Fundamentals of GPS
Basics of GPS uncertainties/precision/errors

1. what kind of noise(errors/uncertainties/unmodeled things affect the GPS precision : AS, SA, orbits, clocks, ionosphere, troposphere, antenna phase centers, centering (tribrachs), etc...

2. how do we evaluate GPS uncertainties ? (difference between formal and a posteriori) can we trust them ?

3. difference between precision and accuracy (internal consistency like repeatability is assertion of precision, comparison with other method affected by different biases is accuracy)

4. reference frames : how do we map ? with what precision ? What influence on results ?
Phase measurement precision

The precision of the phase measurement is affected by the quality of the electronic $\Phi$ noise) but mostly by the precision of the time $t_j$. To get a measurement precision of 3 cm, we need a precision of $10^{-10}$ seconds. Which is never achieved in standard receivers clocks…….

The phase offset $\Phi_{ij}(t_j)$ of this signal is:

$$\Phi_{ij}(t_j) = \Phi_{ij}^{\text{received}}(t_j) - \Phi_{ij}^{\text{local}}(t_j) + n_{ij} + \Phi_{\text{noise}}$$

<table>
<thead>
<tr>
<th>Unknown initial offset</th>
<th>Phase offset</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_j$</td>
<td>time of signal reception at station $j$</td>
</tr>
<tr>
<td>$\Phi_{ij}^{\text{received}}$</td>
<td>phase of signal received at station $j$ coming from satellite $i$</td>
</tr>
<tr>
<td>$\Phi_{ij}^{\text{local}}$</td>
<td>phase of receiver $j$ oscillator</td>
</tr>
<tr>
<td>$\Phi_{\text{noise}}$</td>
<td>random noise of phase measurement</td>
</tr>
<tr>
<td>$n_{ij}$</td>
<td>integer number (n-cycles) representing phase ambiguity</td>
</tr>
</tbody>
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Double differences

One way phases are affected by stations and satellites clock uncertainties

Single differences are affected by stations clock uncertainties
Or
satellites clock uncertainties

Double differences Are free from all clock uncertainties but
⇒ Measurement of distances between points (= baselines)
⇒ Relative positioning
Other perturbation: The Ionosphere

Correct measurement in an empty space

But the ionosphere perturbates propagation of electric wavelength …..

… and corrupts the measured distance

… and the inferred station position
Ionosphere theory

Ionospheric delay $\tau_{\text{ion}}$ depends on:

- ionosphere contains in charged particules (ions and electrons) : $N_e$
- Frequency of the wave going through the ionosphere : $f$

$$\tau_{\text{ion}} = 1.35 \times 10^{-7} \frac{N_e}{f^2}$$
Ionosphere : solution = dual frequency

Problem: \( N_e \) changes with time and is never known

solution: sample the ionosphere with 2 frequencies

\[
\tau_{\text{ion}1} = 1.35 \times 10^{-7} \frac{N_e}{f_1^2} \quad \tau_{\text{ion}2} = 1.35 \times 10^{-7} \frac{N_e}{f_2^2}
\]

\[
\Rightarrow \tau_{\text{ion}2} - \tau_{\text{ion}1} = 1.35 \times 10^{-7} N_e \left( \frac{1}{f_2^2} - \frac{1}{f_1^2} \right)
\]

\[
\Rightarrow N_e = \left[ \tau_{\text{ion}2} - \tau_{\text{ion}1} \right] / 1.35 \times 10^{-7} \left( \frac{1}{f_2^2} - \frac{1}{f_1^2} \right)
\]

Using dual frequency GPS, allow to determine the number \( N_e \) and then to quantify the ionospheric delay on either L1 or L2.

*(in fact, GPS can and is used to make ionosphere Total Electron Containt (TEC) maps of the ionosphere)*
The troposphere (lower layer of the atmosphere) contains water. This also affects the travel time of radio waves.

But the troposphere is not dispersive (effect not inversely proportional to frequency), so the effect cannot be quantified by dual frequency system. Therefore there a position error of 1-50 cm.

Thanks to the presence of many satellites, the effect cancel out (more or less) in average, on the horizontal position. Only remains a vertical error called **Zenith tropospheric delay**.
The tropospheric zenith delay can be estimated from the data themselves... if we measure every 30s on 5 satellites, we have 1800 measurements in 3 hours. We only have 3 unknowns: station lat, lon, and altitude!

So we can add a new one:

1 Zenith delay every 3 hours

The curves show that the estimated Zenith delay vary from 15 cm to 30 cm with a very clear day/night cycle.
Antenna phase center offset and variations

The Antenna phase center is the wire in which the radio wave converts into an electric signal. It’s a “mathematical” point, which exact position depends on the signal alignment with the wire (azimuth and elevation).
Antenna phase center offset and variations

Solution: use **identical** antennas, oriented in the **same direction**

As the signal rotates, the antenna phase centers move.

But they move the same quantity in the same direction if antennas are strictly identical because the incoming signal are the same (satellite is very far away).

Therefore, the **baseline** between stations remains unchanged.

But this works for small baselines only (less than a few 100 km).
Tripod and tribrachs source of errors

The measurement gives the position of the antenna center, we have to tie it to the GPS marker which stays until the next measure.

The antenna has to be leveled horizontally and centered perfectly on the mark. Then:

- Horiz. position of marker = horiz. position of antenna
- Altitude of marker = altitude of antenna – antenna height
10 measurements of the same baseline give slightly different values:

80 km +/- 10 mm

How many measurements are between 80 and 80 + δ?

The histogram curve is a Gaussian statistic.

The baseline **repeatability** is the **sigma** of its Gaussian scatter.
Network of N points (N=9)

(N-1) (=8) baselines from 1st station to all others
(N-2) (=7) baselines from 2nd station to all others

=> subtotal = (N-1)+(N-2)

total number of baselines = (N-1)+(N-2)+…+1
= N(N-1)/2 \( (36 \text{ in that case}) \)
Typical repeatabilities (60 points => ~1800 bsl)

Repeatabilities are much larger than formal uncertainties!
From position to velocity uncertainty

If one measures position $P_1$ at time $t_1$ and $P_2$ at time $t_2$ with precision $\Delta P_1$ and $\Delta P_2$, what is the velocity $V$ and its precision $\Delta V$?

$$V = \frac{(P_2 - P_1)}{(t_2 - t_1)}$$

$$\Delta V = \frac{(\Delta P_2 + \Delta P_1)}{(t_2 - t_1)}$$

Uncertainties don’t add up simply, because sigmas involve probability.

$$\Delta V = \left[ (\Delta P_2)^2 + (\Delta P_1)^2 \right]^{1/2} / (t_2 - t_1)$$
Velocity uncertainties

Expected Precision of the Velocity Estimates

![Graph showing velocity uncertainties over time between measurements.](image)
Velocity ellipses

10 mm/yr

+- 1 mm/yr 95% confidence (3 sigma)

+- 1 mm/yr 80% confidence (1 sigma)

+- 1 mm/yr 90% confidence (2 sigma)

+- 1 mm/an 99% confidence (6 sigma)
Accuracy vs. precision (1)

Fix point:
measure 1 hour every 30 s
=> 120 positions
with dispersion ~+/- 2 cm

5 hours later, measure again
1 hour at the same location
=> Same dispersion but constant offset of 5 cm

Precision = 2 cm
Accuracy = 5 cm
Measure path, 1 point every 10s

=> 1 circle with 50 points

10 circles describe runabout with dispersion ~ 2 cm

Next day, measure again

=> Same figure but constant offset of 6 cm

Precision = 2 cm
Accuracy = 6 cm
Mapping in a reference frame (sketch)
We use data from the IGS network to compute baselines between local network and international reference frame.
Mapping in a reference frame (1)

Constraining campaign positions (and or velocities) to long term positions (and or velocities) works fine … … when station displacement is constant with time

if the station motion is **linear** with time, then estimating the velocity on any time span will give the same value
Mapping in a reference frame (2)

Constraining campaign positions (and or velocities) to long term positions (and or velocities) does not work

... when station displacement is not constant with time

if the station motion is not linear with time, then estimating the velocity on different time span will give different values
Mapping in a reference frame (3)

some stations are better than others …
The GPS reference system: ITRF2000

ITRF (International terrestrial reference frame) is a list of coordinates and velocities for IGS stations. Coordinates and velocities are updated when new data are added. The last realisation was made in 2000 so it is called ITRF2000.

The reference of these motions is such that the sum of all velocities is zero, i.e. there is no net rotation of the Earth surface with respect to the rest of the planet.