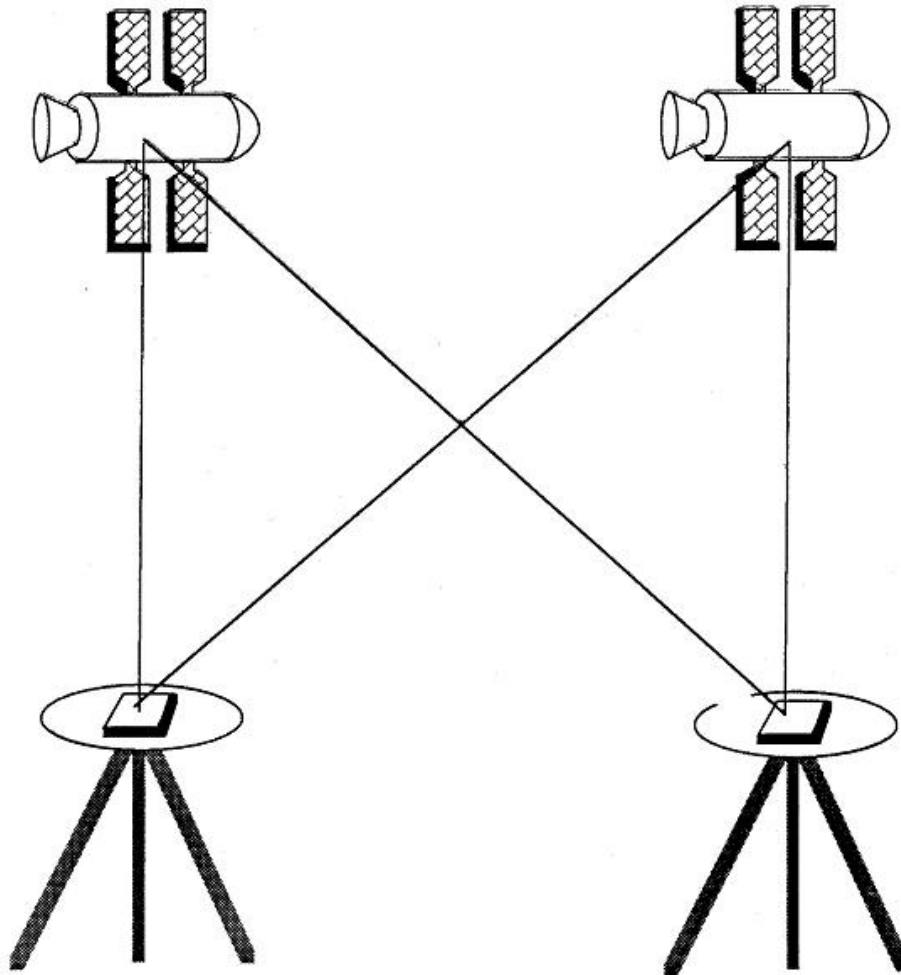


## GPS uncertainties

- Relative/ vs. absolute positioning
- Position precision limitations
- Velocity uncertainties
- Accuracy vs. Precision
- Mapping in a reference frame

## Double differences



Double differences

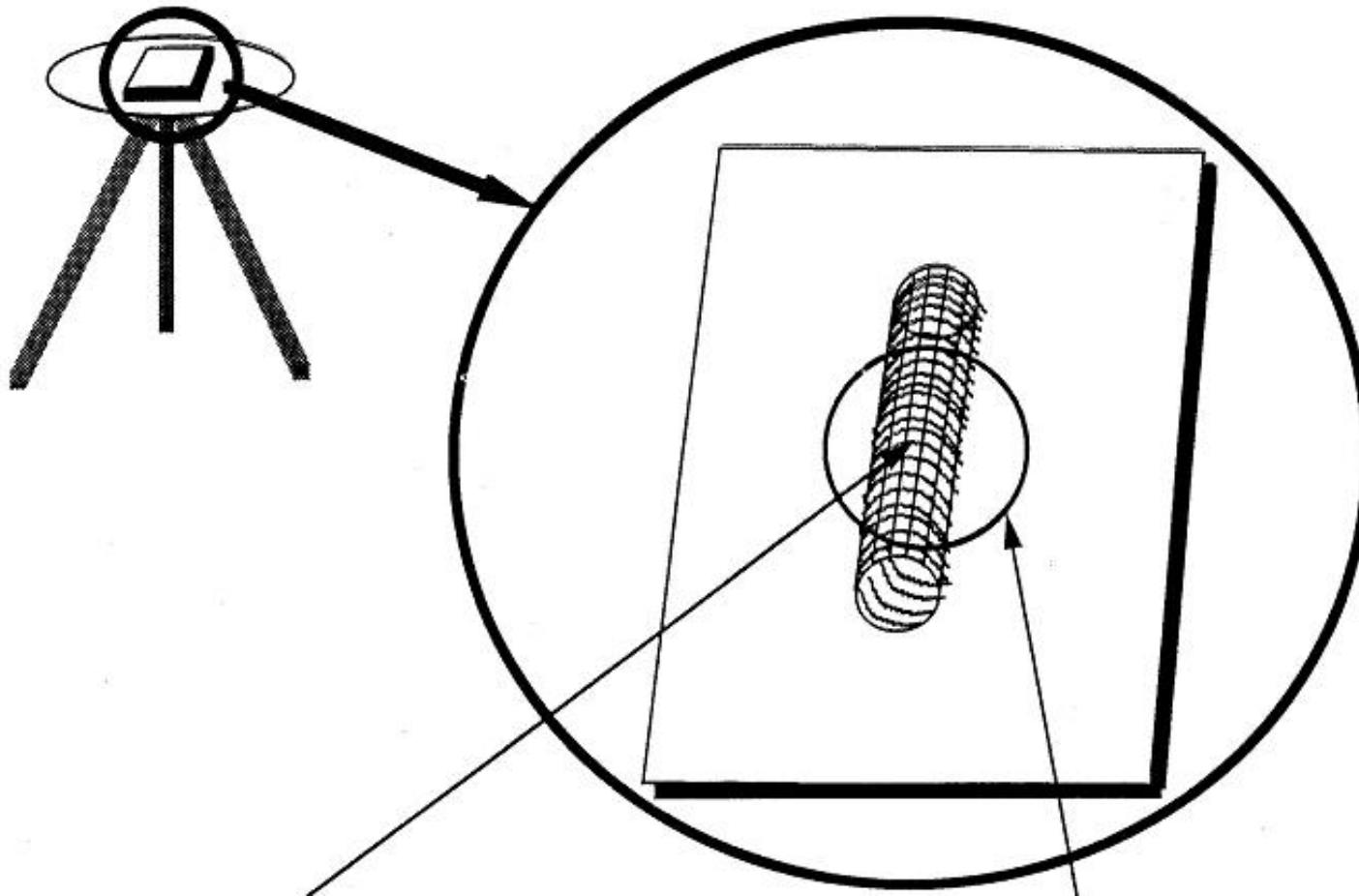
=>

Measurement of point  
distances = **baselines**

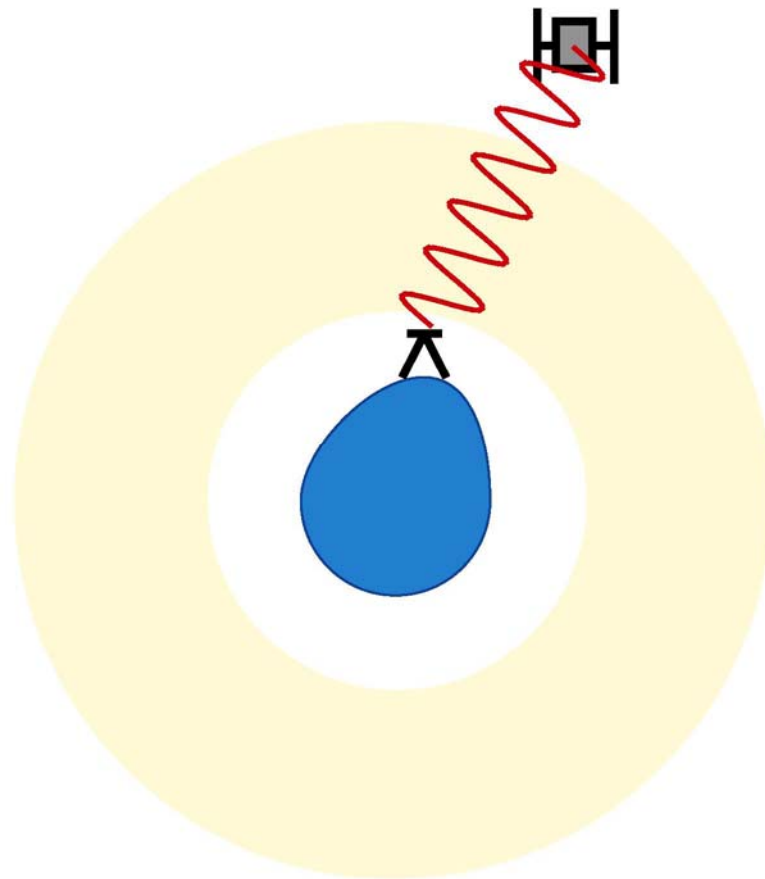
=>

Relative positioning

## Phase center offset and variations



## Ionosphere sketch



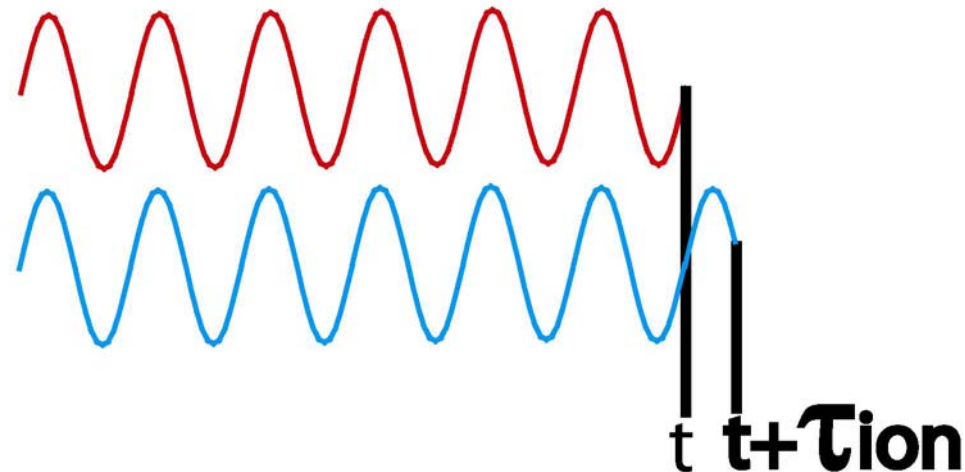
Correct measurement  
in an empty space

But the ionosphere  
perturbates  
propagation of electric  
wavelength .....

... and corrupts the  
measured distance

... and the inferred  
station position

## Ionosphere theory



Ionospheric delay  $\tau_{ion}$  depends on :

- ionosphere contains in charged particules (ions and electrons) :  $N_e$
- Frequency of the wave going through the ionosphere :  $f$

$$\tau_{ion} = 1.35 \cdot 10^{-7} N_e / f^2$$

## Ionosphere : solution = dual frequency

Problem : Ne changes with time and is never known

solution : sample the ionosphere with 2 frequencies

$$\tau_{ion_1} = 1.35 \cdot 10^{-7} \text{ Ne} / f_1^2$$

$$\tau_{ion_2} = 1.35 \cdot 10^{-7} \text{ Ne} / f_2^2$$

$$\tau_{ion_2} - \tau_{ion_1} = 1.35 \cdot 10^{-7} \text{ Ne} (1/f_2^2 - 1/f_1^2)$$

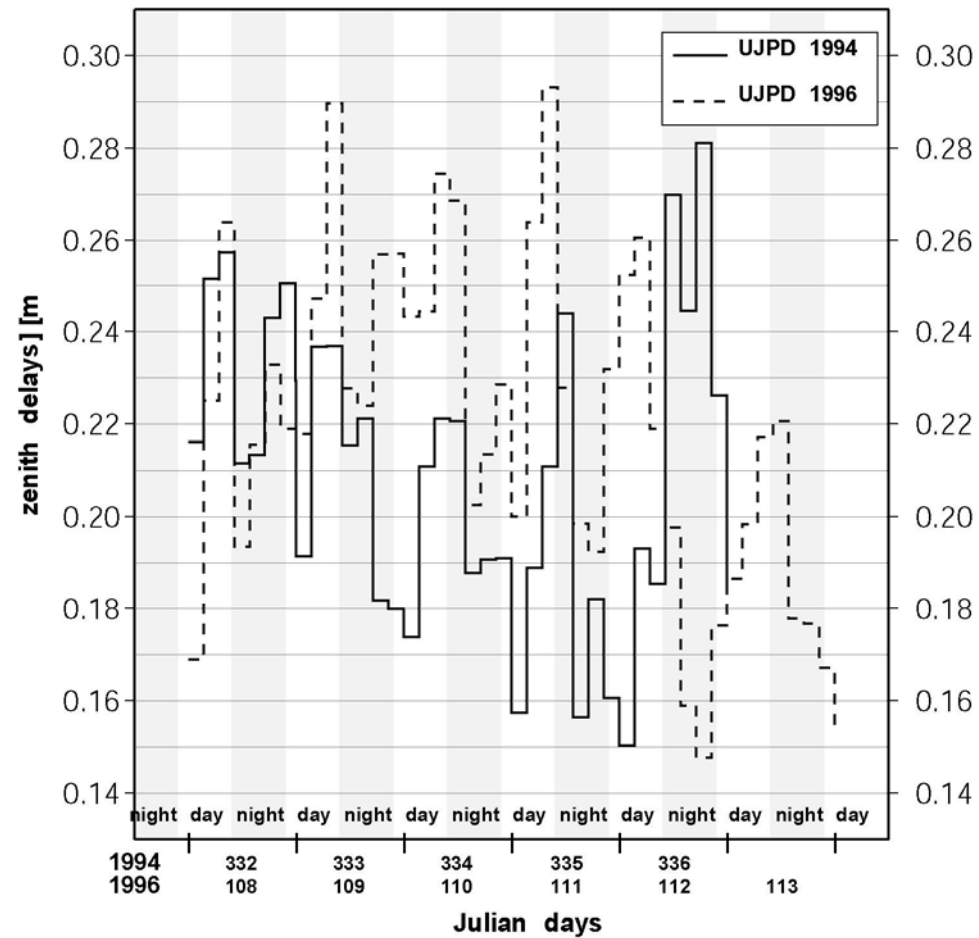
$$\text{Ne} = \left[ \tau_{ion_2} - \tau_{ion_1} \right] / 1.35 \cdot 10^{-7} (1/f_2^2 - 1/f_1^2)$$

Dual frequency GPS to quantify ionospheric delay

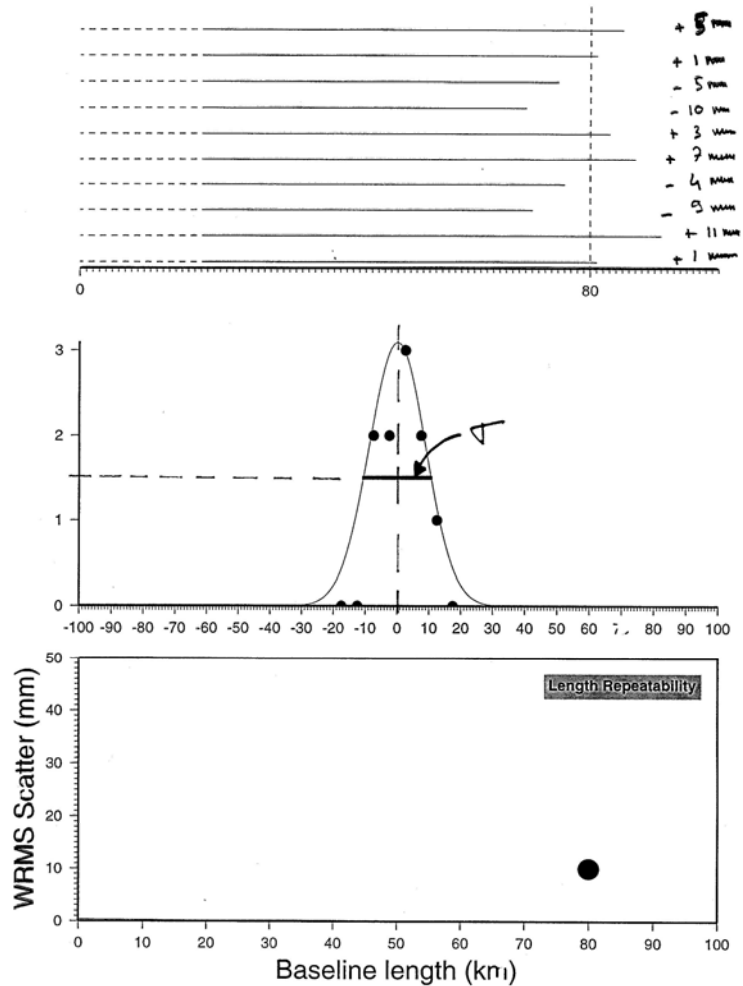
Make ionosphere TEC maps with GPS

# Troposphere

Atmospheric Parameters at Ujung Pendang (Indonesia)

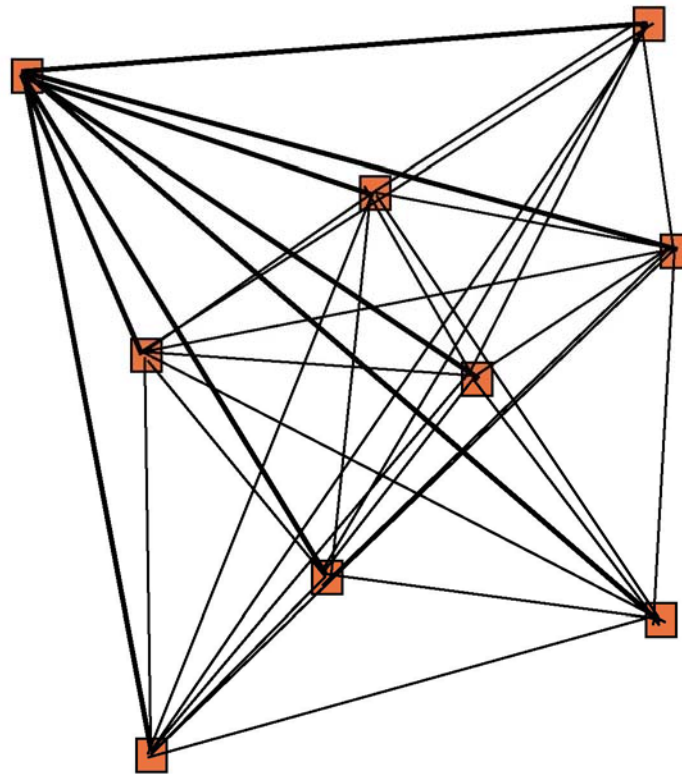


# Precision and repeatability





## Network repeatabilities



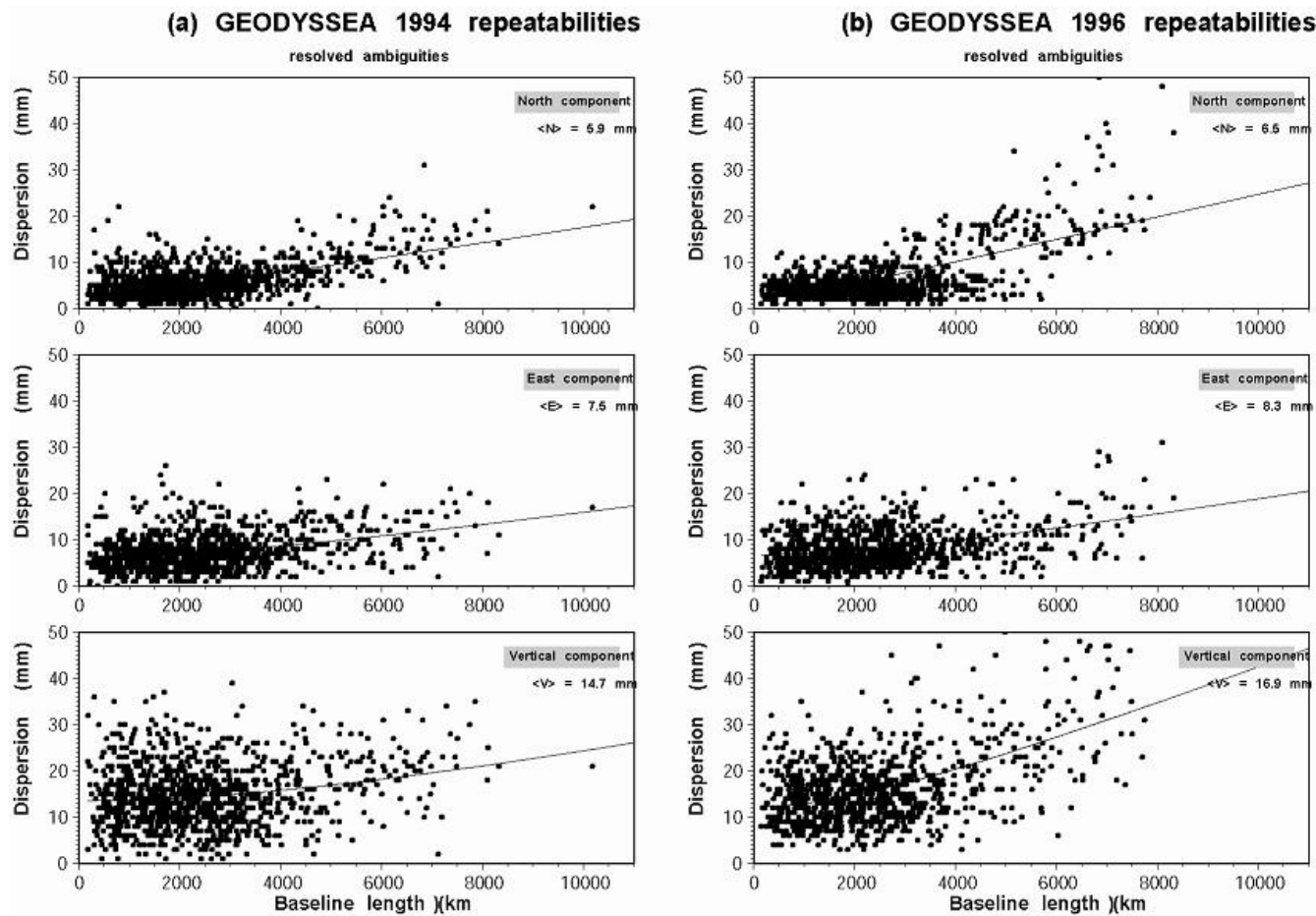
Network of N points  
(N=9)

(N-1) (=8) baselines from  
1st station to all others

(N-2) (=7) baselines from  
2nd station to all others  
=> subtotal = (N-1)+(N-2)

total number of baselines  
= (N-1)+(N-2)+...+1  
=  $N(N-1)/2$  (36 in that case)

## Typical repeatabilities (60 points => ~1800 bsl)



Repeatabilities are much larger than formal uncertainties !

## From position to velocity uncertainty

If one measures position  $P_1$  at time  $t_1$  and  $P_2$  at time  $t_2$  with precision  $\Delta P_1$  and  $\Delta P_2$ , what is the velocity  $V$  and its precision  $\Delta V$  ?

$$V = (P_2 - P_1) / (t_2 - t_1)$$

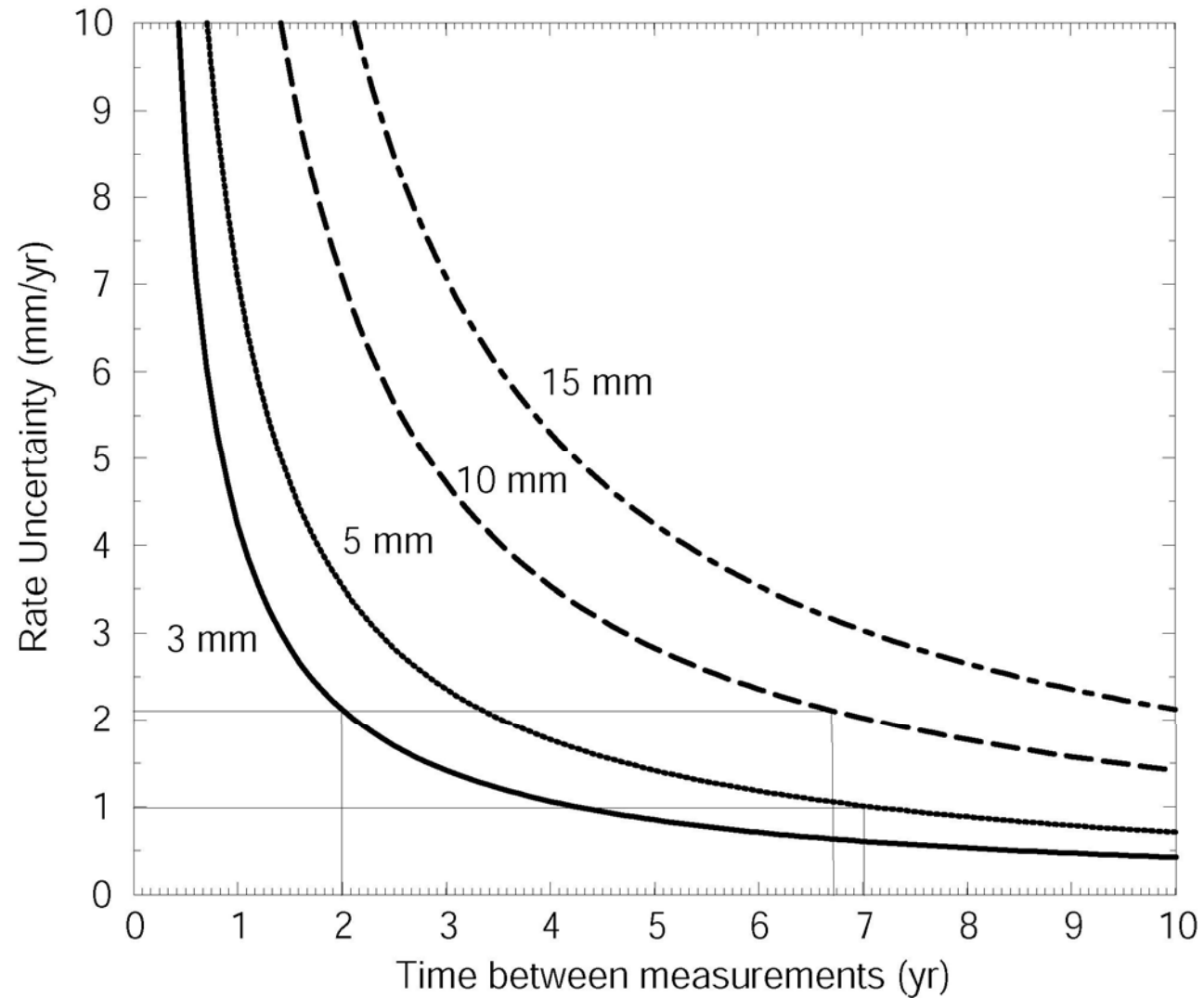
$$\Delta V = (\Delta P_2 + \Delta P_1) / (t_2 - t_1)$$

Uncertainties don't add up simply, because sigmas involve probability.

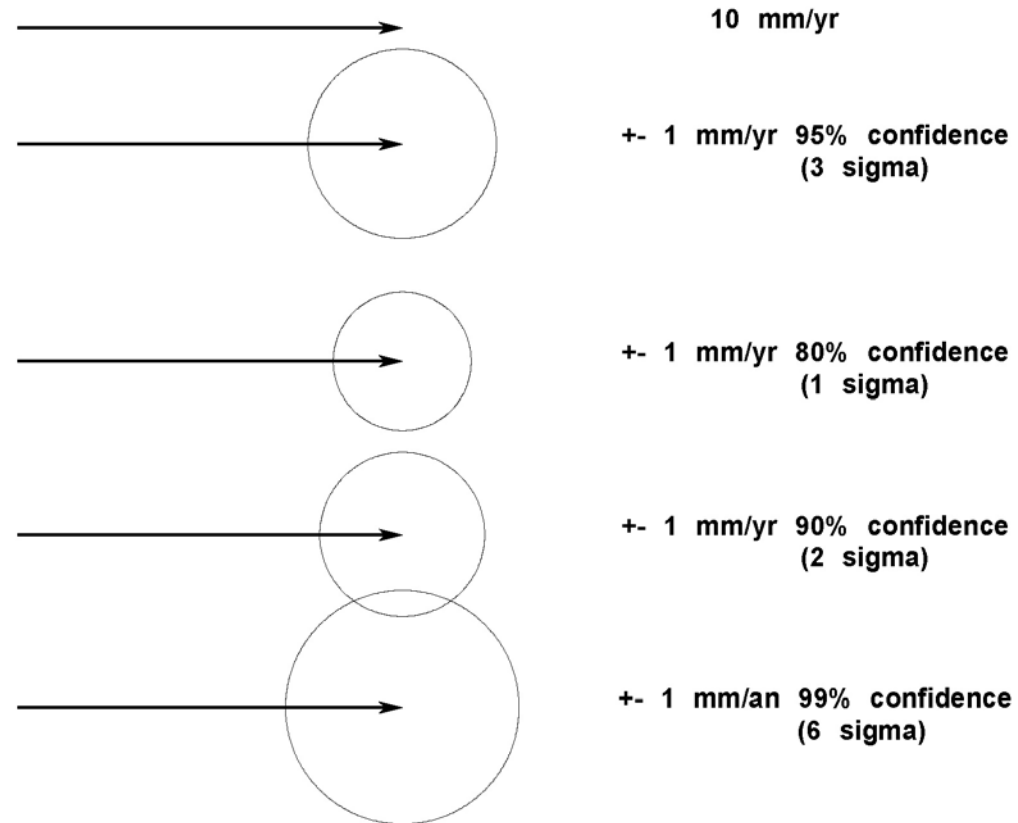
$$\Delta V = [ (\Delta P_2)^2 + (\Delta P_1)^2 ]^{1/2} / (t_2 - t_1)$$

# Velocities uncertainties

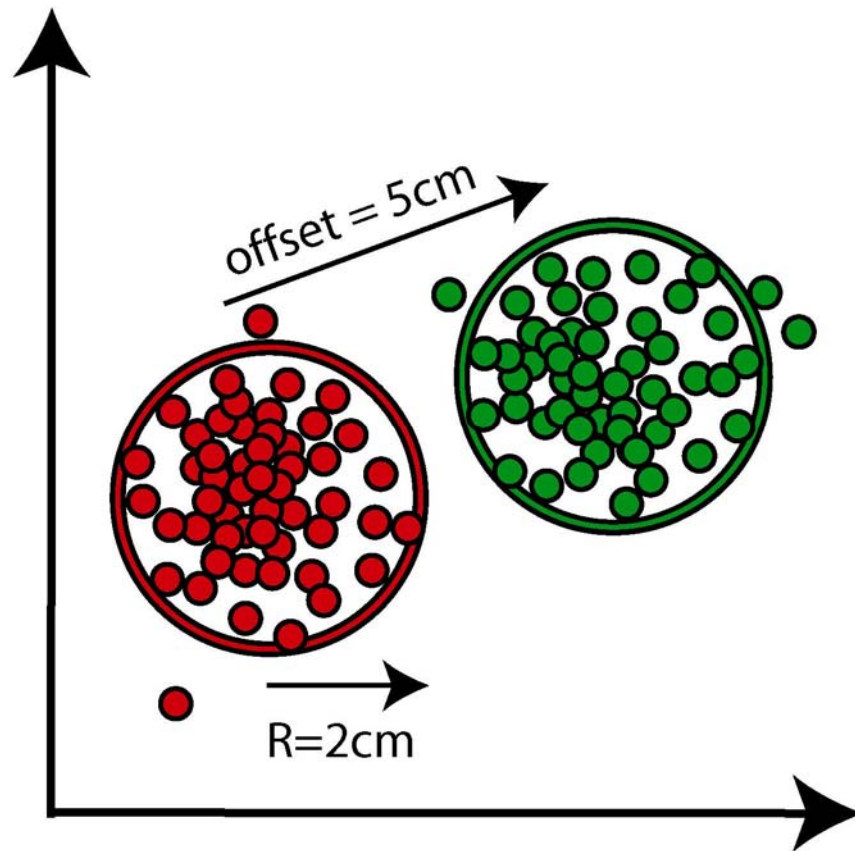
Expected Precision of the Velocity Estimates



## Velocities ellipses



## Accuracy vs. precision (1)



Fix point :  
measure 1 hour every 30 s

=> 120 positions

with dispersion  $\sim \pm 2\text{ cm}$

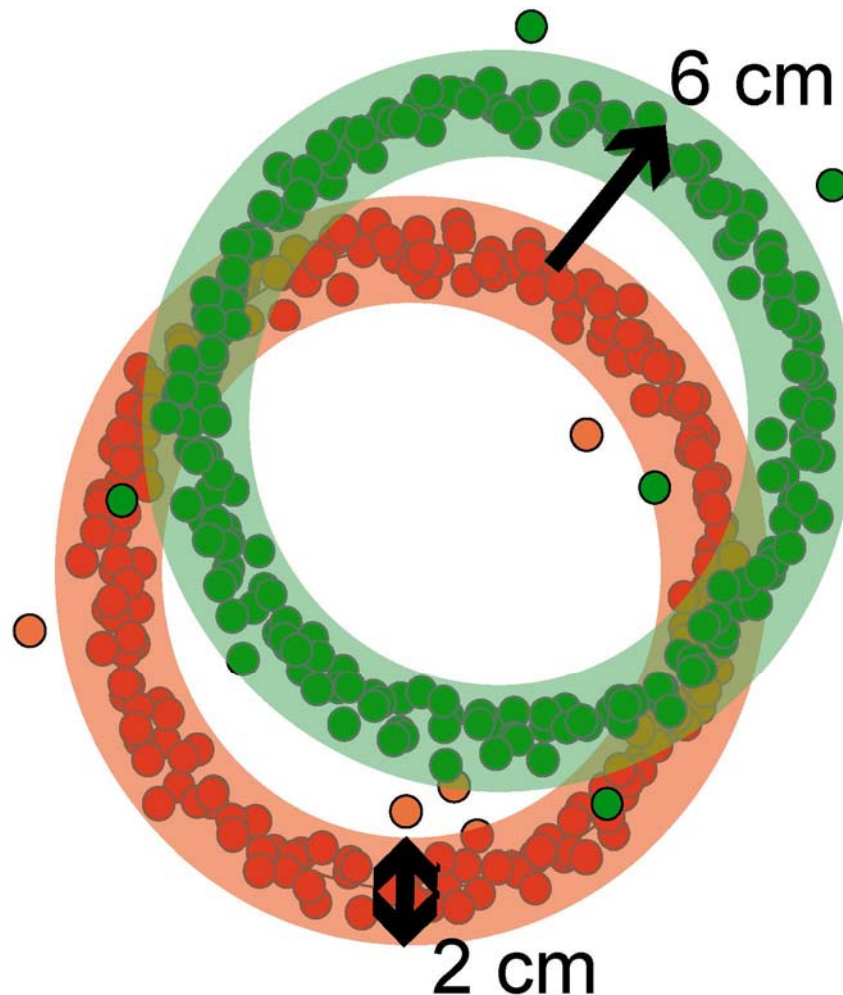
5 hours later, measure again  
1 hour at the same location

=> Same dispersion but  
constant offset of 5 cm

Precision = 2 cm

Accuracy = 5 cm

## Accuracy vs. precision (2)



Measure path, 1 point every 10s

=> 1 circle with 50 points

10 circles describe runabout with dispersion ~ 2 cm

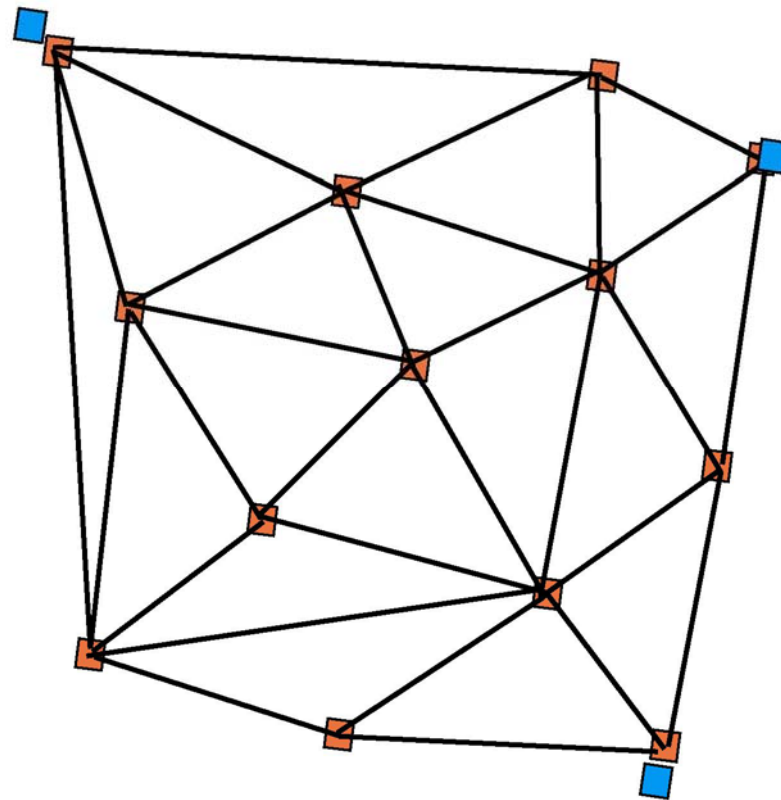
Next day, measure again

=> Same figure but constant offset of 6 cm

Precision = 2 cm

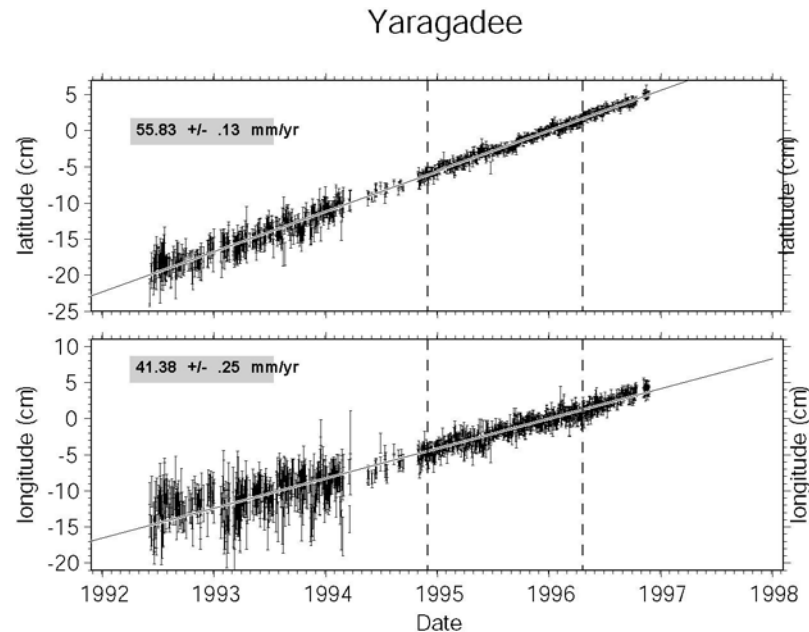
Accuracy = 6 cm

## Mapping in a reference frame (sketch)





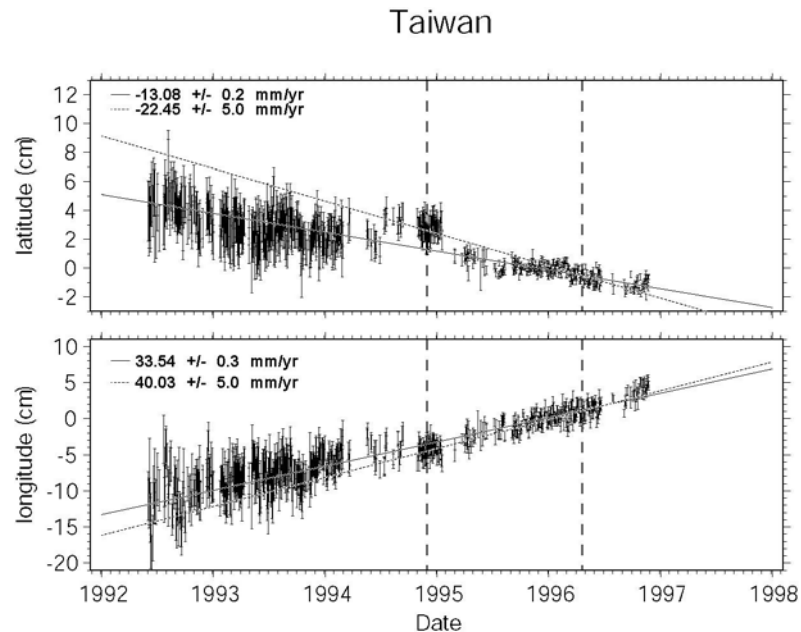
## Mapping in a reference frame (1)



Constraining campaign positions (and or velocities) to long term positions (and or velocities) works fine ...

... when station displacement is constant with time

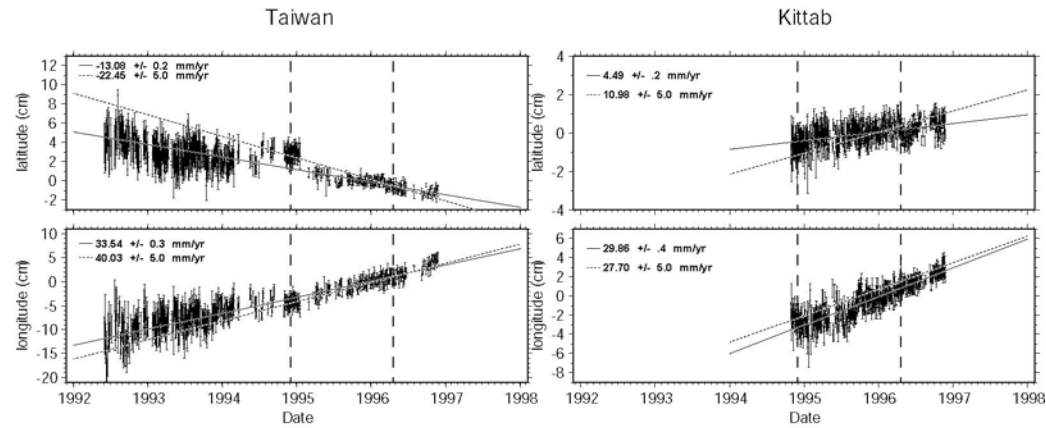
## Mapping in a reference frame (2)



Constraining campaign positions (and or velocities) to long term positions (and or velocities) **does not work**

...  
... when station displacement is **not** constant with time

## Mapping in a reference frame (3)



some stations are better than others ...

