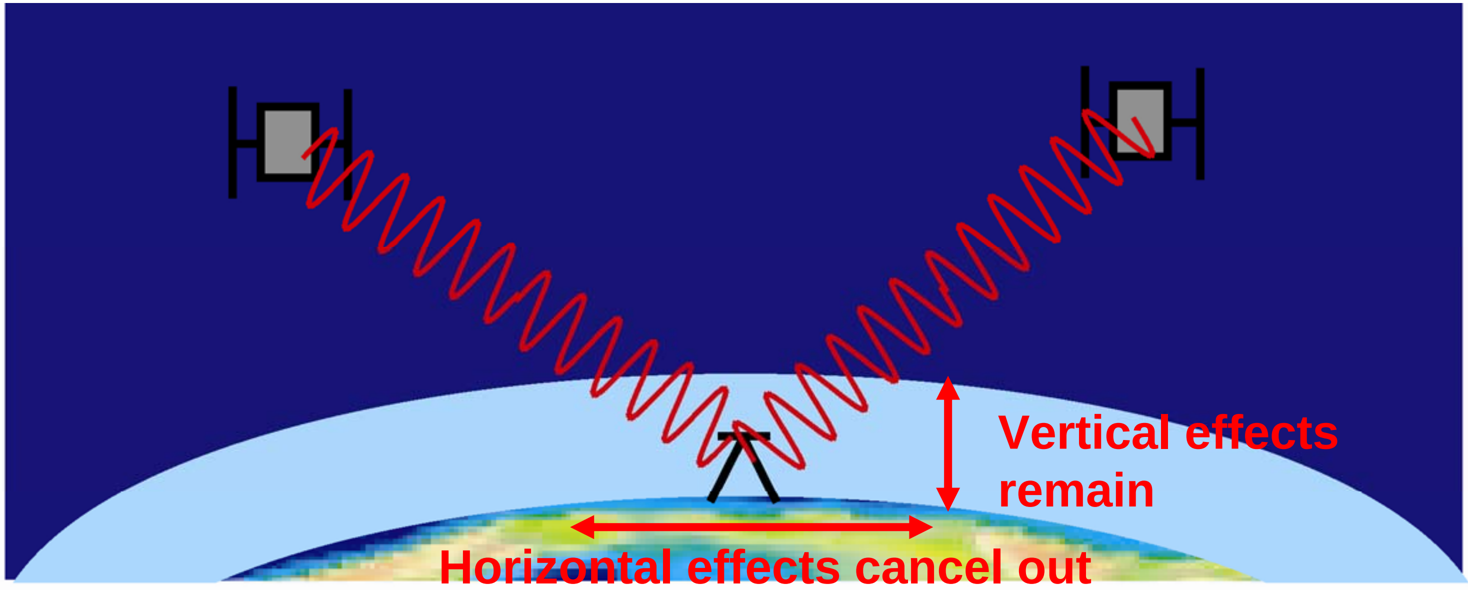
$\Rightarrow \tau ion_2 - \tau ion_1 = 1.35\ 10^{-7}\ Ne\ (1/ f_2^2 - 1/ f_1^2\ )$

$\Rightarrow Ne = \left[\tau ion_2 - \tau ion_1\right] / \ 1.35\ 10^{-7}\ (1/ f_2^2 - 1/ f_1^2\ )$

Using dual frequency GPS, allow to determine the number Ne and then to quantify the ionospheric delay on either L1 or L2.

(*in fact, GPS can and is used to make ionosphere Total Electron Containt (TEC) maps of the ionosphere*)

Second perturbation : The Troposphere

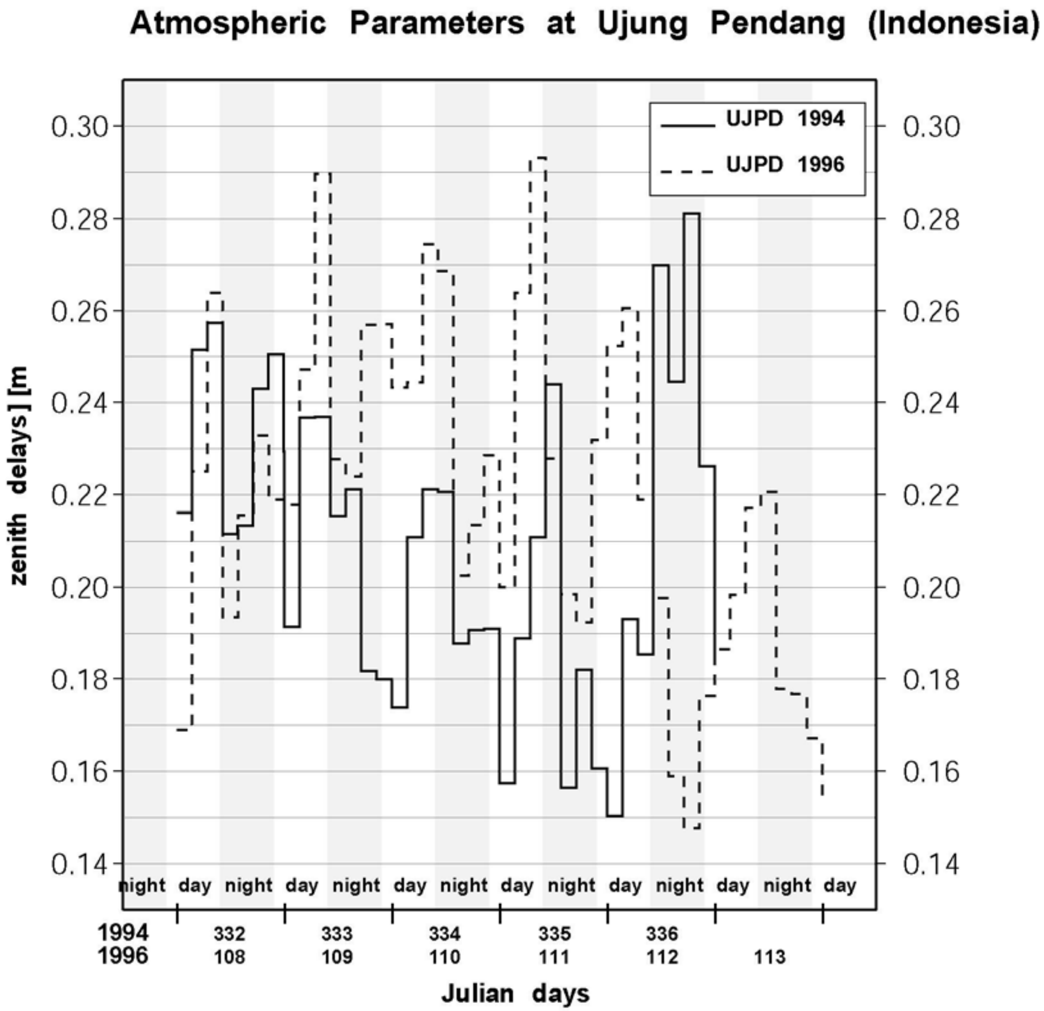


The troposphere (lower layer of the atmosphere) contains water. This also affects the travel time of radio waves.

But the troposphere is not dispersive (effect not inversely proportional to frequency), so the effect cannot be quantified by dual frequency system. Therefore there a position error of 1-50 cm.

Thanks to the presence of many satellites, the effect cancel out (more or less) in average, on the horizontal position. Only remains a vertical error called **Zenith tropospheric delay**

Troposphere zenith delay



The tropospheric zenith delay can be estimated from the data themselves…
if we measure every **30s** on **5** satellites, we have **1800** measurements in **3 hours.**
We only have **3** unknowns : station **lat**, **lon**, and **altitude** !

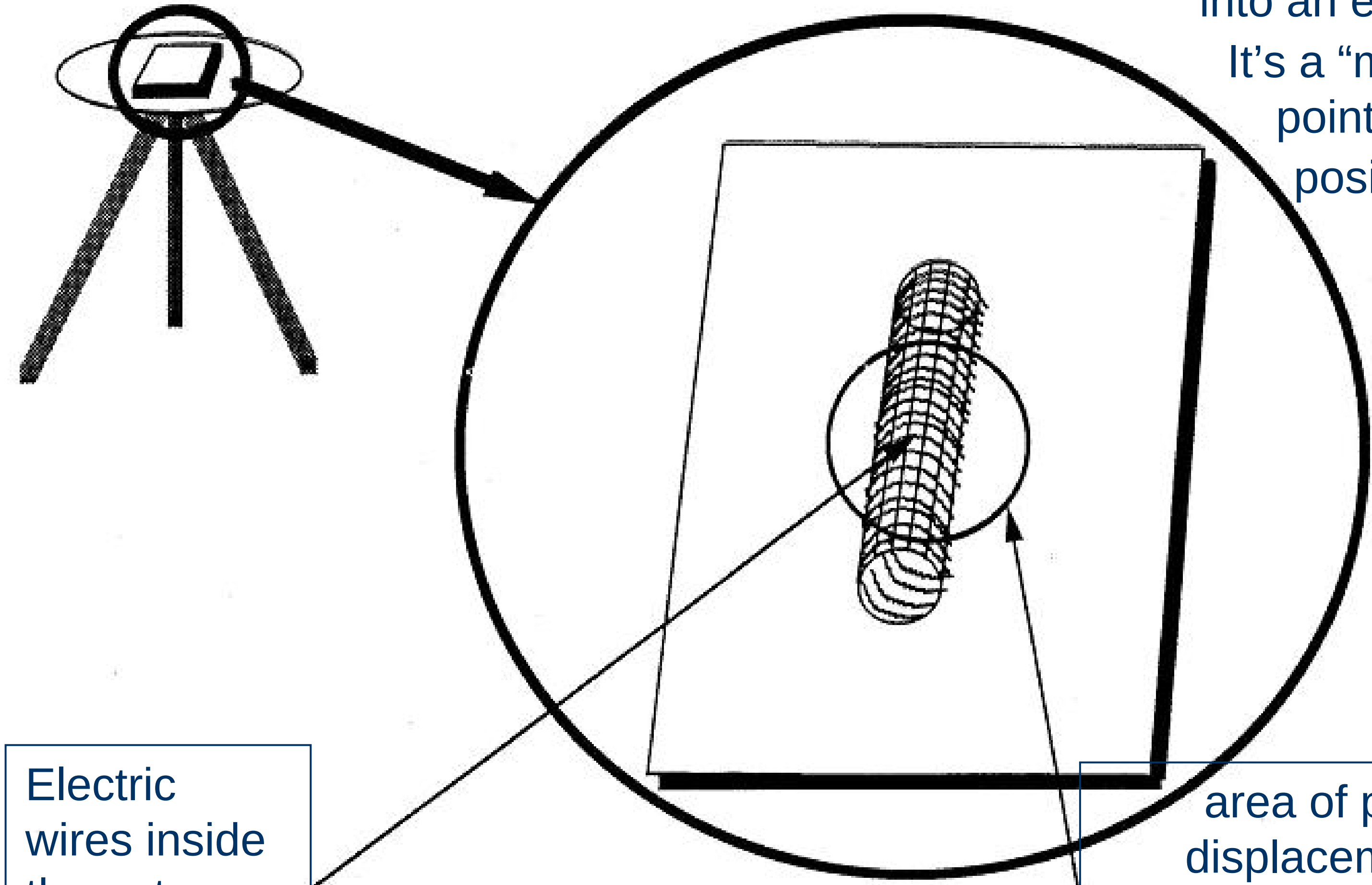
So we can add a new one :
1 Zenith delay every 3 hours

The curves show that the estimated Zenith delay vary from 15 cm to 30 cm with a very clear day/night cycle

Antenna phase center offset and variations

The Antenna phase center is the wire in which the radio wave converts into an electric signal.

It's a "mathematical" point, which exact position depends on the signal alignment with the wire (azimuth and elevation)

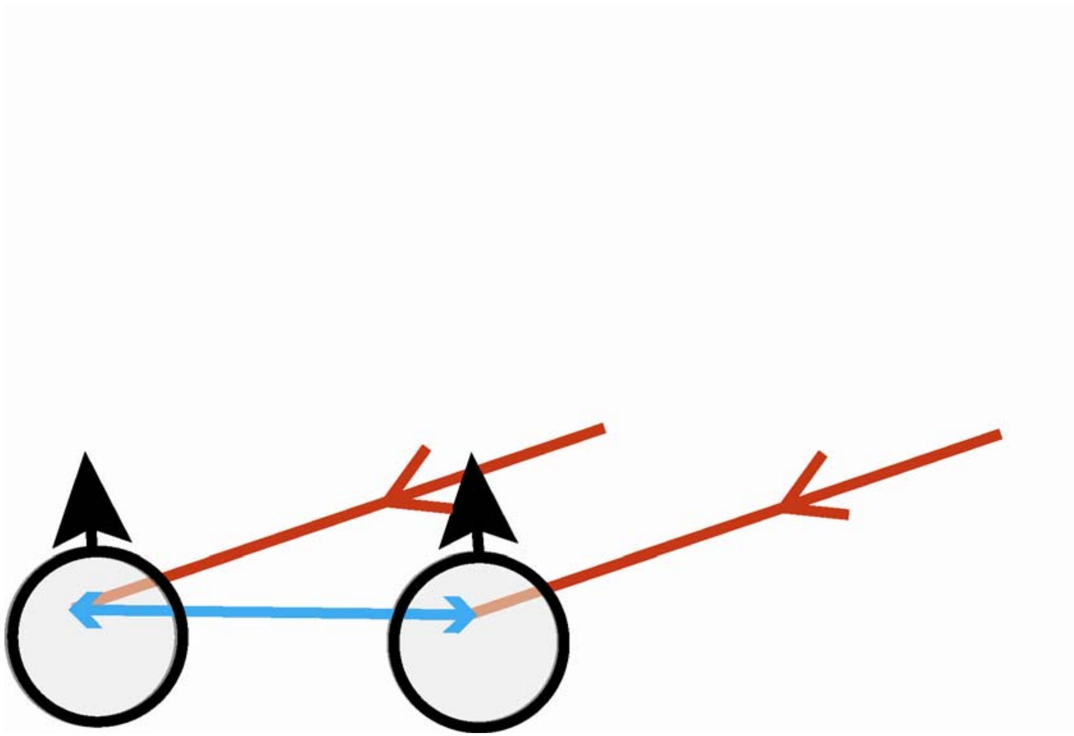


Electric wires inside the antenna

area of phase center displacement (~1 cm)

Antenna phase center offset and variations

Solution : use **identical** antennas, oriented in the **same direction**



As the signal rotates, the antenna phase centers move

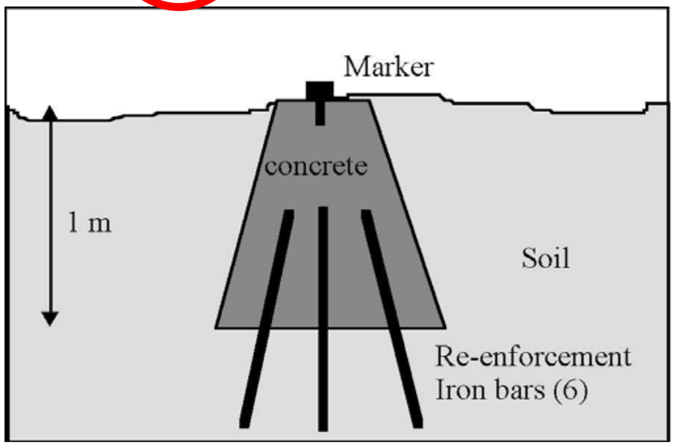
But they move the same quantity in the same direction if antennas are strictly identical because the incoming signal are the same (satellite is very far away)

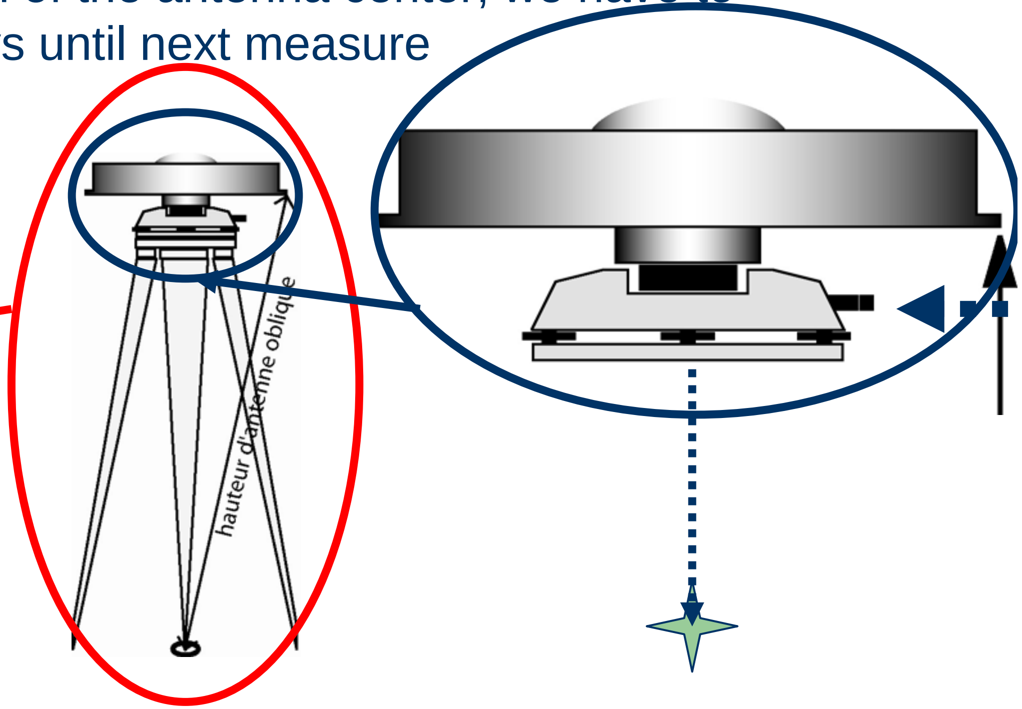
Therefore, the **baseline** between stations remains unchanged

But this works for small baselines only (less than a few 100 km)

Tripod and tribrachs source of errors

The measurement give the position of the antenna center, we have to **tie** it to the GPS marker which stays until next measure



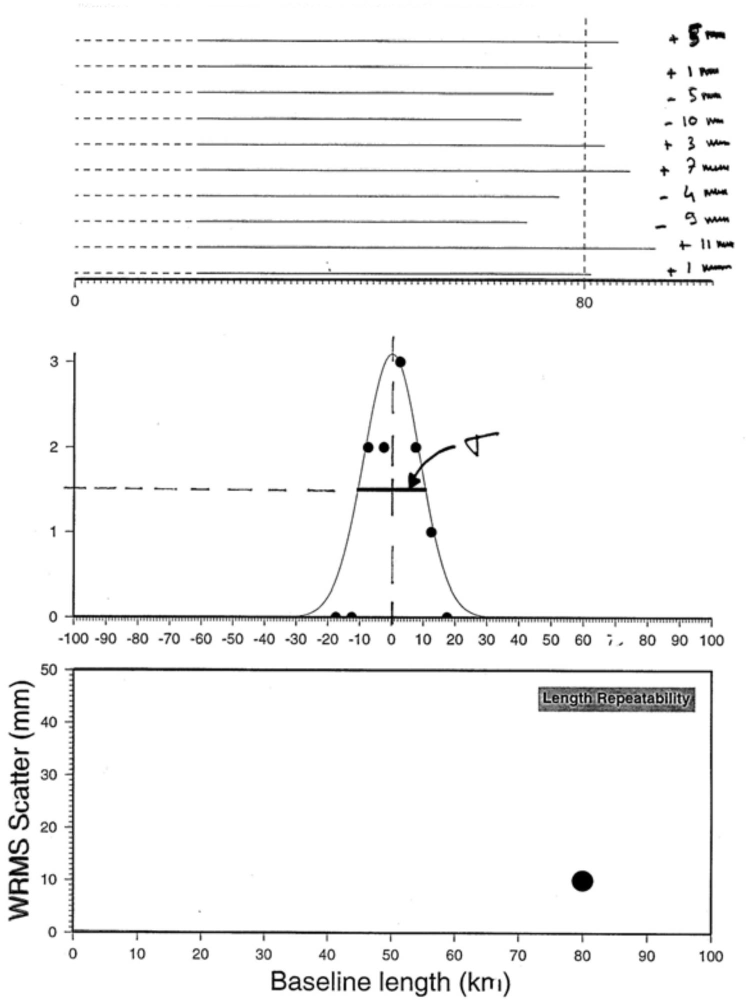


The antenna has to be leveled horizontally and centered perfectly on the mark. Then :

Horiz. position of marker = horiz. position of antenna

Altitude of marker = altitude of antenna – antenna height

Precision and repeatability



10 measurements of the same baseline give slightly different values :

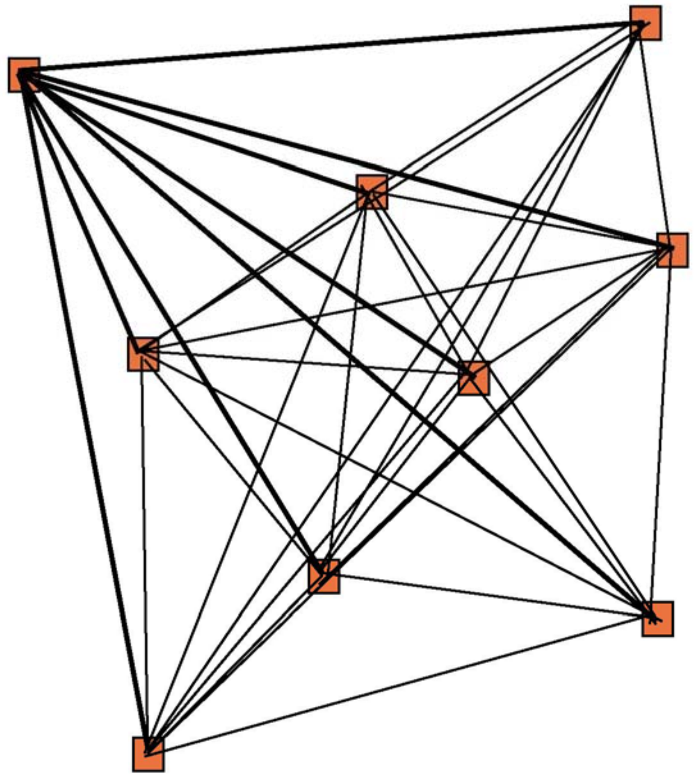
80 km +/- 10 mm

How many measurements are between 80 and 80+δ

The histogram curve is a Gaussian statistic

The baseline **repeatability** is the **sigma** of its Gaussian scatter

Network repeatabilities



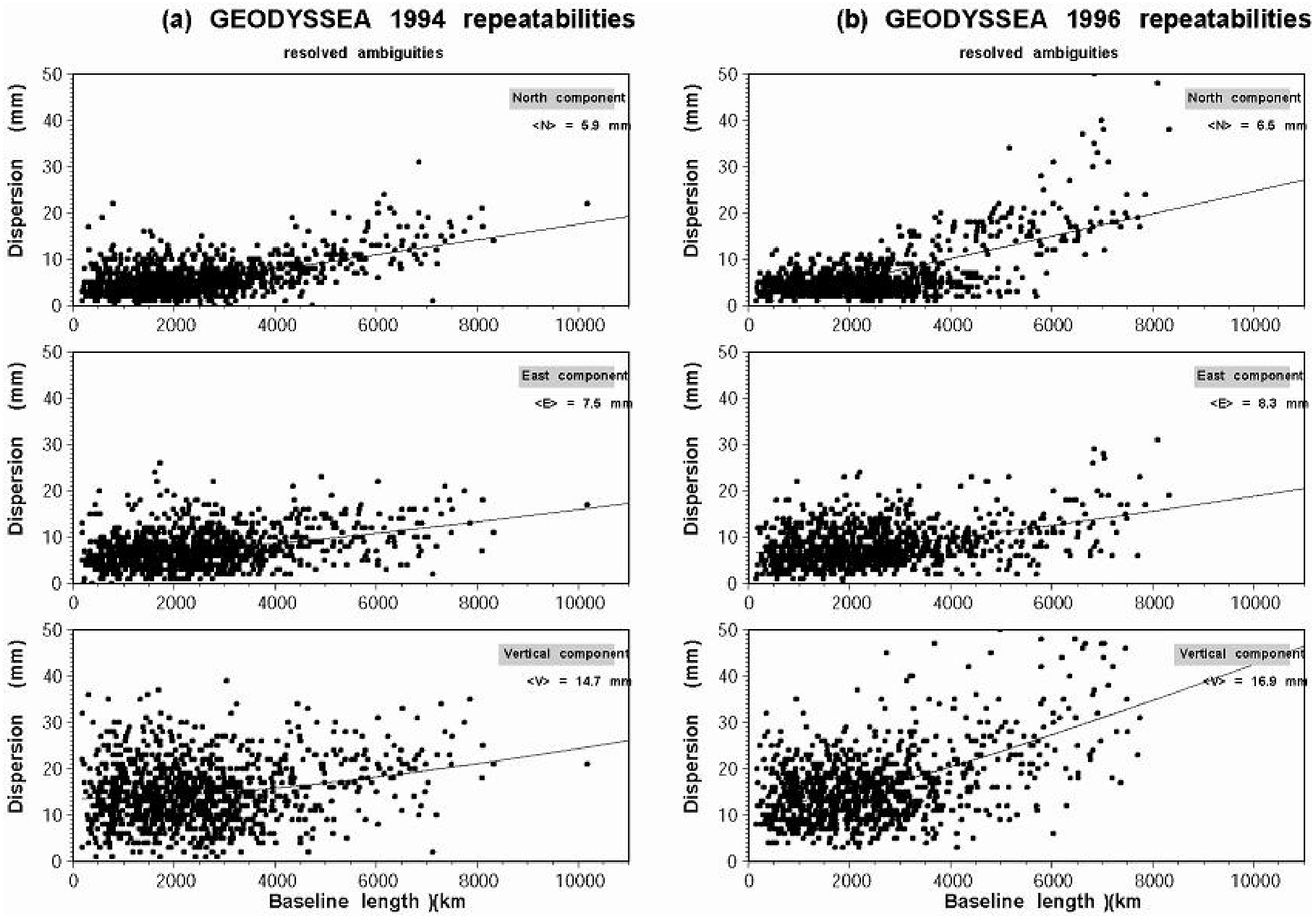
Network of N points
(N=9)

(N-1) (=8) baselines from
1st station to all others

(N-2) (=7) baselines from
2nd station to all others
=> subtotal = (N-1)+(N-2)

total number of baselines
= (N-1)+(N-2)+…+1
= N(N-1)/2 (*36 in that case*)

Typical repeatabilities (60 points => ~1800 bsl)



Repeatabilities are much larger than formal uncertainties !

From position to velocity uncertainty

If one measures position P_1 at time t_1 and P_2 at time t_2 with precision ΔP_1 and ΔP_2, what is the velocity V and its precision ΔV ?

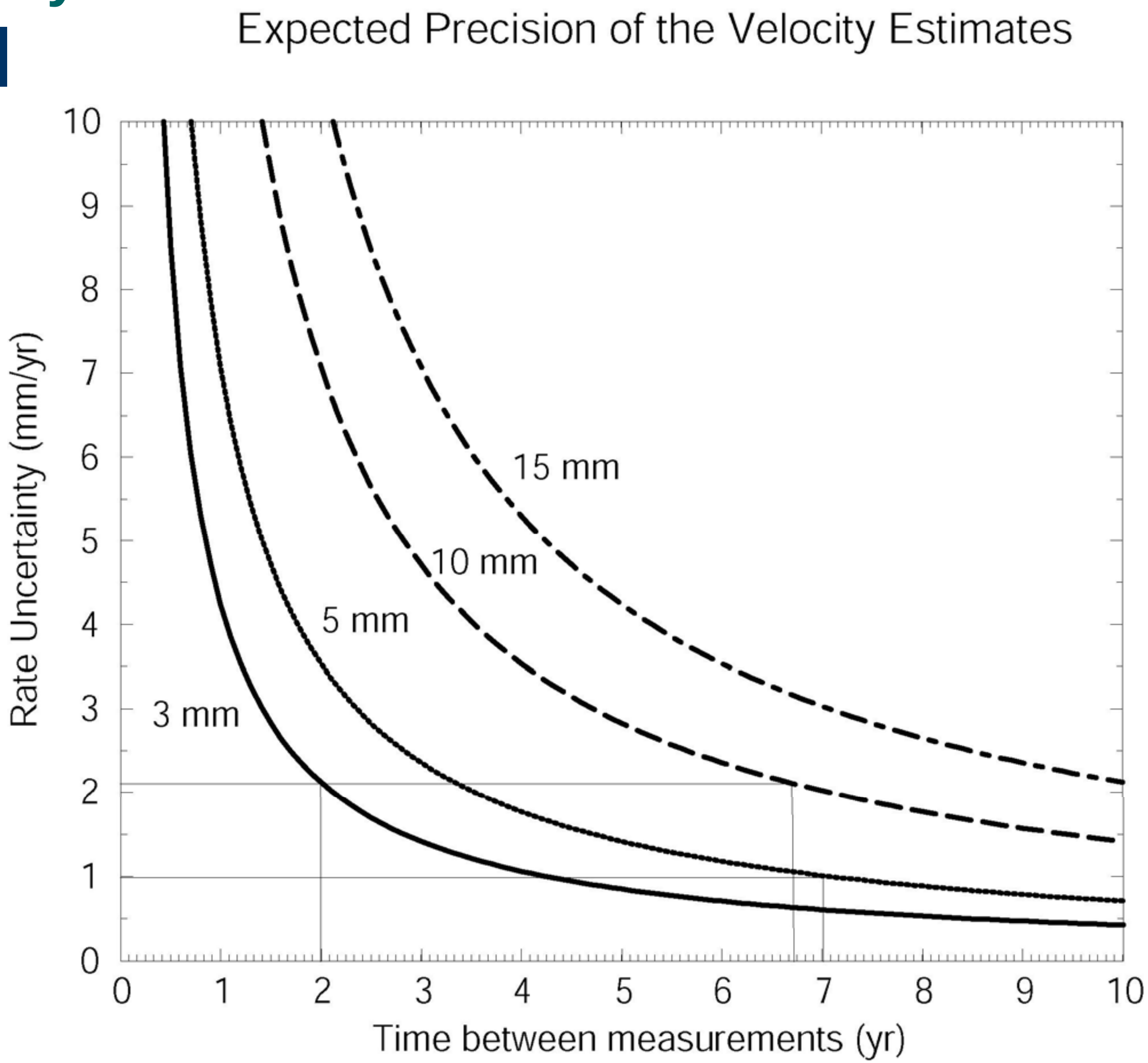
$$\boxed{V = (P_2 - P_1) \; / \; (t_2 - t_1)}$$

$$\boxed{\Delta V = (\Delta P_2 \cancel{+} \Delta P_1) \; / \; (t_2 - t_1)}$$

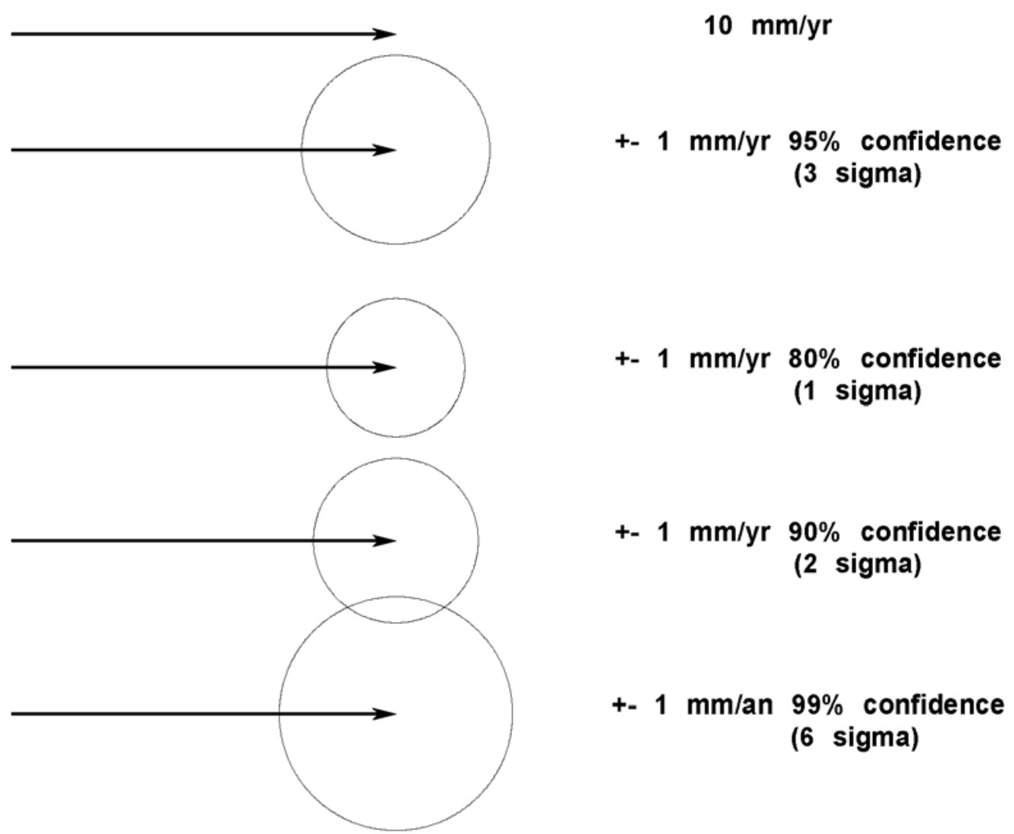
Uncertainties don't add up simply, because sigmas involve probability.

$$\boxed{\Delta V = \left[(\Delta P_2)^2 + (\Delta P_1)^2 \right]^{1/2} \Big/ (t_2 - t_1)}$$

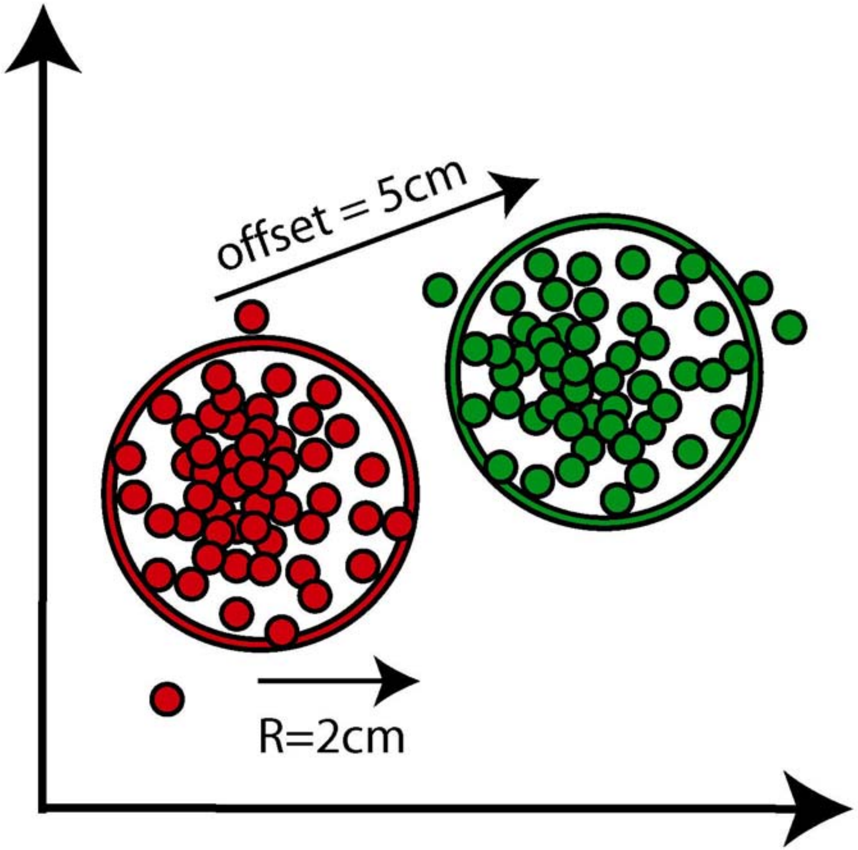
Velocity uncertainties



Velocity ellipses



Accuracy vs. precision (1)



Fix point :
measure 1 hour every 30 s

=> 120 positions

with dispersion ~+/- 2 cm

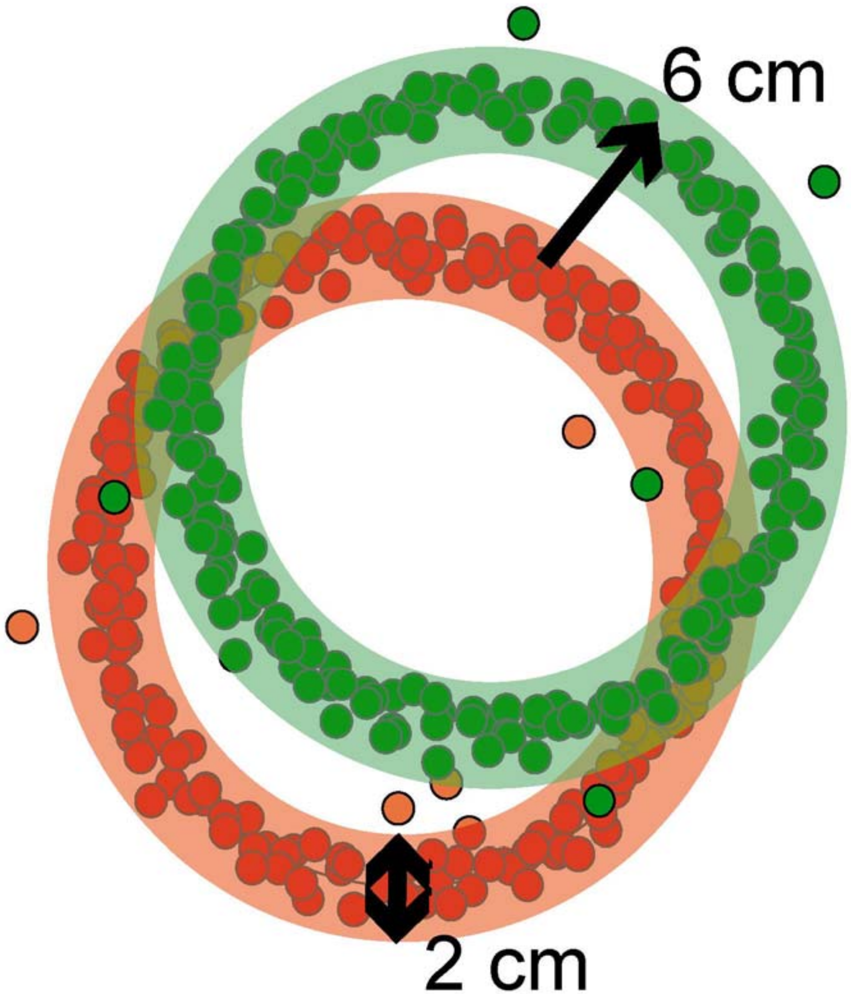
5 hours later, measure again
1 hour at the same location

=> Same dispersion but
constant offset of 5 cm

Precision = 2 cm

Accuracy = 5 cm

Accuracy vs. precision (2)



Measure path, 1 point every 10s

=> 1 circle with 50 points

10 circles describe runabout with dispersion ~ 2 cm

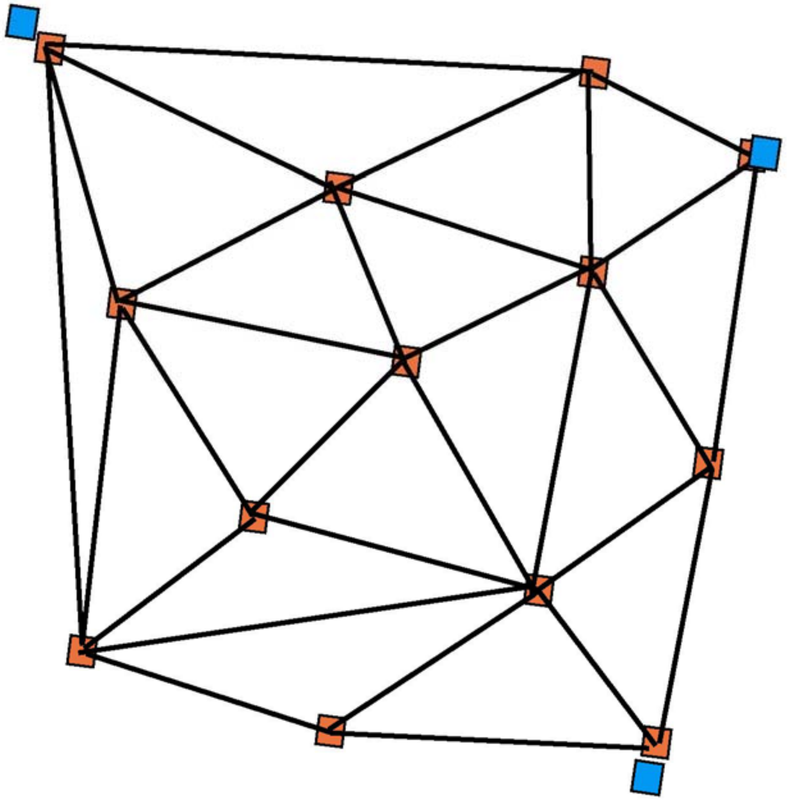
Next day, measure again

=> Same figure but constant offset of 6 cm

Precision = 2 cm

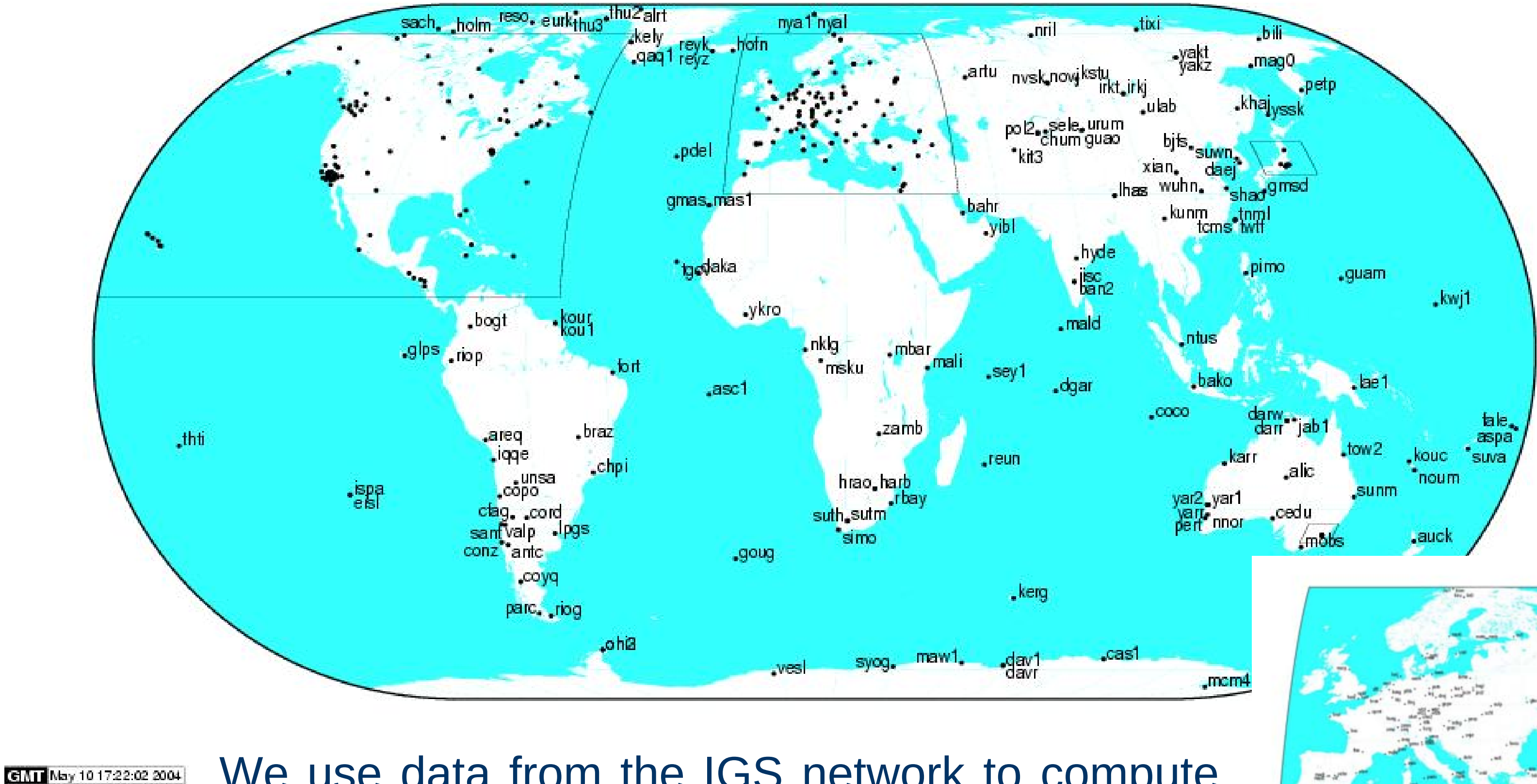
Accuracy = 6 cm

Mapping in a reference frame (sketch)



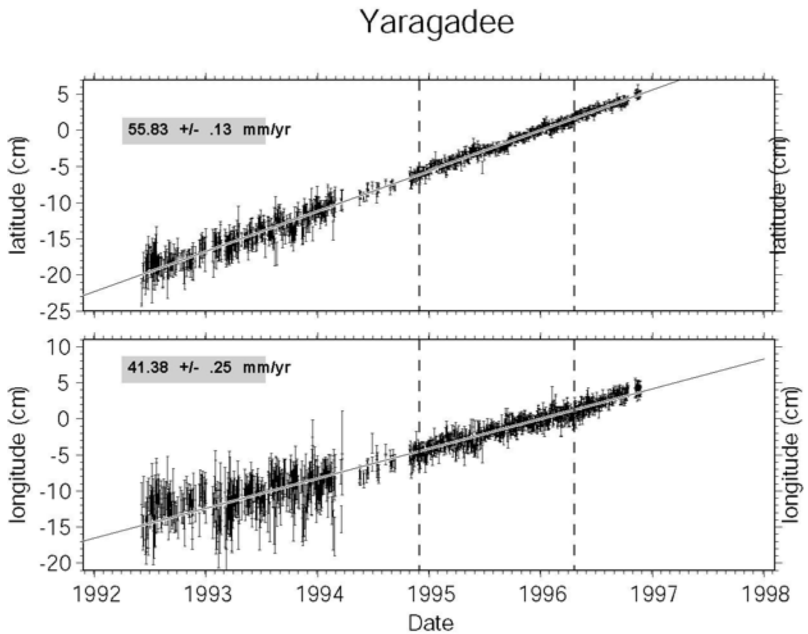
IGS network : 373 global stations

IGS Tracking Network : http://igscb.jpl.nasa.gov



We use data from the IGS network to compute baselines between local network and international reference frame

Mapping in a reference frame (1)

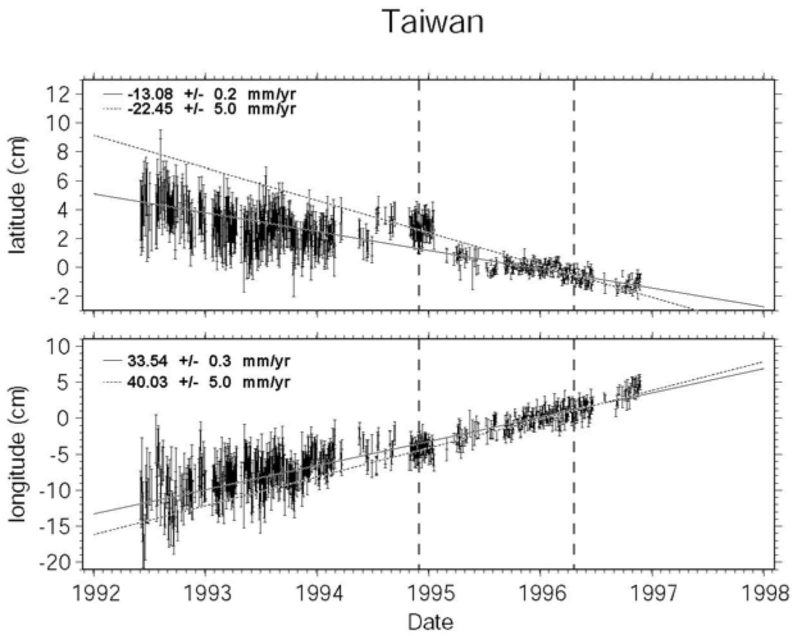


Constraining campaign positions (and or velocities) to long term positions (and or velocities) works fine …

… when station displacement is constant with time

if the station motion is **linear** with time, then estimating the velocity on any time span will give the same value

Mapping in a reference frame (2)

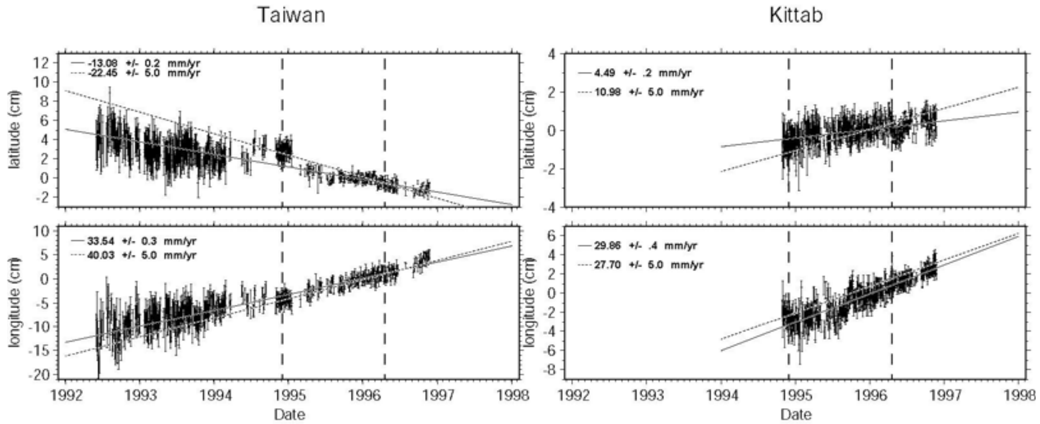


Constraining campaign positions (and or velocities) to long term positions (and or velocities) **does not work** …

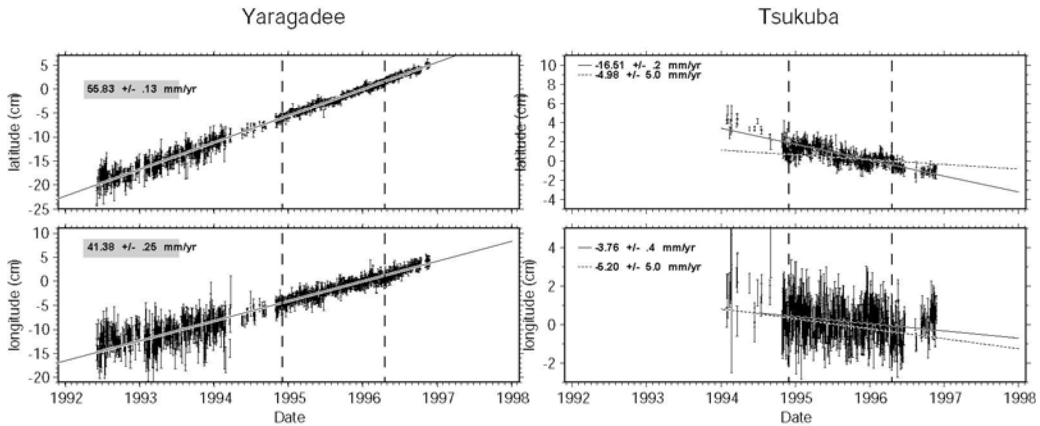
… when station displacement is **not** constant with time

if the station motion is **not linear** with time, then estimating the velocity on **different** time span will give **different** values

Mapping in a reference frame (3)

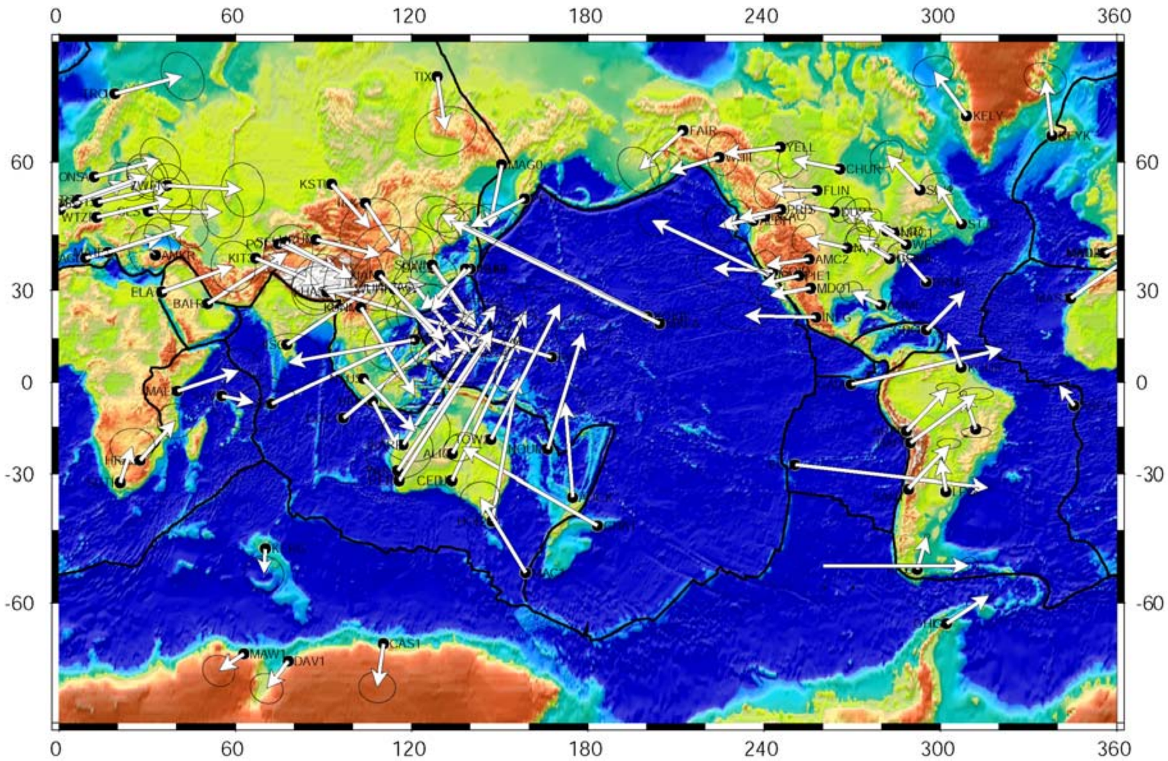


some stations are better than others …



The GPS reference system : ITRF2000

ITRF (International terrestrial reference frame) is a list of coordinates and velocities for IGS stations. Coordinates and velocities are updated when new data are added. The last realisation was made in 2000 so it is called ITRF2000



The reference of these motions is such that the sum of all velocities is zero, i.e. there is no net rotation of the Earth surface with respect of the rest of the planet.